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Method for prediction magnetic silencing of uncertain energy-saturated extended technical objects in prolate spheroidal coordinate system

Aim. Development of method for prediction by energy-saturated extended technical objects magnetic silencing based on magnetostatics geometric inverse problems solution and magnetic field spatial spheroidal harmonics calculated in prolate spheroidal coordinate system taking into account of technical objects magnetic characteristics uncertainties. **Methodology.** Spatial prolate spheroidal harmonics of extended technical objects magnetic field model calculated as magnetostatics geometric inverse problems solution in the form of nonlinear minimax optimization problem based on near field measurements for prediction far extended technical objects magnetic field magnitude. Nonlinear objective function calculated as the weighted sum of squared residuals between the measured and predicted magnetic field COMSOL Multiphysics software package used. Nonlinear minimax optimization problems solutions calculated based on particle swarm nonlinear optimization algorithms. **Results.** Results of prediction extended technical objects far magnetic field magnitude based on extended technical objects spatial prolate spheroidal harmonics of the magnetic field model in the prolate spheroidal coordinate system using near field measurements with consideration of extended technical objects magnetic characteristics uncertainty. **Originality.** The method for prediction by extended technical objects magnetic cleanliness based on spatial prolate spheroidal harmonics of the magnetic field model in the prolate spheroidal coordinate system with consideration of magnetic characteristics uncertainty is developed. **Practical value.** The important practical problem of prediction extended technical objects magnetic silencing based on the spatial prolate spheroidal harmonics of the magnetic field model in the prolate spheroidal coordinate system with consideration of extended technical objects magnetic characteristics uncertainty solved. References 48, figures 2.

Key words: energy-saturated extended technical objects, magnetic field model, magnetic silencing, extended spheroidal coordinate system, spatial extended spheroidal harmonics, prediction, measurements, uncertainty.

Мета. Розробка методу прогнозування магнітної тиші енергонасичених витягнутих технічних об'єктів на основі розв'язку обернених геометричних задач магнітостатики та обчислення просторових сфероїдальних гармонік магнітного поля в витягнутій сфероїдній системі координат з врахуванням невизначеностей магнітних характеристик технічних об'єктів. **Методологія.** Просторові витягнуті сфероїдні гармоніки моделі магнітного поля витягнутих технічних об'єктів розраховані як розв'язок обернених геометричних задач магнітостатики в формі нелінійної задачі мінімаксної оптимізації на основі вимірювань ближнього поля для прогнозування величини магнітного поля витягнутих технічних об'єктів. Нелінійна цільова функція розрахована як зважена сума квадратів залишків між вимірним і прогнозованим магнітним полем, яке обчислено з використанням програмного пакету COMSOL Multiphysics. Розв'язки задач нелінійної мінімаксної оптимізації розраховані на основі алгоритмів нелінійної оптимізації роєм частинок. **Результати.** Результати прогнозування величини дальнього магнітного поля витягнутих технічних об'єктів на основі просторових витягнутих сфероїдальних гармонік моделі магнітного поля в витягнутій сфероїдній системі координат з використанням вимірювань ближнього поля та з врахуванням невизначеності магнітних характеристик витягнутих технічних об'єктів. **Оригінальність.** Розроблено метод прогнозування магнітної тиші витягнутих технічних об'єктів на основі просторових витягнутих сфероїдальних гармонік моделі магнітного поля в витягнутій сфероїдній системі координат з врахуванням невизначеності магнітних характеристик. **Практична цінність.** Вирішено важливу практичну задачу магнітної тиші витягнутих технічних об'єктів на основі просторових витягнутих сфероїдальних гармонік моделі магнітного поля в розширеній сфероїдній системі координат з врахуванням невизначеності магнітних характеристик витягнутих технічних об'єктів. Бібл. 48, рис. 2.

Ключові слова: енергонасичені витягнуті технічні об'єкти, модель магнітного поля, магнітна тиша, витягнута сфероїдна система координат, просторові витягнуті сфероїдні гармоніки, прогноз, вимірювання, невизначеність.

Introduction. The development of energy-saturated technical objects with a given distribution of the generated magnetic field is an urgent problem for many science and industry branches. The strictest requirements for the accuracy of the spatial distribution of the magnetic field are imposed when ensuring the magnetic silencing of spacecraft, the development of anti-mine magnetic protection of naval vessels and submarines, the creation of magnetometry devices including for medical diagnostic devices and other fields [1–5].

The practice of designing of technical objects with standardized levels of their external magnetic field required the development of scientific foundations for their design, production, methodological and metrological support. The foundation of these scientific foundations is the mathematical modeling of a three-dimensional quasi-stationary external magnetic field generated by a technical object.

The main advantage of applying the methods of spatial harmonic analysis to the study and targeted influence on the external magnetic field of technical objects is the maximum simplification of the calculation

of the external magnetic field of a technical object based on a limited number of spatial harmonics.

When energy-saturated technical objects design with a given magnetic field spatiotemporal characteristic, two interrelated problems are solved [6, 7]. First, based on measurements of the real magnetic field of a technical object in the near zone, it is necessary to design a mathematical model of the magnetic field of a technical object, on the basis of which the magnetic field in the far zone can be calculated. This problem is called prediction [6].

To measure the real technical objects magnetic field in the near zone often use point magnetic field sensors [8]. In particular, to measure the magnetic field of ships in the ship magnetism laboratory (France) 39 magnetic field sensors used and located in close proximity to the ship's hull [9]. To design such a mathematical model of a technical object, it is necessary to solve the inverse magnetostatics problem [6]. Based on this mathematical model, the magnetic field in the far zone calculated. For naval vessels a control depth is specified, and for a spacecraft the installation point for the onboard

magnetometer is specified. In addition, for spacecraft, the resulting spacecraft magnetic moment also calculated based on this model.

The second problem is to ensure the specified spatial-temporal characteristics of the magnetic field of a technical object. This is a control problem [6]. Based on the designed mathematical model of the initial magnetic field of a technical object, it is necessary to calculate the spatial location and values of compensating magnetic units in such a way that the resulting magnetic field generated by the technical object satisfies the requirements [10, 11].

The basis of such developments is the design of a mathematical model of the magnetic field of a technical object. If the geometric dimensions of a technical object in an orthogonal coordinate system are approximately the same, then a spherical coordinate system is used.

In particular, the «MicroSAT» spacecraft have the shape of a cube. Moreover, on this spacecraft with the «IonoSAT-Micro» equipment, the on-board magnetometer are coordinate system in the form of a multi-dipole model mounted on rods 2 m long [3]. Therefore mathematical model of such technical object represented with sufficient accuracy on the basis of a spherical.

