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Analytical determination of a quasi-stationary electromagnetic field created by magnetic moments and eddy currents in conducting half-space

Aim. *Study of the distribution of a three-dimensional alternating quasi-stationary electromagnetic field at the surface of conducting half-space with strong skin-effect, the source of which is an arbitrarily oriented magnetic moment.* ***Methodology****. The expressions for non-uniform electromagnetic field with strong skin effect are used for the analysis, which is based on the found exact analytical solution of the general three-dimensional problem and the use of expansion into asymptotic series with respect to a small parameter that is proportional to the ratio of the field penetration depth to the distance between the sources of the external field and the surface of body. Specific expressions at the surface are completely determined by the known field of external sources. In this work, the external magnetic moment field is used.* ***Results****. For strong skin effect, expressions for the electric and magnetic field strength are obtained separately for the components of the magnetic moment oriented perpendicularly and parallel to the flat surface between the dielectric and conducting areas. The features of the electromagnetic field distribution are analyzed depending on the value of introduced small parameter. The results are presented for the module and phase shift of the field strength with respect to the phase of the external field source.* ***Originality****. The expressions found for the electromagnetic field appear to be more general than the use of closed contours with alternating current, since they extend types of external field sources and allow the use of the superposition method instead of integration over the entire contour.* ***Practical value****. The found specific analytical expressions of the electromagnetic field at the surface for the external field of magnetic moments significantly simplify the solution of the problems, since they do not require additional solution of the field equations.* References 20, figures 8.

Key words: **three-dimensional quasi-stationary electromagnetic field, strong skin effect, external field of magnetic moments, asymptotic method, analytical solution**.

Мета*. Дослідження в умовах прояву сильного скін-ефекту розподілу на поверхні електропровідного півпростору тривимірного змінного квазістаціонарного електромагнітного поля, джерелом зовнішнього поля якого є довільно орієнтований магнітний момент.* ***Методологія****. Для аналізу використаний аналітичний розв'язок загальної тривимірної задачі для випадку неоднорідного електромагнітного поля при сильному скін-ефекті і використанні розкладання в асимптотичні ряди по малому параметру, який пропорційний відношенню глибини проникнення поля до відстані між джерелами зовнішнього поля і поверхнею тіла. Конкретні вирази на поверхні повністю визначаються відомим полем зовнішніх джерел, в якості яких використовується поле магнітного моменту.* ***Результати****. Для сильного скін-ефекту отримано вирази для напруженостей електричного і магнітного полів окремо для компонентів магнітного моменту, що орієнтовані перпендикулярно і паралельно до плоскої поверхні між діелектричною і електропровідною областями. Проаналізовано особливості розподілу електромагнітного поля в залежності від величини введеного малого параметру, Результати представлено для модулів і зсуву фаз компонентів напруженостей полів відносно фази джерела зовнішнього поля.* ***Оригінальність****. Знайдені вирази для електромагнітного поля уявляються більш загальними, ніж використання замкнених контурів зі змінним струмом, оскільки розширюють види джерел зовнішнього поля, що враховуються, і дозволяють використати метод суперпозиції замість інтегрування по всьому контуру.* ***Практична цінність****. Знайдені конкретні аналітичні вирази електромагнітного поля на поверхні для зовнішнього поля магнітних моментів значно спрощують вирішення конкретних задач, оскільки не потребують для цього додаткового розв'язку рівнянь поля.* Бібл. 20, рис. 8.

Ключові слова: **тривимірне квазістаціонарне електромагнітне поле, сильний скін-ефект, зовнішнє поле магнітних моментів, асимптотичний метод, аналітичний розв'язок.**

Introduction. The interaction of an alternating electromagnetic field with electrically conducting bodies is accompanied by the manifestation of the skin effect. In high-frequency and short-time pulsed electromagnetic processes, there is a strong skin effect, then the current and electromagnetic field are concentrated in the thin surface layer of the body. In this case, the formulation of mathematical models for calculating the electromagnetic field is significantly simplified. The simplest mathematical model of the ideal skin effect can be imagined, when the characteristic dimensions of the conducting body L significantly exceed the field penetration depth δ. Here, it is enough to consider a stationary problem for a body with ideal electrical conductivity and, accordingly, zero field penetration depth $\delta \to 0$ [1, 2]. In this case, the normal component of the magnetic and the tangential component of the electric fields are equal to zero.

The further development of approximate models of the electromagnetic field penetration into conducting medium at $\delta \neq 0$ is associated, first of all, with studies based on the impedance boundary condition [3, 4].

In the mathematical model proposed by M. Leontovich back in the middle of the 20th century [3], the model with ideal electrical conductivity remained the starting point. From this model, the magnetic field strength tangent to the surface of the conducting body was determined. The bounded depth of field penetration was calculated using the concept of the impedance boundary condition, where the magnetic field strength at the surface of the body is related to the electric field strength by a specific ratio. It is assumed that electromagnetic field locally penetrates into metallic body in the same way as uniform field penetrates into conducting half-space. The model is approximate and one of the issues is determining the limits of the model's application. A detailed analysis of many years of research in the development of the concept of the impedance boundary condition is presented, for example, in [5, 6].

