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# Refined calculation of energy modes of a frequency-regulated induction motor

***Purpose.*** *To obtain analytical dependencies for the precise calculation of the stator current of a frequency-regulated three-phase short-circuited induction motor and to estimate the components of its main electrical power losses, which are spent on the transportation magnetic power losses (to the magnetization circuit) and additional power losses (through the motor air gap), as well as with using the obtained refined dependencies to research the electromagnetic processes and energy modes of the frequency-regulated induction motor when its speed and load change.* ***Methodology.*** *The method of generalized vectors is used for the refined calculation of the electromagnetic processes and energy modes of the frequency-regulated induction motor.* ***Results.*** *Based on the catalog data and parameters of the induction motor's equivalent replacement circuit, also the specified values of its useful rotational torque and speed, refined analytical dependencies were obtained for the calculation of the main electromagnetic power losses of the frequency-regulated induction motor, which take into account the influence of all types of power losses, which present in it, as well as – power losses spent on transporting magnetic losses (to the magnetization circuit) and additional losses (through the air gap of the motor). With the help of the obtained dependencies, the energy modes (including main power consumption and electromagnetic power losses, efficiency factor, power factor) of the frequency-regulated induction motor in the driving and generator modes of its operation in relation to the first (at speeds not higher than the nominal) and the second (at speeds above the nominal) speed control zones for the operating ranges of the motor useful rotational torque and speed changes were calculated.* ***Originality.*** *A refined analytical calculation dependence has been obtained for determining the active projection of the generalized stator current vector of a frequency-regulated induction motor, which takes into account the presence of additional power losses and the component of electrical losses caused by the transportation of additional power losses through the air gap of the motor; an analytical dependence is also proposed for determining the increment of the mentioned active projection, which is due to the transportation of magnetic power losses to the motor magnetization circuit.* ***Practical value.*** *Analytical calculation dependencies are proposed for the quantitative assessment of errors (as a percentage of mentioned values) in steady-state modes for determining the main electromagnetic power losses of the frequency-regulated induction motor, caused by the absence (in comparison with relevant studies from known publications) of taking into account additional and magnetic power losses, as well as – the influence of electrical component losses caused by the transportation of the mentioned power losses through the air gap or to the magnetization circuit of the motor, respectively.* References 16, tables 5, figures 2.

*Key words:* **induction motor, frequency regulation, electromagnetic power losses, steady-state energy modes.**

***Мета.*** *Отримати аналітичні залежності для уточненого розрахунку статорного струму частотно-регульованого трифазного короткозамкненого асинхронного двигуна і оцінити складові його основних електричних втрат потужності, котрі викликані транспортуванням магнітних втрат потужності (до контуру намагнічування) і додаткових втрат потужності (через повітряний проміжок двигуна), а також дослідити з використанням отриманих уточнених залежностей усталені електромагнітні процеси й енергетичні режими частотно-регульованого асинхронного двигуна при зміні його швидкості та навантаження.* ***Методологія.*** *При уточненому розрахунку електромагнітних процесів та енергетичних режимів частотно-регульованого асинхронного двигуна застосовано метод узагальнених векторів.* ***Результати.*** *Виходячи з каталожних даних та параметрів еквівалентної схеми заміщення асинхронного двигуна, а також заданих значень його корисного обертового моменту та швидкості, отримані для розрахунку основних електромагнітних втрат потужності частотно-регульованого асинхронного двигуна уточнені аналітичні залежності, в яких враховується вплив усіх видів присутніх у ньому втрат потужності, а також – втрат потужності, що викликані транспортуванням магнітних втрат (до контуру намагнічування) та додаткових втрат (через повітряний проміжок двигуна). За допомогою отриманих залежностей були розраховані енергетичні режими (у тому числі – основні споживана потужність та електромагнітні втрати потужності, коефіцієнт корисної дії, коефіцієнт потужності) частотно-регульованого асинхронного двигуна при двигуневому та генераторному режимах його роботи стосовно першої (при швидкостях не вище номінальної) і другої (при швидкостях вище номінальної) зон регулювання швидкості для робочих діапазонів зміни корисного обертового моменту і швидкості двигуна.* ***Наукова новизна.*** *Отримано уточнену аналітичну розрахункову залежність для визначення активної проекції узагальненого вектора статорного струму частотно-регульованого асинхронного двигуна, в котрої враховується наявність в ньому додаткових втрат потужності та складової електричних втрат, яка викликана транспортуванням додаткових втрат потужності через повітряний проміжок двигуна; також запропонована аналітична залежність для визначення прирощення вказаної активної проекції, яке обумовлено транспортуванням магнітних втрат потужності до контуру намагнічування двигуна.* ***Практична цінність.*** *Запропоновано аналітичні розрахункові залежності для кількісної оцінки в усталених режимах похибок (у відсотках від уточнених значень) щодо визначення основних електромагнітних втрат потужності частотно-регульованого асинхронного двигуна, обумовлених при розрахунку цих втрат відсутністю (в порівнянні з відповідними дослідженнями з відомих публікацій) урахування додаткових і магнітних втрат потужності, а також – впливу електричних складових втрат, викликаних транспортуванням згаданих втрат потужності через повітряний проміжок або до контуру намагнічування двигуна відповідно.* Бібл. 16, табл. 5, рис. 2.