The KS5MF2 spacecraft and the MS-2-8 spacecraft of the «Sich 2» family also have cube shape [3]. However onboard magnetometer LEMI-016 located at 0.35 m distance from from the sensor KPN CSP.

Then mathematical model of such technical object designed based on spherical coordinate system, but in the form of a multipole model taking into account dipoles, quadrupole and octupole harmonics [12].

If the technical object has an extended shape with a predominant size along one coordinate in the orthogonal coordinate system, then it is necessary to use an elongated spheroidal coordinate system [13–16].

Over the past decade, the development of small satellites, nanosatellites such as «CubeSats», has increased exponentially for Earth observation and deep space missions in the Solar System, mainly due to their lower cost and faster development. The GS-1 spacecraft of the «CUBESAT» family has geometric dimensions 0.371 m by 0.114 m by 0.11 m, so the length more than 3 times other dimensions [3]. Then mathematical model of such technical object presented on the basis of prolate spheroidal coordinate system [12].

Strict requirements are also imposed on the maximum level of the magnetic field created by naval vessels, minesweepers and submarines near their hull [17–19]. Sea Minesweepers has a length more than 6 times the other dimensions. Naval vessels are even more elongated so the length is more than 10 times the other dimensions.

To compensate magnetic field of naval vessels and submarines compensation windings system used in three orthogonal coordinates – longitudinal, transverse and vertical directions and solenoid windings [17–19]. A feature of the technical objects under consideration is the uncertainty of the magnetic characteristics of their elements and their change in different operating modes [2]. Naturally for such elongated extended technical objects mathematical models of the magnetic field must be designed in a prolate spheroidal coordinate system [13–16].

The aim of the work is to develop the method for prediction by energy-saturated extended technical objects magnetic silencing based on solution of geometric inverse magnetostatics problems and calculation magnetic field spatial prolate spheroidal harmonics in a prolate spheroidal coordinate system taking into account the technical objects magnetic characteristics uncertainties.

Direct geometric magnetostatics problems of an energy-saturated extended technical object in prolate spheroidal coordinate system. Usually, to ensure the specified magnetic characteristics, units in a special low-magnetic design are installed at such important technical objects [1–5]. At the same time, at the stage of production and adjustment of high-grade low-magnetic units, compensating magnetic elements are installed in the form of permanent magnets or electromagnet windings. In this case, the magnitudes of the magnetic moments of such units become incredibly small. However, in the magnetic field of such units, harmonics of higher orders appear – quadrupoles, octopoles, etc.

The practical solution complexity of these problems is associated with the need to use a sufficient number of integral characteristics of the magnetic field, which could serve as a quantitative criterion for the quality of the field distribution. The method that would allow in practice to use the integral characteristics of the magnetic field – spatial harmonics, and to associate them with the parameters of the technical object – remain insufficiently developed. The need to develop such method is confirmed by one of the latest standards of the European Space Agency ECSS-E-HB-20-07A (2012), which recommends using them to ensure the magnetic cleanliness of space vehicles as integral characteristics of the spatial distribution of the magnetic field its spherical harmonics [2].

The application of spatial harmonic analysis methods is based on the study of the harmonic composition of the magnetic field. The result of this application is the transition from the measured values of the induction vector to the integral characteristics of the magnetic field harmonics – multipole coefficients. Then, based on the obtained values of the multipole coefficients, the magnetic field can be described in the entire external region. The accuracy of the description depends both on the accuracy of determining the multipole coefficients themselves and on the number of spatial harmonics used in the expansion of the source function. The choice of the type of basic solutions and coordinate system depends on the specific conditions of the problem being solved and makes it possible to analytically describe the magnetic field of a wide class of technical objects.

The basis for integral characteristics and methods for their control near objects of extended shape are methods of spatial harmonic analysis in an elongated spheroidal coordinate system, where the shape of the coordinate surfaces makes it possible to bring the description area closer to the surface of the object itself.

Let's consider a mathematical model of an extended energy-saturated technical object in an elongated spheroidal coordinate system ξ, η, φ [16]. A feature of the prolate spheroidal coordinate system is the use of the parameter c , which determines the linear scale for the unit vectors of all three coordinates. The value of the

parameter c is equal to half the interfocal distance of the spheroid, the foci of which lie on the z axis at points $\pm c$.

The solution to the Laplace equation of the scalar magnetic potential for the external region outside the coordinate surface $\xi = \text{const}$ containing the source in accordance with [16] written in the following form

$$U = \sum_{n=1}^{\infty} \sum_{m=0}^n Q_n^m(\xi) \begin{Bmatrix} c_n^m \cdot \cos(m\varphi) \\ s_n^m \cdot \sin(m\varphi) \end{Bmatrix} P_n^m(\eta), \quad (1)$$

where P_n^m , Q_n^m are associated Legendre functions of the first and second kind, respectively, with degree n and

$$\begin{aligned} H_{\xi} &= -\frac{\sqrt{\xi^2-1}}{c\sqrt{\xi^2-\eta^2}} \sum_{n=1}^{\infty} \sum_{m=0}^n \frac{dQ_n^m(\xi)}{d\xi} \{c_n^m \cos(m\varphi) + s_n^m \sin(m\varphi)\} P_n^m(\cos(\eta)); \\ H_{\eta} &= -\frac{\sqrt{1-\eta^2}}{c\sqrt{\xi^2-\eta^2}} \sum_{n=1}^{\infty} \sum_{m=0}^n Q_n^m(\xi) \frac{dP_n^m(\cos(\eta))}{d\eta} \{c_n^m \cos(m\varphi) + s_n^m \sin(m\varphi)\}; \\ H_{\varphi} &= \frac{m}{c\sqrt{(\xi^2-1)(1-\eta^2)}} \sum_{n=1}^{\infty} \sum_{m=0}^n Q_n^m(\xi) P_n^m(\cos(\eta)) \{c_n^m \sin(m\varphi) - s_n^m \cos(m\varphi)\}. \end{aligned} \quad (2)$$