The development of effective methods for solving 3D problems of electromagnetic field theory in a fairly general formulation is a topical problem, despite significant progress in the use of numerical calculation methods. Analytical methods of solving similar problems still have a certain appeal for theoretical research. This is due to their positive aspects. First, there is a wide range of objects where specialized analytical or combined numerical and analytical approaches remain effective.

Such objects include, in particular, systems whose geometric features are characterized by a different nature of the field change in space – a rapid change near concentrated sources of the field or near the interface between media and a much slower one in another region of space with a much larger volume. Secondly, the availability of an analytical solution makes it possible to obtain general features of the formation of a 3D field, and make it possible to carry out an in-depth analysis of the causes and features of physical processes. There is also an opportunity to develop well-grounded approaches to 3D modeling of complex electromagnetic systems. Finally, analytical solutions provide a certain set of exactly solvable problems, which can be a benchmark for comparison when developing other methods for calculating systems of more complex geometry, where obtaining analytical solutions is impossible.

The book [7] presents the exact analytical solution obtained by the author and published in separate articles for alternating and pulsed electromagnetic fields created by a system of spatial contours with a current of arbitrary configuration, located near a magnetized conducting body with a flat surface, where eddy currents are induced. The solution for closed contours was found in the integral form for vector and scalar potentials, magnetic and electric field strengths in dielectric and conducting media without restrictions on the geometry of the contours, medium properties and field frequency. The obtained solution made it possible, in particular, to establish the following general features of field formation [8]. The components of current density and electric field strength, perpendicular to the surface, have a zero value in the entire conducting half-space. The result is also a boundary condition for the normal component of the electric field strength in the dielectric medium, which is completely determined by the known field of external sources. Another consequence of the exact solution is the conclusion that non-uniform electromagnetic field, when penetrating into conducting half-space, always decays with depth faster than uniform field.

Simplification of computational procedures is also necessary for the analytical solution, especially when solving optimization and inverse problems of field theory. The calculation is simplified significantly not only for ideal skin effect at $\delta \to 0$, but also for the strong skin effect in its extended sense, when the distance r between the sources of external field and observation points at the body surface is limited. An effective technique is the expansion of potentials and field vectors into an asymptotic series [9, 10] with respect to the small parameter $\varepsilon = \mu\delta / \left(\sqrt{2}r\right) < 1$, where μ is the relative magnetic permeability of the conducting medium. This representation also allows to draw further conclusions regarding the general features of the 3D electromagnetic field formation. In particular, it was established that at the flat boundary the field strength is determined not only by the value of the components of known external field, as in the model of ideal skin effect, but also by its derivatives along the coordinate perpendicular to the media interface. Thus, the effect of field non-uniformity at the surface is determined, and the distribution of the field at the surface does not require the solution of additional boundary value problems.

The peculiarity of the applied power asymptotic series of the Poincaré type [11, 12] is the limited number of their members N. This is due to the error of determining each term of the series, which increases with the increase in the value of the parameter ε and the number of the series term n. Therefore, there is such a number of terms at which the error is minimal and further increasing their number only increases the error. In [9, 10], issues of the limits of application of the asymptotic method for the general case of an arbitrary external field, error analysis, determine of the number of terms of limited series, as well as their optimal number are presented. In addition, when calculating the value of the field due to the error in determining the terms of the series, their values are taken into account with a weight function, the value of which depends on the error estimate.

In [9], in particular, it is shown that calculations with sufficient accuracy can be performed for the value of the small parameter $\varepsilon \le 0.3$. This condition is fulfilled in many technological processes, where it is necessary to ensure a strong interaction between the electromagnetic field of inductor and the conducting body. For example, in devices for high-frequency induction heating of flat metal products [13, 14], the distance between the inductor and the body usually does not exceed $h = 3$ cm. In this case, at $\varepsilon = 0.3$, for example, for brass products ($\mu = 1$, $\gamma = 1.25 \cdot 10^{7}$ Ω^{-1} m^{-1}) at $h = 0.03$ m calculations can be performed for frequencies $f = \omega/2\pi \ge 125$ Hz. In equipment for exposure to a strong field to improve the mechanical properties of metal products [15, 16], the distance is $h = 0.01$-0.02 m. In this case, at $h = 0.01$ m for aluminum ($\mu = 1$, $\gamma = 3.71 \cdot 10^{7}$ Ω^{-1} m^{-1}) permissible frequencies are $f \ge 380$ Hz. It should be noted that the devices for the technological processes indicated here are examples of objects in the development of which the calculation methods under investigation can be used.

Sources of the external field can be not only contours with alternating current. In the general case, sources of the external field can also be represented by a system of magnetic moments [17]. This representation is even more convenient, since in the quasi-stationary approximation the contour must be closed and cannot be divided into parts [18]. At the same time, the principle of superposition is valid for the magnetic field of the system of magnetic moments. Each magnetic moment $\boldsymbol{m}$ is a individual field source whose vector $\boldsymbol{A}$ ($\nabla \cdot \boldsymbol{A} = 0$) and scalar φ_m magnetic potentials in a non-magnetic medium

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m} \times \boldsymbol{r}}{r^3} = -\frac{\mu_0}{4\pi}\boldsymbol{m} \times \nabla\frac{1}{r};$$
$$\varphi_m = \frac{\boldsymbol{m} \cdot \boldsymbol{r}}{4\pi r^3} = -\frac{1}{4\pi}\boldsymbol{m} \cdot \nabla\frac{1}{r} \qquad (1)$$

determine the same magnetic field strength $\boldsymbol{H}$

$$\boldsymbol{H} = \frac{1}{\mu_0}\nabla \times \boldsymbol{A} = -\nabla\varphi_m = \frac{1}{4\pi}\left[\frac{3\boldsymbol{m} \cdot \boldsymbol{r}}{r^4}\frac{\boldsymbol{r}}{r} - \frac{\boldsymbol{m}}{r^3}\right], \qquad (2)$$

here the vector $\boldsymbol{r}$ is directed from the point of source (moment) to the observation point.