*Ключові слова:* **асинхронний двигун, частотне регулювання, електромагнітні втрати потужності, усталені енергетичні режими**.

**Introduction.** As a result of the widespread industrial implementation of frequency-regulated (f-r) short-circuited induction motors (IMs) in various branches of the economy today and due to the observed increase in the price of electric energy, the task of precise definition has become very relevant and practically in demand (including with the use of modern powerful computing means) of instantaneous energy indicators (power losses and their components, consumed power, efficiency) for the above motors in the operating ranges of changes in their speed and load. Only by solving this task will it become possible to develop and further implement energy-efficient control (which ensures the minimization of power losses or energy consumption) for the specified motors.

Despite the existing considerable number of well-known publications devoted to the calculation and research of the energy modes of the f-r IMs, all of them do not fully take into account the presence of all components of power losses, which are actually present in a real induction motor. In particular, in the monographs [1, 2], an idealized representation of the f-r IM is considered, in which magnetic and additional motor power losses are not taken into account at all. In monographs [3–7] and well-known articles [8, 9], only additional motor power losses are not taken into account when calculating and researching energy modes of the IM; moreover, when assessing the impact of magnetic power losses, the component of electrical losses caused by them is also taken into account, which is spent on transporting magnetic power losses to the magnetization circuit of the motor. In contrast to the publications listed above, in the article [10], which investigates the energy modes of the f-r IM in start-braking modes, at the same time, all existing (including magnetic and additional) motor power losses are taken into account in the calculations; however, this does not take into account components of electrical losses spent on transporting magnetic and additional power losses to the magnetization circuit or through the motor air gap.

In contrast to all previous publications, in the article [11], devoted to the study of the energy modes of the f-r IM during acceleration and braking, all types of power losses present in it (including magnetic and additional) are simultaneously taken into account, as well as the component of electrical losses is taken into account which is spent on transporting magnetic power losses to the magnetization circuit; however, this does not take into account the component of electrical losses spent on transporting additional power losses through the air gap of the motor.

Known publications [12, 13] present calculation analytical dependencies for determining additional IM power losses, which in the nominal mode of operation of the motor make up 0.5 % of its input power consumption, and in modes different from the nominal, are characterized by their change in proportion to the second degree of module of the generalized motor stator current vector.

According to the monograph [4], additional IM power losses are created by the joint action of two components: «no load stray losses» and «load stray losses». Known methods discussed in [14, 15] are used for the experimental determination of these components. Permissible values of additional power losses for induction motors are set by International Standards: IEEE112 – for the USA; IEC34-2 – for Europe and IEC37 – for Japan [16].

It follows from the conducted analysis that at present there are no refined analytical dependencies for the calculation and study of the energy modes of the f-r IMs, which would simultaneously take into account all types of power losses (electrical in the stator and rotor windings, mechanical, magnetic and additional) of the motor and all components of its electrical losses in the stator winding (which are spent on the creation of electromagnetic torque and motor power and the aforementioned transportation of magnetic and additional motor power losses). Also, currently there are no analytical dependencies for the refined calculation of the stator current, which take into account the influence of these components of the stator current, which are spent on transporting magnetic and additional power losses to the magnetization circuit or through the air gap of the IM.