It is more convenient to carry out practical measurements and calculations of magnetic field strength components in the orthogonal coordinate system, the transition to which for the strength components is carried out using the formulas:

$$\begin{aligned} H_x &= \xi \cdot \frac{\sqrt{1-\eta^2}}{\sqrt{\xi^2-\eta^2}} \cdot \cos(\varphi \cdot H_{\xi}) - \\ &- \eta \cdot \frac{\sqrt{\xi^2-1}}{\sqrt{\xi^2-\eta^2}} \cdot \cos(\varphi \cdot H_{\eta}) - \sin(\varphi \cdot H_{\varphi}); \\ H_y &= \xi \cdot \frac{\sqrt{1-\eta^2}}{\sqrt{\xi^2-\eta^2}} \cdot \sin(\varphi \cdot H_{\xi}) - \\ &- \eta \cdot \frac{\sqrt{\xi^2-1}}{\sqrt{\xi^2-\eta^2}} \cdot \sin(\varphi \cdot H_{\eta}) + \cos(\varphi \cdot H_{\varphi}); \\ H_z &= \eta \cdot \frac{\sqrt{\xi^2-1}}{\sqrt{\xi^2-\eta^2}} \cdot H_{\xi} + \xi \cdot \frac{\sqrt{1-\eta^2}}{\sqrt{\xi^2-\eta^2}} \cdot H_{\eta}, \end{aligned} \quad (3)$$

where the coordinates x, y, z in the orthogonal coordinate system are related to coordinates ξ, η, φ in the elongated spheroidal coordinate system by the following relation

$$Q_n^m(\xi) = \frac{P_n(\xi)}{2} \cdot \ln\left(\frac{\xi+1}{\xi-1}\right) - \sum_{k=1}^n \frac{1}{k} \cdot \sum_{\lambda=0}^m C_m^{\lambda} \cdot P_{k-1}^{\lambda}(\xi) \cdot P_{n-k}^{m-\lambda}(\xi) + \frac{(1-\delta(m,0))}{2} \times \dots \times \sum_{q=0}^{m-1} C_m^q \cdot P_n^q(\xi) \cdot (m-q)! \cdot \frac{(\xi-1)^{m-q} - (\xi+1)^{m-q}}{(-1)^{m-q-1}(\xi^2-1)^{\frac{m-q}{2}}}; \quad (6)$$

$$Q_n^m(\xi) = \frac{(\xi^2-1)^{\frac{m}{2}} n! m!}{2^{n+1}} \sum_{k=0}^m \frac{(k+n)! \Omega(m-k, \xi)}{k!(m-k)!} \sum_{\lambda=k}^n \frac{(\xi-1)^{n-\lambda} (\xi+1)^{\lambda-k}}{\lambda!(n+k-\lambda)! (\lambda-k)! (n-\lambda)!} \sum_{k=1}^n \frac{1}{k} \sum_{\lambda=0}^m C_m^{\lambda} P_{k-1}^{\lambda}(\xi) P_{n-k}^{m-\lambda}(\xi);$$

where

$$\Omega(v, \xi) = \begin{cases} v=0 & \ln\left(\frac{\xi+1}{\xi-1}\right) \\ v \neq 0 & (-1)^{v-1} (v-1)! \frac{(\xi-1)^v - (\xi+1)^v}{(\xi^2-1)^v} \end{cases}, \quad C_n^k = \frac{n!}{(n-k)! k!}.$$

order m ; c_n^m, s_n^m – constant coefficients characterizing the amplitudes of external spheroidal harmonics of the magnetic field.

Similar to spherical harmonic analysis, the magnetic field strength of a source limited by a spheroidal surface specified by ξ_0 outside the spheroid can be determined as the sum of the projections of the magnetic field strength of spheroidal spatial harmonics, determined by the corresponding coefficients. For spheroidal projections, the magnetic field strength in the external region will take the form

$$\begin{aligned} x &= c \cdot \sqrt{\xi^2-1} \cdot \sqrt{1-\eta^2} \cdot \cos\varphi; \\ y &= c \cdot \sqrt{\xi^2-1} \cdot \sqrt{1-\eta^2} \cdot \sin\varphi; \\ z &= c \cdot \xi \cdot \eta; \end{aligned} \quad \begin{aligned} \xi &\in [1, \infty[; \\ \eta &\in [0, 1]; \\ \varphi &\in [0, 2\pi]; \end{aligned} \quad (4)$$

Practical harmonic analysis in elongated spheroidal coordinate system based on (2) requires the calculation of associated Legendre polynomials of the first and second kind. Polynomials of the second kind calculated using the well-known formula with a limitation on the number of terms of the infinite series [16]

$$\begin{aligned} Q_n^m(\xi) &= \frac{(-1)^m \cdot (2)^{m-1} \cdot (\xi^2-1)^{m/2}}{\xi^{n+m+1}} \times \\ &\times \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{n}{2} + \frac{m}{2} + k + \frac{1}{2}\right)! \cdot \Gamma\left(\frac{n}{2} + \frac{m}{2} + k + 1\right)}{\Gamma(k+1) \cdot \Gamma\left(n+k+\frac{3}{2}\right)} \cdot \xi^{2k}. \end{aligned} \quad (5)$$

The region $\xi \in [\xi_0, 4]$ places strict demands on the accuracy of $Q_n^m(\xi)$ function calculations. In the work [16] algorithms for direct calculation $Q_n^m(\xi)$ obtained in the form of finite sums

Direct geometric magnetostatics problems of energy-saturated units in spherical coordinate system.

Usually inside extended energy-saturated technical object main sources of the magnetic field in form of N units of energy-saturated units installed. The sources of the magnetic field on naval vessels and submarines are electric motors, generators, distribution boards, semiconductor converters, transformers, etc. At the design and manufacturing stage, each such unit undergoes mandatory testing for its magnetic cleanliness. As a rule, these units represent compact devices and, therefore, mathematical models of the magnetic field of these units are conveniently represented in spherical coordinate systems [22–28].

Let us assume that during the testing process, for each magnetic unit, a model of its magnetic field is determined in a spherical coordinate system in the form of a Gaussian series

$$U = \frac{1}{4\pi} \cdot \sum_{n=1}^{\infty} \left(\frac{1}{r}\right)^{n+1} \cdot \sum_{m=0}^n \left(g_n^m \cdot \cos(m\varphi) + h_n^m \cdot \sin(m\varphi) \cdot P_n^m(\cos\theta) \right). \quad (7)$$

In this case, the spherical coordinates r , φ , and θ models of the magnetic field of the unit n are associated with the magnetic center of this units, and the spatial harmonics g_n^m , h_n^m are a comprehensive characteristic of the magnetic field generated by this unit n only.