Each contour with current I_0, for the electromagnetic field of which calculation expressions are given in [7], can be replaced by a surface S resting on a closed contour with field sources in the form of a double layer of magnetic charges (magnetic moments) $\mathrm{d}\boldsymbol{m} = \mu_0 I_0 \mathrm{dS}\boldsymbol{n}$, where $\mu_0 I_0 \boldsymbol{n}$ is the surface density vector of the distributed magnetic moment directed along the normal $\boldsymbol{n}$ to the surface (Fig. 1).

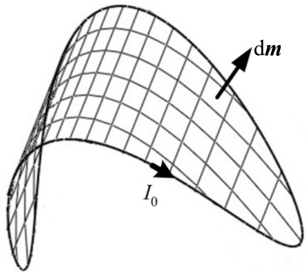


Fig. 1. Replacement of a contour with a current by the surface of a double layer of magnetic charges (magnetic moments)

Now, in contrast to the found expressions for the contour, the calculations allow the application of the principle of superposition with the summation of the fields created by the system of magnetic moments covering the surface *S*. As the distance from the source of the field to the observation point increases, the number of magnetic moments that provide the required accuracy decreases. At a considerable distance, the field source can be represented by one total magnetic moment that creates the field (1), (2).

Currently, despite the fairly general nature of the use of the field of magnetic moments, there are not enough specific studies of their application to represent the field of external sources in the asymptotic method of calculating the electromagnetic field. Therefore, the study of the possibility of their application in the practice of analytical calculations of 3D quasi-stationary fields is a topic problem.

The goal of the work is to obtain specific calculation relationships and to study the features of the distribution for non-uniform quasi-stationary electromagnetic field at the surface of conducting half-space created by external source in the form of one magnetic moment and eddy currents in conducting medium which change in time according to a sinusoidal law under the conditions of strong skin effect.

Mathematical model. It is assumed that near the conducting half-space with electrical conductivity γ and relative magnetic permeability μ at a distance h in dielectric non-magnetic medium there is a magnetic moment $\dot{\boldsymbol{m}} = \dot{\boldsymbol{m}}_{\|} + \dot{\boldsymbol{m}}_{\perp}$ that varies in time according to a sinusoidal law with cyclic frequency ω (Fig. 2). Here and in the below, complex amplitudes are denoted by a dot above the corresponding symbol.

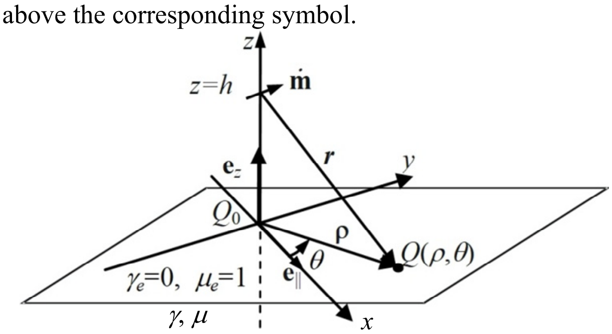


Fig. 2. The location of the magnetic moment nearby conducting half-space

In the general case, the magnetic moment is arbitrarily oriented relative to the media interface: the component $\dot{\boldsymbol{m}}_{\|} = \dot{m}_{\|}\boldsymbol{e}_{\|}$ is oriented parallel to the surface along the unit vector $\boldsymbol{e}_{\|}$; the component $\dot{\boldsymbol{m}}_{\perp} = \dot{m}_z \boldsymbol{e}_z$ is oriented along the unit vector normal to the surface $\boldsymbol{e}_z$.

The solution of the problem for the electromagnetic field at the surface between dielectric and conducting areas based on the exact solution for the system: «an arbitrary spatial contour with sinusoidal current as a source of the external field – a conducting half-space» [7]. It was shown in [8] that in the case of strong skin effect in the extended sense when $\varepsilon < 1$, taking into account the external field non-uniformity at the surface of conducting body with flat surface, the electric and magnetic field strengths are completely determined by the magnetic field strength of external sources $\dot{\boldsymbol{H}}_0$. The resulting expressions in the form of expansion into limited asymptotic series for the tangential and normal components of the electric $\dot{\boldsymbol{E}} = \dot{\boldsymbol{E}}_{\|} + \dot{\boldsymbol{E}}_{\perp}$ and magnetic $\dot{\boldsymbol{H}} = \dot{\boldsymbol{H}}_{\|} + \dot{\boldsymbol{H}}_{\perp}$ field strengths are as follows:

- Tangential components of fields that are the same at the surface in dielectric and conducting media:

$$\dot{\boldsymbol{E}}_{\|}(z=0) = \varsigma \sum_{n=0}^{N} 2a_n(\mu)\left(\frac{\varepsilon\, r}{\sqrt{j}}\right)^n \left\{\frac{\partial^{(n)}}{\partial z^n}\boldsymbol{e}_z \times \dot{\boldsymbol{H}}_{0\|}\right\}\Bigg|_{z=0} ; \quad (3)$$

$$\dot{\boldsymbol{H}}_{\|}(z=0) = -\sum_{n=0}^{N+1} 2a_{n-1}(\mu)\left(\frac{\varepsilon\, r}{\sqrt{j}}\right)^n \left\{\frac{\partial^{(n)}\dot{\boldsymbol{H}}_{0\|}}{\partial z^n}\right\}\Bigg|_{z=0} , \quad (4)$$

where $\zeta = p/\gamma$ is the surface impedance, $p = \sqrt{j\omega\mu\mu_0\gamma}$ is the propagation constant, j is the imaginary unit. In (3), (4) it is taken into account that $\varepsilon r/\sqrt{j} = \mu/p$. The $a_n(\mu)$ are the coefficients of the Taylor series expansion of the function

$$1/w = \sum_{n=0}^{\infty} a_n(\mu)\left(\chi/\sqrt{j}\right)^n , \text{ where } w(\chi) = \frac{\chi}{\sqrt{j}} + \sqrt{1 + \left(\frac{\chi}{\mu\sqrt{j}}\right)^2} ,$$

it is assumed $a_{-1} = -1$. The number N of terms of the limited asymptotic series is determined, first of all, by the value of small parameter ε [9, 10].

- Normal components of the electric $\dot{\boldsymbol{E}}_{\perp}^{+}$, $\dot{\boldsymbol{E}}_{\perp}^{-}$ and magnetic $\dot{\boldsymbol{H}}_{\perp}^{+}$, $\dot{\boldsymbol{H}}_{\perp}^{-}$ fields at different sides of the surface in the dielectric (z = 0+0) and conducting (z = 0–0) media are different.

At any point of the electrically conducting medium, the component of the electric field intensity directed perpendicular to the surface is equal to zero for arbitrary values of the parameter ε and then at the surface of the body in dielectric medium the electric field strength is completely determined by the induced electric field of external sources which is considered as known

$$\dot{\boldsymbol{E}}_z(z<0) = 0 ; \quad \dot{E}_{\perp}^{+} = -2j\omega\dot{A}_{0z}(z=0) , \quad (5)$$

where $\dot{A}_{0z}(z=0)$ is the normal component of the vector potential of the magnetic field of external sources.

Taking into account the continuity of the normal component of the magnetic flux density, the expressions for the normal components of the strength on different sides of the surface are

$$\dot{\boldsymbol{H}}_{\perp}^{+} = \mu\dot{\boldsymbol{H}}_{\perp}^{-} = -\sum_{n=0}^{N} 2a_n(\mu)\left(\frac{\varepsilon\, r}{\sqrt{j}}\right)^{n+1}\left\{\frac{\partial^{(n+1)}\dot{\boldsymbol{H}}_{0\perp}}{\partial z^{n+1}}\right\}\Bigg|_{z=0}. \qquad (6)$$

The zero term of the asymptotic series in (3), (4) corresponds to approximate model in which it is assumed that the field at the surface is uniform and the normal component of the magnetic field strength is zero. At the same time, the value of the tangential magnetic field at the surface corresponds to the magnetic field at the surface of a body with ideal conductivity and, accordingly, zero field penetration depth $\delta\to 0$.

Expressions (3) – (6) take into account the non-uniformity of the field of external sources. This is evidenced by the presence in the expressions of derivatives with respect to coordinates directed perpendicular to the surface of interface media. The influence of the non-uniformity of the electromagnetic field for tangential components is revealed in terms of the series with numbers $n \geq 1$. The normal component of the external field is already taken into account in the first term of the asymptotic series.

The given expressions of electromagnetic field distribution are approximate. In the calculation examples presented below, the value of the small parameter does not exceed the assumed value $\varepsilon = 0.3$. The number of terms of the asymptotic series was equal to four ($N = 3$). At the same time, according to the estimates made in [9, 10], the relative error in determining the field strengths did not exceed the value $\Delta_N = 5\cdot 10^{-3}$.

The electric and magnetic components of the field at the flat surface of the conducting body are calculated below separately according to expressions (3), (4), (6) for the components of the magnetic moment oriented along the normal and parallel to the surface.

Electromagnetic field of magnetic moment $\boldsymbol{m}_{\perp}$ oriented along the normal to the surface. The external magnetic field of the magnetic moment $\dot{\boldsymbol{m}}_{\perp} = \dot{m}_z\boldsymbol{e}_z$ has axial symmetry, and it is convenient to write its expression in the cylindrical coordinate system (ρ, θ, z) with standard basis vectors ($\boldsymbol{e}_\rho$, $\boldsymbol{e}_\theta$, $\boldsymbol{e}_z$) directed along the corresponding coordinates. Then, in accordance with (2), the magnetic moment creates the following field at arbitrary point of the space

$$\dot{\boldsymbol{H}}_0 = \frac{1}{4\pi}\left[3\frac{(\dot{\boldsymbol{m}}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\dot{\boldsymbol{m}}}{r^3}\right] = \frac{\dot{m}_z}{4\pi}\left[3\frac{(z-h)^2}{r^5} - \frac{1}{r^3}\right]\boldsymbol{e}_z + \frac{\dot{m}_z}{4\pi}3\frac{(z-h)\rho}{r^5}\boldsymbol{e}_\rho. \qquad (7)$$

The parameter ε depends on the distance r between the source and observation points. At the point Q_0 at the surface directly below the magnetic moment the distance $r = h$ is minimal and parameter ε takes its maximum value. Accordingly, at this point the error of the approximate calculation method is the largest.