Then the components H_r , H_φ , H_θ of the magnetic field strengths generated by all N units at the point with coordinates r , φ , and θ in the spherical coordinate system associated with the geometric center of this unit n calculated from spherical spatial harmonics, taken with the corresponding multipole coefficients:

$$\begin{aligned} H_r &= \sum_{n=1}^{\infty} \sum_{m=0}^n \frac{n+1}{r^{n+2}} \left\{ g_n^m \cos(m\varphi) + h_n^m \sin(m\varphi) \right\} P_n^m(\cos\theta); \\ H_\theta &= - \sum_{n=1}^{\infty} \sum_{m=0}^n \frac{1}{r^{n+2}} \left\{ g_n^m \cos(m\varphi) + h_n^m \sin(m\varphi) \right\} \frac{dP_n^m(\cos\theta)}{d\theta}; \quad (8) \\ H_\varphi &= \sum_{n=1}^{\infty} \sum_{m=0}^n \frac{m}{r^{n+2}} \left\{ g_n^m \sin(m\varphi) - h_n^m \cos(m\varphi) \right\} \frac{P_n^m(\cos\theta)}{\sin\theta}. \end{aligned}$$

Typically, when testing the magnetic characteristics of unit measurements are carried out in the orthogonal coordinate system x , y , z , which are related to the spherical coordinates r , φ , and θ the following dependence

$$\begin{aligned} x &= r \cdot \sin(\vartheta) \cos(\varphi); \\ y &= r \cdot \sin(\vartheta) \cdot \sin(\varphi); \Rightarrow \vartheta \in [0, \pi]; \\ z &= r \cdot \cos(\vartheta); \end{aligned} \quad (9)$$

Then, using the calculated values (6) of the magnetic field strength components H_r , H_φ , H_θ in spherical coordinates r , φ , and θ calculate the components H_x , H_y , H_z of this magnetic field strength vector in the rectangular coordinate system x , y , z , associated with the magnetic units centers [29–34]

$$\begin{aligned} H_x &= H_r \sin(\theta_0) \cos(\varphi_0) + H_\theta \cos(\theta_0) \cos(\varphi_0) - H_\varphi \sin(\varphi_0); \\ H_y &= H_r \sin(\theta_0) \sin(\varphi_0) + H_\theta \cos(\theta_0) \sin(\varphi_0) + H_\varphi \cos(\varphi_0); \\ H_z &= H_r \cos(\theta_0) - H_\theta \sin(\theta_0). \end{aligned} \quad (10)$$

Naturally, some of the units are also extended sources of a magnetic field and it is advisable to describe them in the spheroidal coordinate system ξ , η and φ associated with the centers of these blocks. In particular, on a naval vessels such elongated energy-saturated units are distribution boards, semiconductor converters, etc., installed in a row. The components H_x , H_y , H_z of magnetic field generated by these extended energy-saturated units calculated using (8) for direct geometric magnetostatics problems in an elongated spheroidal coordinate system.

Let us set the coordinates ξ_k , η_k , φ_k of k – points for calculating magnetic field strengths in an elongated spheroidal coordinate system ξ , η , φ associated with the center of the technical object. Based on (9) calculated the coordinates x_k , y_k , z_k of these points in the orthogonal coordinate system associated with the center of the technical object. Let's set the coordinates x_n , y_n , z_n of n location points of energy-saturated units in the same orthogonal coordinate system, also associated with the center of the technical object.

Let us choose the directions of the axes of the orthogonal coordinate systems of individual units parallel to the directions of the axes of the orthogonal coordinate system associated with the center of the technical object.

Then, based on (8) the components H_{rkn} , $H_{\varphi kn}$, $H_{\theta kn}$ of the magnetic field strength vector, generated by separate n units in a current with coordinates x_k , y_k , z_k can be calculated in spherical coordinate systems associated with the magnetic centers of individual units. When calculating the component H_{rkn} , $H_{\varphi kn}$, $H_{\theta kn}$ of the magnetic field strength vector it is necessary to use the following values of the spherical coordinates r_k , φ_k , and θ_k of the location of the magnetic field measurement point with coordinates r_k , φ_k , and θ_k relative to the location point of units n with coordinates r_n , φ_n , and θ_n in (2)

$$\begin{aligned} r_{kn} &= \sqrt{((x_k - x_n)^2 + (y_k - y_n)^2 + (z_k - z_n)^2)}; \\ \cos(\theta_{kn}) &= (z_k - z_n) / r_{kn}; \\ \text{tg}(\varphi_{kn}) &= (y_k - y_n) / (x_k - x_n). \end{aligned} \quad (11)$$

Then, from the calculated based on (8) components H_{rkn} , $H_{\varphi kn}$, $H_{\theta kn}$ of the magnetic field strength vector generated by individual n units at a point with coordinates x_k , y_k , z_k in spherical coordinate systems associated with the magnetic centers of individual blocks, the components H_{rkn} , $H_{\varphi kn}$, $H_{\theta kn}$ of the magnetic field strength vector generated by these n units in a current with coordinates x_k , y_k , z_k in a rectangular coordinate system associated with the magnetic center of a technical object according can be calculated based on (8). Naturally, in (2) it is necessary to use the values r , φ , and θ coordinates in the spheroidal coordinate system calculated from (2).

From the obtained components H_{rkn} , $H_{\varphi kn}$, $H_{\theta kn}$ of the magnetic field strength vector generated by n blocks in a point with coordinates x_k , y_k , z_k in a rectangular coordinate system associated with the magnetic center of a technical object, the components H_{xkn} , H_{ykn} , H_{zkn} can be calculated total magnetic field generated by all N energy-saturated objects at the measurement point with coordinates x_k , y_k , z_k in a rectangular coordinate system associated with the center of the technical object, in the form of the following sums of components H_{xkn} , H_{ykn} , H_{zkn} magnetic field strength generated by individual n energy-saturated blocks

$$H_{xk} = \sum_{n=1}^{n=N} H_{xkn}; \quad H_{yk} = \sum_{n=1}^{n=N} H_{ykn}; \quad H_{zk} = \sum_{n=1}^{n=N} H_{zkn}. \quad (12)$$

Prediction geometric inverse magnetostatics problems of an energy-saturated extended technical object in prolate spheroidal coordinate system. A feature of the energy-saturated extended technical objects are the uncertainty of the magnetic characteristics of their elements and their change in different operating modes [35–40]. Let us introduce the vector \mathbf{G} of uncertainties of the parameters of energy-saturated extended technical object unit's magnetic cleanliness [41–44]. It should be noted that the values of external spheroidal harmonics c_n^m , s_n^m of magnetic field model (2), (3) of an elongated energy-saturated object in an elongated spheroidal coordinate system ξ , η , φ and the values of the spatial spherical harmonics g_n^m , h_n^m of magnetic field model (8) of the spherical coordinates r , φ , and θ for all N units of the energy-saturated extended technical object determined in the course of testing the magnetic cleanliness of all energy-saturated extended technical object units depend on the operating modes of the microsatellite and, therefore, are functions of the components of the vector \mathbf{G} of uncertainties of the parameters of the magnetic cleanliness of the energy-saturated extended technical object units.

Then for a given value of the vector \mathbf{G} of uncertainties of the parameters of the magnetic cleanliness of energy-saturated extended technical object units, given coordinates r_n , φ_n , and θ_n of spatial arrangement of N energy-saturated extended technical object units with given values of the spatial spherical harmonics g_n^m , h_n^m of magnetic field model (8) components $H_{xk}(\mathbf{G})$, $H_{yk}(\mathbf{G})$, $H_{zk}(\mathbf{G})$ of the magnetic field generated by all N units of the energy-saturated extended technical object at the point with coordinates r_k , φ_k and θ_k calculated based on (6) – (8). Since the results of measuring the magnetic field depend on the operating modes of the V , the components $H_{xk}(\mathbf{G})$, $H_{yk}(\mathbf{G})$, $H_{zk}(\mathbf{G})$ of the measurement vector also are functions of the vector \mathbf{G} .

Let us now consider the prediction geometric inverse magnetostatics problems of calculating the values of external spheroidal harmonics c_n^m , s_n^m of magnetic field model (2) of an elongated energy-saturated object in an elongated spheroidal coordinate system ξ , η , φ , associated with the magnetic center of an energy-saturated object based on the results of measurements of magnetic field $H_{xk}(\mathbf{G})$, $H_{yk}(\mathbf{G})$, $H_{zk}(\mathbf{G})$ generated by all N energy-saturated units this energy-saturated extended technical object at the K point with coordinates r_k , φ_k and θ_k [36, 37].

For convenience, we will perform the calculations in an orthogonal coordinate system associated with the center of the technical object so that instead of the measured values of the magnetic field $H_{\xi k}(\mathbf{G})$, $H_{\eta k}(\mathbf{G})$, $H_{\varphi k}(\mathbf{G})$ in an elongated spheroidal coordinate system at points ξ_k , η_k , φ_k we will use the magnetic field components $H_{xk}(\mathbf{G})$, $H_{yk}(\mathbf{G})$, $H_{zk}(\mathbf{G})$ in an orthogonal system with coordinates x_k , y_k , z_k . Naturally, the coordinates ξ_k , η_k , φ_k in the spheroidal coordinate system and the coordinates x_k , y_k , z_k in the orthogonal coordinate

system correspond to the same points K of measurement of the magnetic field of a technical object.

Let us introduce the vector $\mathbf{Y}_M(\mathbf{G})$, components $H_{xk}(\mathbf{G})$, $H_{yk}(\mathbf{G})$, $H_{zk}(\mathbf{G})$ of which are the measured values of the magnetic field in an orthogonal system with coordinates x_k , y_k , z_k , at the K measurement points [10, 48].

Let us introduce the vector \mathbf{X} of the desired parameters of the mathematical model of the energy-saturated extended technical object, the components of which are the desired coordinates x_n , y_n , z_n in the orthogonal coordinate system associated with the center of the technical object of spatial arrangement of N energy-saturated extended units as well as the desired values of c_n^m , s_n^m of external spheroidal harmonics of this units.

Then for a given vector value \mathbf{X} and for a given vector value \mathbf{G} , the components $H_{xkn}(\mathbf{X}, \mathbf{G})$, $H_{ykn}(\mathbf{X}, \mathbf{G})$, $H_{zkn}(\mathbf{X}, \mathbf{G})$ of the magnetic field in the orthogonal coordinate system generated by these N energy-saturated extended units at measurement points with coordinates x_k , y_k , z_k calculated based on (8). When calculating these components $H_{xkn}(\mathbf{X}, \mathbf{G})$, $H_{ykn}(\mathbf{X}, \mathbf{G})$, $H_{zkn}(\mathbf{X}, \mathbf{G})$ of magnetic field parameter c_{kn} and the corresponding angles ξ_{kn} , η_{kn} , φ_{kn} location of measurement points with coordinates x_k , y_k , z_k from points and location of n blocks with coordinates x_n , y_n , z_n an elongated spheroidal coordinate system it is necessary to calculate in the form of solving the following system of equations

$$\begin{aligned} x_k - x_n &= c_{kn} \sqrt{\xi_{kn}^2 - 1} \cdot \sqrt{1 - \eta_{kn}^2} \cdot \cos(\varphi_{kn}); \\ y_k - y_n &= c_{kn} \sqrt{\xi_{kn}^2 - 1} \cdot \sqrt{1 - \eta_{kn}^2} \cdot \sin(\varphi_{kn}); \\ z_{kn} - z_{kn} &= c_{kn} \cdot \xi_{kn} \cdot \eta_{kn}. \end{aligned} \quad (13)$$

$\xi_{kn} \in [1, \infty[;$
 $\eta_{kn} \in [0, 1];$
 $\varphi_{kn} \in [0, 2\pi];$

Then, based on the calculated components $H_{xkn}(\mathbf{X}, \mathbf{G})$, $H_{ykn}(\mathbf{X}, \mathbf{G})$, $H_{zkn}(\mathbf{X}, \mathbf{G})$ of the magnetic field generated by each element n , the components $H_{xk}(\mathbf{X}, \mathbf{G})$, $H_{yk}(\mathbf{X}, \mathbf{G})$, $H_{zk}(\mathbf{X}, \mathbf{G})$ of the resulting magnetic field generated by all N units at the measurement points K can be calculated similarly to (10).

Let us introduce the vector $\mathbf{Y}_C(\mathbf{G})$, components of which are the calculated values components $H_{xk}(\mathbf{X}, \mathbf{G})$, $H_{yk}(\mathbf{X}, \mathbf{G})$, $H_{zk}(\mathbf{X}, \mathbf{G})$ of the magnetic field at the K measurement points with the coordinates x_k , y_k , z_k .

For vector \mathbf{X} of the desired parameters and for vector \mathbf{G} of parameters uncertainties of the mathematical model of the spacecraft magnetic field, then, based on (3) – (8) the initial nonlinear equation $\mathbf{Y}_C(\mathbf{G})$ for the spacecraft multipole magnetic dipole model calculated

$$\mathbf{Y}_C(\mathbf{X}, \mathbf{G}) = \mathbf{F}(\mathbf{X}, \mathbf{G}), \quad (14)$$

where the vector nonlinear function $\mathbf{F}(\mathbf{X}, \mathbf{G})$ obtained on the basis of expression (3) – (8) with respect to the vector \mathbf{X} of unknown variables, whose components are desired coordinates x_n , y_n , z_n of spatial arrangement N energy-saturated extended units as well as the desired values of values c_n^m , s_n^m of external spheroidal harmonics of this units.