For further analysis, we will use the single maximum value of the small parameter $\varepsilon_m = \mu\delta/\left(h\sqrt{2}\right) = \varepsilon(r/h)$. Using the h and ε_m values, the surface impedance is found to be $\varsigma = \sqrt{j\omega\mu_0 h\varepsilon_m}$.

By substituting the value of the external magnetic field (7) into the expressions for the field strengths (3), (4), (6), we obtain their distribution along the radial coordinate ρ at the surface of the conducting body, depending on the value of the small parameter ε_m, as well as the height of the location h and cyclic field frequency ω.

The normalized values of the electric and magnetic field strengths are entered as follows:

$$\dot{\boldsymbol{E}}_{\|}^{*} = \dot{\boldsymbol{E}}_{\|}\Bigg/\left(\frac{\mu_0\dot{m}_z\omega}{4\pi h^2}\right) \quad \text{and} \quad \dot{\boldsymbol{H}}^{*} = \dot{\boldsymbol{H}}\Bigg/\left(\frac{\dot{m}_z}{4\pi h^3}\right).$$

The expression for the normalized value of the tangential component of the electric field strength takes the following form:

$$\dot{\boldsymbol{E}}_{\|}^{*} = \boldsymbol{e}_\theta\sqrt{j}\,6\frac{\rho}{h}\varepsilon_m\sum_{n=0}^{N}a_n\left(\frac{\varepsilon_m}{\sqrt{j}}\right)^n h^{n+4}\frac{\partial^{(n)}}{\partial z^n}\left(\frac{z-1}{r^5}\right)\Bigg|_{z=0}. \qquad (8)$$

In turn, the normalized values of the tangential and normal to the surface components of the magnetic field strength are revealed

$$\dot{\boldsymbol{H}}_{\|}^{*} = -\boldsymbol{e}_\rho\,6\frac{\rho}{h}\sum_{n=0}^{N+1}a_{n-1}\left(\frac{\varepsilon_m}{\sqrt{j}}\right)^n h^{n+4}\frac{\partial^{(n)}}{\partial z^n}\left(\frac{z-1}{r^5}\right)\Bigg|_{z=0}; \qquad (9)$$

$$\dot{\boldsymbol{H}}_{\perp}^{*} = \boldsymbol{e}_z 2\sum_{n=0}^{N}a_n\left(\frac{\varepsilon_m}{\sqrt{j}}\right)^{n+1} h^{n+4}\frac{\partial^{(n+1)}}{\partial z^{n+1}}\left(\frac{2(z-1)^2-\rho^2}{r^5}\right)\Bigg|_{z=0}. \qquad (10)$$

From the presented dependencies, it can be seen that in this case of axisymmetric electromagnetic field, the electric field strength has only one azimuthal component, and the magnetic field strength is represented by radial and normal components.

In Fig. 3–5, the dependencies of the distribution of the normalized components of the complex-value amplitudes of the electric and magnetic field strengths are given as $\dot{E}_\theta^{*} = \pm\left|\dot{E}_\theta^{*}\right|\exp\left(j\varphi_{E\theta}\right)$ and $\dot{H}_k^{*} = \pm\left|\dot{H}_k^{*}\right|\exp\left(j\varphi_{Hk}\right)$ where $k = \rho$, z. The sign «–» before the module of the complex-value amplitude is used to indicate the opposite to the selected direction of the field vector component. In this case, at the same time, the phase shift angle relative to the phase of the magnetic moment changes by π.

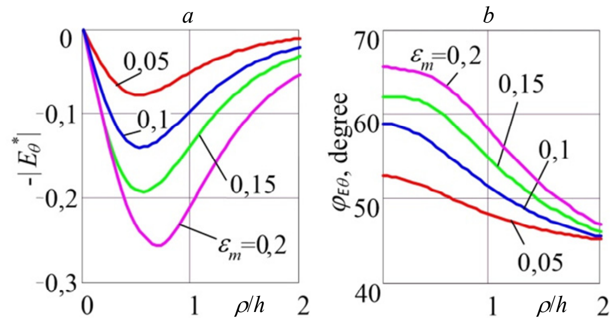


Fig. 3. Distribution along the surface of the module $-\left|\dot{E}_\theta^{*}\right|$ (*a*) and the phase shift angle $\varphi_{E\theta}$ (*b*) of the tangential component of the electric field strength for the source $\dot{\boldsymbol{m}}_{\perp}$