Naturally that the vector nonlinear function $\mathbf{F}(\mathbf{X}, \mathbf{G})$ also is a function of the vector \mathbf{G} of uncertainties of the parameters of microsatellite units magnetic cleanliness.

Let us introduce the $\mathbf{E}(\mathbf{X}, \mathbf{G})$ vector of the discrepancy between the vector $\mathbf{Y}_M(\mathbf{G})$ of the measured

magnetic field and the vector $Y_C(X, G)$ of the predicted by model (3) magnetic field

$$E(X, G) = Y_M(G) - Y_C(X, G) = Y_M(G) - F(X, G). \quad (15)$$

The nonlinear vector objective function (12) is obtained on the basis of expression (3) with respect to the vector X of unknown variables, whose components are the values coordinates unknown variables, whose components are desired coordinates x_n, y_n, z_n of spatial arrangement N energy-saturated extended units as well as the desired values of values c_n^m, s_n^m of external spheroidal harmonics of this units and the vector G of uncertainties of the parameters of the magnetic cleanliness of microsatellite units.

This approach is standard when designing robust mathematical model of the spacecraft magnetic field, when the coordinates of the spatial arrangement and the magnitudes of the magnetic moments of the dipoles are found from the conditions for minimizing the vector of the discrepancy between the vector of the measured magnetic field and the vector of the predicted by model magnetic field, but for the «worst» the vector of uncertainty parameters of the spacecraft magnetic moments are found from the conditions for maximizing the same vector of the discrepancy between the vector of the measured magnetic field and the vector of the predicted by model magnetic field.

As a rule, when optimizing the nonlinear objective function (15), it is necessary to take into account restrictions as vector inequalities [41–44]

$$G(X, G) \leq G_{\max}. \quad (16)$$

Then the magnitude of the magnetic field at any point P_j of the far zone of a technical object with coordinates x_j, y_j, z_j can be calculated and predicted based on the obtained model (9).

Inverse problem solving method. In course of geometric inverse magnetostatic's problem solving it is necessary to repeatedly solve the direct problem (2) during the iterative calculation of spatial harmonics g_n^m, h_n^m of nanosatellite initial magnetic field mathematical model when prediction geometric inverse magnetostatic's problem solution (3) calculated and during iterative calculation component g_n^m, h_n^m of spherical harmonics of the magnetic field generated by this C compensating magnetic units with unknown coordinates r_c, φ_c and θ_c when control geometric inverse magnetostatic's problem (15) solution calculated.

Components of the vector games (15) are nonlinear functions of the vector X of required parameters and the vector G of uncertainty parameters of geometric inverse magnetostatic's problem for prediction and control by nanosatellite magnetic cleanliness taking into account direct problem uncertainties and calculated by COMSOL Multiphysics software.

Typically geometric inverse magnetostatic's problem for prediction and control by technical objects magnetic cleanliness comes down to solving minimizing optimization problem [6, 7]. When geometric inverse magnetostatic's problem for prediction and control by technical objects magnetic cleanliness taking into account direct problem uncertainties worst-case design approach usually used to impart robustness designed prediction and control by nanosatellite magnetic cleanliness taking into

account direct problem uncertainties. In this cases solution of both prediction and control geometric inverse problem reduced to solving a game in which vector X of required parameters – first player minimizes game payoff (15), but vector G of direct problem uncertainties – second player tries to maximize same game payoff (15).

A feature of the problem of calculated solution under consideration is the multi-extremal nature of game payoff (15) so that the considered region of possible solutions contains local minima and maxima. This due to fact that when minimizing the induction level of the resulting magnetic field at one point in the shielding space, the induction level at another point in this space increases due to under compensation or overcompensation of the original magnetic field. Therefore, to calculate the solution to the vector game under consideration, it is advisable to use stochastic multi-agent optimization algorithms [35, 36].

The main approach to multiobjective optimization is to search for the Pareto set, which includes all solutions that are not dominated by other solutions. To find non-dominated solutions, it is convenient to use specially calculated ranks. However, this raises the problem of comparing several solutions that have the same rank values. To adapt the PSO method in relation to the problem of finding Pareto-optimal solutions on the set of possible values of a vector criterion, it is most simple to use binary preference relations that determine the Pareto dominance of individual solutions.

Inverse problem solving algorithm. Consider algorithm for calculating the solution of the vector game (15). The works [35–37] consider various approaches to computing solutions to vector games based on various heuristic approaches. Unlike works [35, 36], in this work, in order to find a unique solution to a vector game from a set of Pareto-optimal solutions, in addition to the vector payoff (15) we will also use information about the binary relationships of preferences of local solutions relative to each other [38–40].

To calculate one single global solution to the vector game (15) individual swarms exchange information with each other during the calculation of optimal solutions to local games. Information about the global optimum obtained by particles from another swarm used to calculate the speed of movement of particles from another swarm, which allows us to calculate all potential Pareto-optimal solutions [41–46].

In the standard particles swarm optimization algorithm the particle velocities change is carried out according to linear laws [6, 7]. To increase the speed of finding a global solution, special nonlinear algorithms of stochastic multi-agent optimization recently proposed in [45, 46], in which the motion of i particle of j swarm described by the following expressions

$$v_{ij}(t+1) = w_{1j}v_{ij}(t) + c_{1j}r_{1j}(t)H(p_{1ij}(t) - \varepsilon_{1ij}(t))\left[y_{ij}(t) - x_{ij}(t)\right] + c_{2j}r_{2j}(t)H(p_{2ij}(t) - \varepsilon_{2ij}(t))\left[y_j^*(t) - x_{ij}(t)\right]; \quad (17)$$

$$u_{ij}(t+1) = w_{2j}u_{ij}(t) + c_{3j}r_{3j}(t)H(p_{3ij}(t) - \varepsilon_{3ij}(t))\left[z_{ij}(t) - \delta_{ij}(t)\right] + c_{4j}r_{4j}(t)H(p_{4ij}(t) - \varepsilon_{4ij}(t))\left[z_j^*(t) - \delta_{ij}(t)\right]; \quad (18)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1); \quad (19)$$

$$g_{ij}(t+1) = \delta_{ij}(t) + u_{ij}(t+1),$$

where $x_{ij}(t)$, $g_{ij}(t)$ and $v_{ij}(t)$, $u_{ij}(t)$ is the position and velocity of i particle of j swarm.

In (17) – (19) $y_{ij}(t)$, $z_{ij}(t)$ and $y_j^*(t)$, $z_j^*(t)$ – the best local and global positions of the i -th particle, found respectively by only one i -th particle and all the particles of j swarm. Moreover, the best local position $y_{ij}(t)$ and the global position $y_j^*(t)$ of the i particle of j swarm are understood in the sense of the first player strategy $x_{ij}(t)$ for minimum of component $E_i(\mathbf{X}, \mathbf{G})$ of the vector payoff (15). However, the best local position $z_{ij}(t)$ and the global position z_j^* of the i particle of j swarm are understood in the sense of the second player strategy $g_{ij}(t)$ for maximum of the same component $E_i(\mathbf{X}, \mathbf{G})$ of the vector payoff (15).

Four independent random numbers $r_{1j}(t)$, $r_{2j}(t)$, $r_{3j}(t)$, $r_{4j}(t)$ are in the range of $[0, 1]$, which determine the stochastic particle velocity components.

Positive constants c_{1j} , c_{2j} and c_{3j} , c_{4j} determine the cognitive and social weights of the particle velocity components.

The Heaviside function H is used as a switching function of the motion of a particle, respectively, to the local $y_{ij}(t)$, $z_{ij}(t)$ and global $y_j^*(t)$, $z_j^*(t)$ optimum.

Switching parameters of the cognitive p_{1ij} , p_{3ij} and social p_{2ij} , p_{4ij} components of the particle velocity to the local and global optimum taken in the form of increments changes in the payoff (15) for players' strategies $x_{ij}(t)$, $g_{ij}(t)$ when moving to local and global optimum respectively.

Random numbers $\varepsilon_{1ij}(t)$, $\varepsilon_{2ij}(t)$, $\varepsilon_{3ij}(t)$ and $\varepsilon_{4ij}(t)$ determine the switching parameters of the particle motion, respectively, to local and global optima.

To improve solution finding process quality, the inertia coefficients w_{1j} , w_{2j} used.

In random search, the motion of the particle is carried out in the direction of the maximum growth of the component of the objective function, found in the process of random search. In general, this direction serves as an estimate of the direction of the gradient in a random search. Naturally, such an increment of the objective function serves as an analogue of the first derivative – the rate of change of the objective function.

To take these constraints into account when searching for solutions, we used special particle swarm optimization method for constrained optimization problems [38–40]. To take these binary preference relations into account when searching for solutions, we used special evolutionary algorithms for multiobjective optimizations [45, 46].

Simulation results. The real magnetic signature of naval vessels and submarines are a secret [45–48]. That's why as an example, consider modeling the magnetic signature of an extended energy-saturated object with the following initial data [6]. The sources of the magnetic field are 16 dipoles located at points with coordinates $x = \pm 39$ m and ± 13 m with $y = \pm 4$ m and $z = 3.5$ m. These dipoles have different values of the magnetic moment components M_x , M_y and M_z along the three axes of the rectangular coordinate system.

The magnetic field levels were calculated in the interval $x = -100$ m and $x = +100$ m for three values $y = 0$ and $y = \pm 20$ m. Thus, three components of the magnetic field strength were calculated at 303 points, so that the total number

measurements amounted to 909. In this case, calculations were performed for two values $z = 19$ m and $z = 60$ m.

In the example under consideration, the object is elongated along the X coordinate, which corresponds to the real location of the rectangular coordinate system axes of the object in question. However, formulas (2), (3) for calculating the magnetic field in an elongated spheroidal system are given for the case when the object is elongated along the Z coordinate.

Let us consider the model in the form of one magnetic field source located at the beginning of the coordinates. The accuracy of approximation of the magnetic field of an elongated object depends on the number of harmonics taken into account in the magnetic field model.

Let us first consider the approximation in the form of one first harmonic. During the optimization process, the value of the parameter $c = 58.03$ and the magnitude of the first harmonic $c_1^0 = -1.75586$, $c_1^1 = -0.226512$, $s_1^1 = -0.440788$ calculated.

Figure 1 shows magnetic signature projections taking into account only one first harmonic, for three coordinates of the a) $Y = -20$ m, $Z = 19$ m; b) $Y = 0$, $Z = 19$ m and c) $Y = 20$ m, $Z = 19$ m. Note that the components of the magnetic field change most strongly for the passage characteristic at the center of the technical object with coordinates $Y = 0$, $Z = 19$ m.

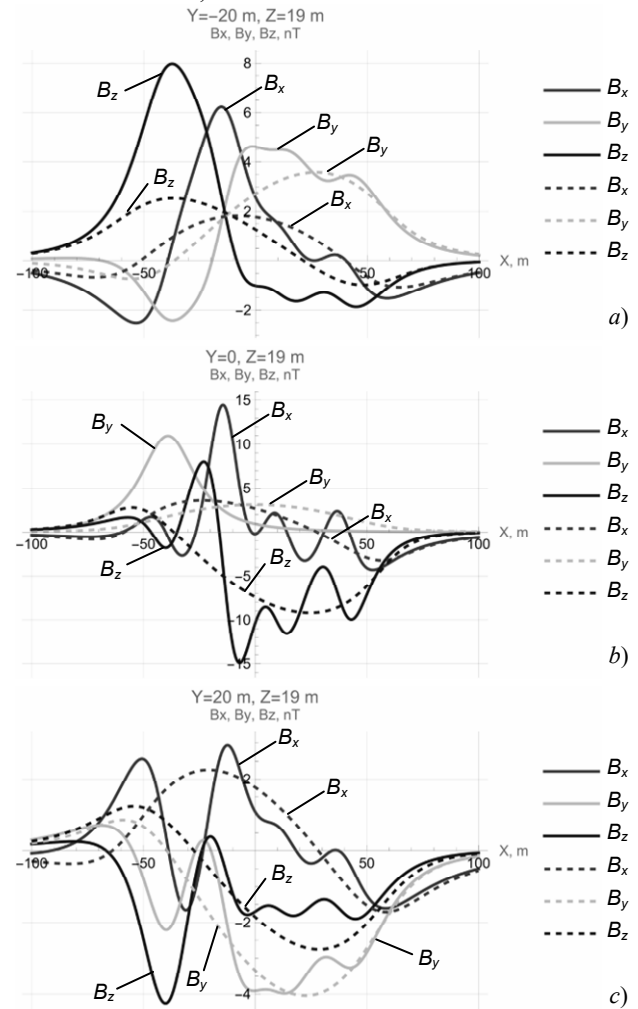


Fig. 1. Magnetic signature projections taking into account only one first harmonic of a real object (solid lines) and models (dashed lines)