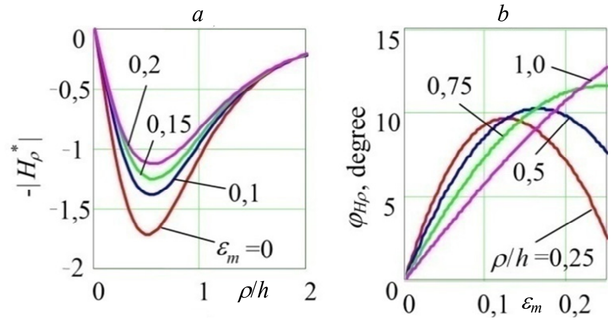


Fig. 4. Radial component of the magnetic field strength: dependence of the module $-\left|\dot{H}_\rho^*\right|$ on the radial coordinate (*a*); dependence of the phase shift angle $\varphi_{H\rho}$ on the parameter ε_m (*b*)

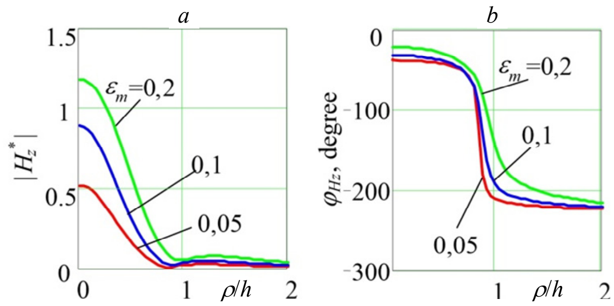


Fig. 5. Distribution along the surface of the module $\left|\dot{H}_z^*\right|$ (*a*) and phase shift angle φ_{Hz} (*b*) of the normal component of magnetic field strength

For an ideal skin effect at $\varepsilon_m \to 0$, the tangential strength of the electric field, as well as the normal component of the magnetic field, are equal to zero. In this case, the tangential strength of the magnetic field is equal to double value of the tangential component of external magnetic field [9]. Such field is used in a simplified model of the diffusion of a locally uniform field into a conducting body. In this case of the field of the magnetic moment, the phase shift is equal to zero, the tangential component is directed towards the radial coordinate (9), reaches its maximum value at the points of the circle of radius $\rho = h/2$ and is equal to

$$\dot{H}_{\|\max}^* = -6 \cdot 0{,}5 \Big/ \sqrt{0{,}5^2 + 1} = -1{,}717\,.$$

From the data presented in Fig. 3–5 it can be seen that with an increase in the parameter ε, that is, with increase in the influence of the external electromagnetic field non-uniformity during its diffusion into the conducting half-space, the character of the field distribution over the surface changes.

The tangential component of the electric field (Fig. 3) is no longer zero and increases with the growth of the parameter ε. On the contrary, the tangent component of the magnetic field strength decreases with increasing ε. At the same time, as can be seen from Fig. 4,*b*, the dependence on the parameter ε of the phase shift has a non-monotonic character.

For non-uniform field at the bounded thickness of the skin layer, that is, in the case of $\varepsilon > 0$, the normal component of the magnetic field strength is no longer zero. Note that even for the parameter $\varepsilon \cong 0.2$, the normal component $\left|\dot{H}_z^*\right|$ becomes commensurate with the tangential component $\left|\dot{H}_\rho^*\right|$ and neglecting this field component in simplified models can lead to significant calculation errors. We also note that the normal component of the magnetic field becomes insignificant at the distance of $\rho/h \geq 0.8$. In the area $\rho/h \geq \approx 0.8 \div 1.0$, the phase shift of the normal component of field changes sharply by approximately 180°. This means that in this area there is change in the direction of normal component of field compared to the direction in the area $\rho/h \leq 0.8$.

Note also that the magnetic field is elliptically polarized. This is evidenced by the fact that, as can be seen from Fig. 4,*b* and Fig. 5,*b* that the phases of the mutually perpendicular magnetic field components $\dot{H}_\rho^*$ and $\dot{H}_z^*$ are differ from each other.

Electromagnetic field of magnetic moment $\boldsymbol{m}_\|$, oriented parallel to the surface. In contrast to the previous case of normally oriented magnetic moment, the magnetic field of magnetic moment $\dot{\boldsymbol{m}}_\| = \dot{m}_\| \boldsymbol{e}_\|$ oriented parallel to the surface of media interface is convenient to write in the Cartesian coordinate system (x, y, z), the x and y axes of which lie at the surface, and the x axis is directed along the projection of the vector $\dot{\boldsymbol{m}}_\|$ to flat surface (Fig. 2). The standard basis vectors of the coordinate system are ($\boldsymbol{e}_x$, $\boldsymbol{e}_y$, $\boldsymbol{e}_z$).

The external magnetic field of the considered magnetic moment in the Cartesian coordinate system has all three components

$$\dot{\boldsymbol{H}}_0 = \frac{\dot{m}_\|}{4\pi}\left[\frac{2x^2 - y^2 - (z-h)^2}{r^5}\boldsymbol{e}_x + \frac{3xy}{r^5}\boldsymbol{e}_y + \frac{x(z-h)}{r^5}\boldsymbol{e}_z\right]. \quad (11)$$

After substituting (11) into expressions (3), (4), (6) for the electric and magnetic field strengths at the surface of media interface, we obtain