Let us consider the approximation in the form of five harmonic. During the optimization process, the value of the parameter $c = 36.5236$ and the magnitude of the five harmonic

$$c_1^0 = -1.17987, c_1^1 = -0.524211, s_1^1 = -1.28567;$$

$$c_2^0 = 7.0228, c_2^1 = 2.34334, c_2^2 = -0.0767858, s_2^1 = -3.32643,$$

$$s_2^2 = -0.0375657;$$

$$c_3^0 = -64.5892, c_3^1 = -3.96332, c_3^2 = -0.195524,$$

$$c_3^3 = -0.000181004, s_3^1 = -4.36204, s_3^2 = -0.0730327,$$

$$s_3^3 = 0.00376364;$$

$$c_4^0 = -112.13, c_4^1 = 3.10697, c_4^2 = -0.0777715,$$

$$c_4^3 = -0.00120079, c_4^4 = 0.0000413822,$$

$$s_4^1 = 7.89786, s_4^2 = 0.0941992, s_4^3 = 0.00197418,$$

$$s_4^4 = -0.0000144349;$$

$$c_5^0 = 75.3116, c_5^1 = -0.894198, c_5^2 = 0.0634926,$$

$$c_5^3 = 0.000540308, c_5^4 = 3.49849 \cdot 10^{-6}, c_5^5 = -7.31748 \cdot 10^{-8},$$

$$s_5^1 = 3.46386, s_5^2 = -0.0377943, s_5^3 = 0.000859992,$$

$$s_5^4 = 6.01948 \cdot 10^{-6}, s_5^5 = -9.39689 \cdot 10^{-8} \text{ calculated.}$$

Figure 2 shows magnetic signature projections taking into account five harmonic, for three coordinates of the magnetic field passage sections: a) $Y = -20$ m, $Z = 19$ m; b) $Y = 0$, $Z = 19$ m and c) $Y = 20$ m, $Z = 19$ m.

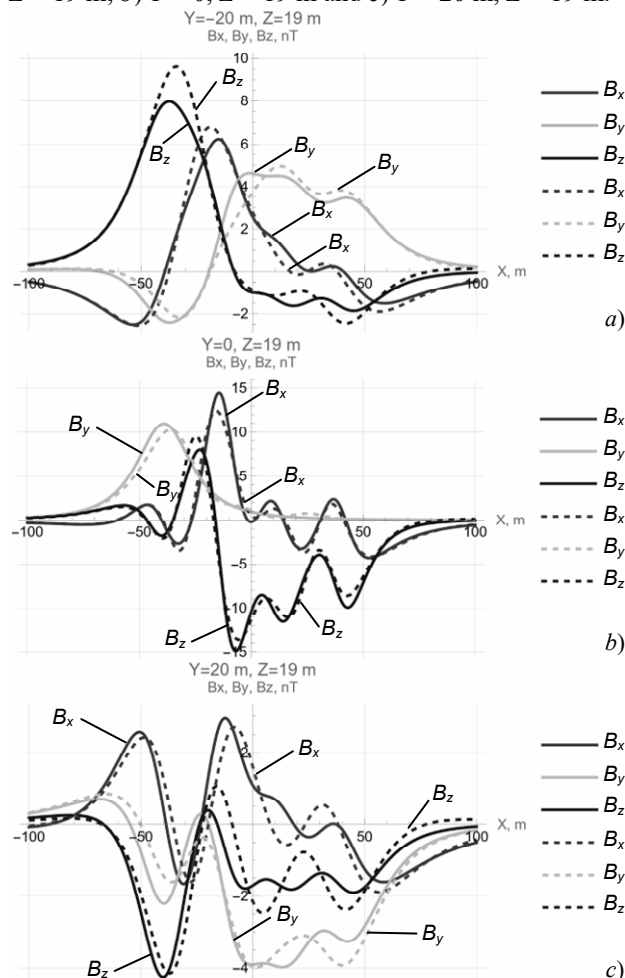


Fig. 2. Magnetic signature projections taking into account five harmonic of a real object (solid lines) and models (dashed lines)

Naturally, that when taking into account the five harmonics shown in Fig. 2 accuracy of the magnetic field prediction is significantly higher than when taking into account only one first harmonic, shown in Fig. 1.

A further increase in accuracy can be achieved both by increasing the number of harmonics taken into account and by increasing the number of magnetic field sources.

Moreover, the coordinates of these sources also need to be calculated in the form of a solution to the optimization problem.

Thus, in the example under consideration, the initial magnetic field of the technical object generated in the form of 16 dipoles located in the space of a technical object. Using these 16 dipoles, the «measured» magnetic field was calculated at 303 points in the near zone of a technical object.

Based on developed method for these 909 values of the «measured» magnetic field in the near zone predictive model designed in the form of one elongated magnetic field source with five spatial spheroidal elongated harmonics. Based on this developed prediction model the magnetic field of a technical object in the far zone calculated in an elongated spheroidal coordinate system to fulfill the technical requirements that apply to the level of the external magnetic field of a technical object.

Conclusions.

1. The method for prediction by extended technical objects magnetic silencing based on spatial prolate spheroidal harmonics magnetic field model in the prolate spheroidal coordinate system with consideration of magnetic characteristics uncertainty developed.

2. Prediction extended technical objects magnetic silencing calculated as solution of geometric inverse magnetostatics problems in the form of nonlinear minimax optimization problem based on near field measurements for prediction far extended technical objects magnetic field magnitude. Nonlinear objective function calculated as the weighted sum of squared residuals between the measured and predicted magnetic field COMSOL Multiphysics software package used. Solutions of this nonlinear minimax optimization problems calculated based on particle swarm nonlinear optimization algorithms.

3. The developed method used to design of prediction extended technical objects magnetic silencing based on the spatial prolate spheroidal harmonics of the magnetic field model in the prolate spheroidal coordinate system with consideration of extended technical objects magnetic characteristics uncertainty. A further increase in accuracy can be achieved both by increasing the number of harmonics taken into account and by increasing the number of magnetic field sources.

4. Obtained prediction model based on experimentally measured magnetic field in the near zone will be used to calculate the magnetic field in the far zone and to solve the control problem by extended technical objects magnetic silencing in the prolate spheroidal coordinate system of monitoring magnetic silence.

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