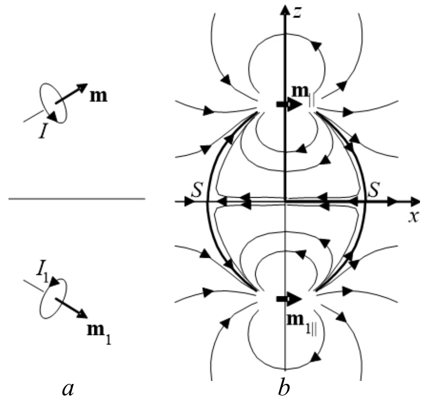
$$\dot{\boldsymbol{E}}_\| = \frac{\mu_0 \dot{m}_\| \omega}{4\pi h^2}\sqrt{j}\, 2\varepsilon_m \times$$
$$\times \left\{ \begin{aligned} &\boldsymbol{e}_y \sum_{n=0}^{N} a_n \left(\frac{\varepsilon_m}{\sqrt{j}}\right)^n h^{n+3} \frac{\partial^{(n)}}{\partial z^n}\left(\frac{2x^2-y^2}{r^5} - \frac{(z-1)^2}{r^5}\right)\Bigg|_{z=0} - \\ &-\boldsymbol{e}_x \sum_{n=0}^{N} a_n \left(\frac{\varepsilon_m}{\sqrt{j}}\right)^n h^{n+3} \frac{\partial^{(n)}}{\partial z^n}\left(\frac{3xy}{r^5}\right)\Bigg|_{z=0} \end{aligned} \right\}; \quad (12)$$

$$\dot{\boldsymbol{H}}_\| = \frac{\dot{m}_\|}{4\pi h^3} 2 \times$$
$$\times \left\{ \begin{aligned} &-\boldsymbol{e}_x \sum_{n=0}^{N} a_{n-1} \left(\frac{\varepsilon_m}{\sqrt{j}}\right)^n h^{n+3} \frac{\partial^{(n)}}{\partial z^n}\left(\frac{2x^2-y^2}{r^5} - \frac{(z-1)^2}{r^5}\right)\Bigg|_{z=0} + \\ &+\boldsymbol{e}_y \sum_{n=0}^{N} a_{n-1} \left(\frac{\varepsilon_m}{\sqrt{j}}\right)^n h^{n+3} \frac{\partial^{(n)}}{\partial z^n}\left(\frac{3xy}{r^5}\right)\Bigg|_{z=0} \end{aligned} \right\}; \quad (13)$$

$$\dot{\boldsymbol{H}}_\perp = \frac{\dot{m}_\|}{4\pi h^3} \boldsymbol{e}_z 2 \sum_{n=0}^{N} a_n \left(\frac{\varepsilon_m}{\sqrt{j}}\right)^{n+1} h^{n+4} \frac{\partial^{(n+1)}}{\partial z^{n+1}}\left(\frac{x(z-1)}{r^5}\right)\Bigg|_{z=0}. \quad (14)$$

In this case, the electromagnetic field at the surface of the half-space is symmetrical about the x axis. The components of the electric $\dot{E}_{\|x}$ and magnetic $\dot{H}_{\|y}$ field strengths have even symmetry, the components $\dot{E}_{\parallel y}$, $\dot{H}_{\parallel x}$, $\dot{H}_{\perp z}$ of the electromagnetic field have odd symmetry relative to the x axis.

An understanding of the electromagnetic field formation can be obtained if we first consider the ideal skin effect. In this case, it is sufficient to consider the formation of a magnetic field only in dielectric medium. The method of mirror images can be used to calculate the magnetic field of the moment located above the media interface. In the case of ideal skin effect, the general solution of the problem for finding the magnetic field is reduced to taking into account the current of the source and mirrored from the surface of the contour with the oppositely directed current [7]. This representation for magnetic moments, in contrast to currents, is reduced to the same direction of the tangential components and the opposite direction of the normal components of the initial $\boldsymbol{m}$ and mirrored $\boldsymbol{m}_1$ moments [17] (Fig. 6,*a*).



a *b*

Fig. 6. The structure of the magnetic field of the magnetic moment located over half-space with ideal conductivity

The structure of the magnetic field of two equally directed magnetic moments $\boldsymbol{m}_{\parallel}$ and $\boldsymbol{m}_{1\parallel}$ (Fig. 6,*b*) is convenient to analyze by defining critical points [19, 20] at which the vector field is zero. Let's find such points in the vertical plane $y = 0$. The field component perpendicular to this plane is zero $H_y = 0$. Due to the symmetry of the two magnetic moments, the field component perpendicular to the x axis and directed along the z axis is also zero $H_z = 0$. It remains to find the zero value of the H_x component on the x axis. Both magnetic moments have the same H_x field components. As a result, we get for this component

$$H_x = \boldsymbol{H}\cdot\boldsymbol{e}_x = 2\frac{1}{4\pi}\left[\frac{3\left(\boldsymbol{m}_{\parallel}\cdot\boldsymbol{r}\right)}{r^4}\frac{\left(\boldsymbol{r}\cdot\boldsymbol{e}_x\right)}{r} - \frac{\boldsymbol{m}_{\parallel}\cdot\boldsymbol{e}_x}{r^3}\right] = \frac{m_{\parallel}}{2\pi}\left(\frac{3x^2}{r^5} - \frac{1}{r^2}\right) = \frac{m_{\parallel}}{2\pi}\frac{2x^2 - h^2}{r^5}. \qquad (15)$$

It follows from this that the critical points of the field in the plane $y = 0$ are located at the points $x = \pm h/\sqrt{2}$ of the axis parallel to the direction of the magnetic moments. These are critical points of the hyperbolic type (saddle) S, through which the separatrixs pass – the field lines (shown by bold lines), which separate areas with different character of field formation. In Fig. 6,*b* for clarity, magnetic field lines are shown not only above the surface of the body, but also in the area $z < 0$, where the field is absent in the case of an ideal skin effect.

Figure 7 illustrates the dependence of the various components of the magnetic field strength on the coordinates at the plane and on the value of the small parameter ε_m, the difference from zero of which indicates the influence of the bounded value of the penetration depth of the non-uniform electromagnetic field. In all figures, the dependencies for the ideal skin effect $\varepsilon = 0$ are shown by bold curves.

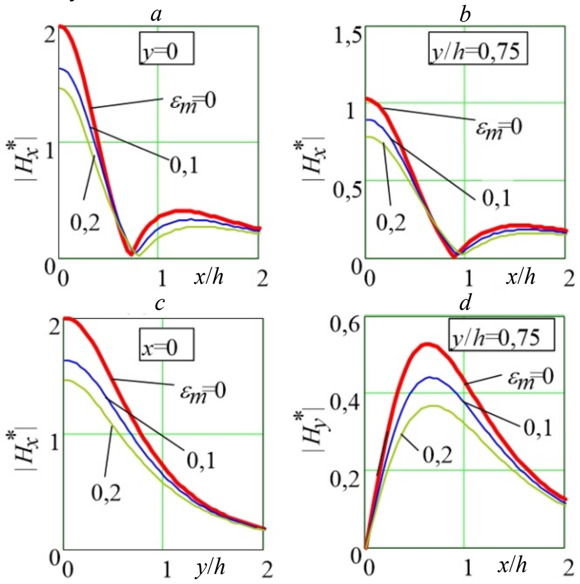


Fig. 7. Distribution of the tangential component of the magnetic field at the media interface for the source $\dot{\boldsymbol{m}}_{\parallel}$

As can be seen from Fig. 6,*b*, at $\varepsilon_m = 0$, the tangential component of the magnetic field strength at the surface of the body changes direction when passing through critical points S. This feature of the field distribution is also shown in Fig. 7,*a*. When $\varepsilon_m > 0$, when the eddy currents no longer flow along the surface, but occupy a certain layer of finite thickness, a general tendency to decrease the magnetic field is observed. Here, the position of the critical hyperbolic point practically does not change.

When moving away from the plane $y = 0$, the longitudinal (parallel to the direction of the magnetic moment) component of the magnetic field strength decreases (Fig. 7,*c*). The zero value of this component still exists. However, not all components of the field strength are equal to zero at the corresponding points of the surface when $y \neq 0$. The tangential strength component perpendicular to the x axis will be different from zero (Fig. 7,*d*). This feature is illustrated in Fig. 8.

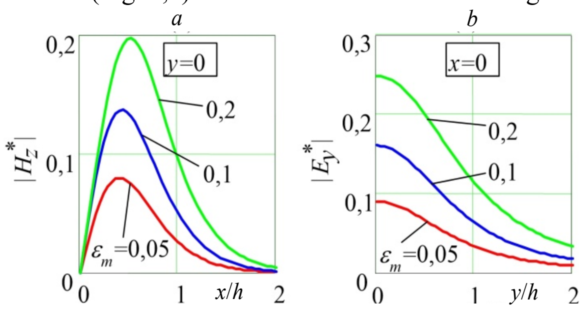


Fig. 8. Distribution over the surface of the modules of the normal component of the magnetic field (*a*) and the tangential component of the electric field (*b*) for the source $\boldsymbol{m}_{\parallel}$

The normal component of the magnetic field strength (Fig. 8,*a*) remains insignificant compared to the maximum value of the tangential component of the magnetic field (Fig. 7,*a*). But in the area near critical points of the field, the normal component becomes dominant.

The electric field strength under the external field of the horizontal magnetic moment (Fig. 8,*b*) is comparable in value to the electric field created by the action of the magnetic moment oriented normal to the surface (Fig. 5).

Conclusions. From the presented results for both the normally oriented magnetic moment and the moment directed parallel to the media interface, it follows that mathematical models with ideal skin effect at $\delta\rightarrow 0$ have a limited scope of application. In the case of non-uniform field of external sources, when the field penetration depth is commensurate with the distance between the source and conducting body, it is necessary to use more correct mathematical models for electromagnetic field. Analytical approaches using the expansion of the field into asymptotic series based on the introduced small parameter ε are convenient way of describing the electromagnetic field.

The specific expressions found for the electromagnetic field at the surface between the dielectric and conducting half-space under the action of arbitrarily oriented magnetic moment appear to be more general than the use of closed contours with alternating current, since they expand the types of considered external field sources and allow the use of the superposition method instead of integration over the whole contour.

In cases that allow the use of the conducting half-space model with strong skin effect, specific expressions for the field at the surface are found, which are completely determined by the known field of external sources (in this case, the field of magnetic moments). This significantly simplifies the solution of the corresponding problems, since there is no need to separately solve the field equations.

Further development of research can be aimed at determining the field under the action of other types of sources of a non-uniform external field, finding the impedance boundary condition for such fields, and finally, as a general program, spreading the applied approach to systems with curvilinear surfaces of media interface.

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