**The goal of the article.** To obtain analytical dependencies for the refined calculation of the stator current of a frequency-regulated three-phase short-circuited induction motor and to estimate the components of its electrical power losses, which are spent on the transportation of magnetic power losses (to the magnetization circuit) and additional power losses (through the air gap of the motor), as well as to investigate with using the obtained refined dependencies, the steady-state electromagnetic processes and energy modes of the frequency-regulated induction motor with changes in its speed and load.

**Obtained results.** During the calculations and studies, the assumption was made:

a) the three-phase stator winding of the motor is symmetrical;

b) the air gap of the motor is the same along the entire inner circuit of the stator;

c) the magnetization curve of steel is linear (that is, the value of the internal inductances of the motor does not depend on the currents);

d) the internal parameters of the induction motor (inductances and active resistances brought to its calculated operating temperature [12]) were considered unchanged;

e) the main (caused by the first harmonic components of the phase stator currents) power losses of the f-r IM are taken into account, which include the following loss components [12]:

- electrical $\Delta P_{e.s}$ in the three-phase stator winding;
- electrical $\Delta P_{e.r}$ in short-circuited rotor winding;
- magnetic $\Delta P_{i.r}$ (in the steel of the stator and rotor core);
- mechanical $\Delta P_{ad}$ (spent on friction in bearings and self-ventilation);
- additional $\Delta P_{ad}$ (caused by pulsations of magnetic flux density in the teeth of the stator and rotor).

In order to further simplify the analytical dependencies describing the electromechanical and energy parameters of the f-r IM mode, we will use the method of generalized vectors [1] and the relative system of units generally accepted for AC electric machines [4, 10, 11].

Mentioned electrical (in stator $\Delta P_{e.s}$ and rotor $\Delta P_{e.r}$ windings), magnetic $\Delta P_{i.r}$, mechanical $\Delta P_{mech}$ and additional $\Delta P_{ad}$ types of main power losses, as well as main electromagnetic $\Delta P_{em}$ and total $\Delta P_{m}$ IM power losses (which will be called simply «losses» below) are determined from known analytical dependencies [1, 10, 11]:

$$\begin{cases} \Delta P_{e.s} = R_s \cdot I_s^2, \quad \Delta P_{e.r} = \beta \cdot T_{em} = k_r^2 \cdot R_r \cdot I_{1\Sigma y}^2, \\ \Delta P_{ir} = \Delta P_{ir.n} \cdot \left(\Psi_m / \Psi_{mn}\right)^2 \cdot \left(\omega_1 / \omega_{1n}\right)^{\lambda}, \\ \Delta P_{ad} \approx \Delta P_{ad.n} \cdot \left(I_s / I_{sn}\right)^2 = R_{ad} \cdot I_s^2, \quad \beta = \omega_1 - \omega, \\ \Delta P_{ad.n} = \dfrac{0.005 \cdot P_{2n}}{\eta_n}, \quad R_{ad} = \dfrac{\Delta P_{ad.n}}{I_{sn}^2}, \\ \Delta P_{mech} = \Delta P_{mech.n} \cdot \left(\omega / \omega_n\right)^2, \\ \Delta P_{em} = \Delta P_{e.s} + \Delta P_{e.r} + \Delta P_{ir} + \Delta P_{ad}, \\ \Delta P_m = \Delta P_{em} + \Delta P_{mech}, \quad k_r = L_m / \left(L_m + L_{\sigma r}\right), \end{cases} \tag{1}$$

where $R_s$ is the active resistance of the IM phase stator winding; $R_r$ is the equivalent (reduced to the three-phase winding) phase active resistance of the short-circuited rotor winding of the motor; $I_s$ and $I_{sn}$ are, respectively, the instantaneous and nominal value of the module of the generalized vector of the IM stator current (and in the adopted relative system of units: $I_{sn}$ = 1 pu [4]); $k_r$ is the coupling factor of the rotor [1]; $L_m$ and $L_{\sigma r}$ are the magnetization inductance and dissipation inductance of the IM rotor, respectively; $R_{ad}$ is the equivalent active resistance to take into account additional motor losses [10, 11]; $P_{2n}$ and $\eta_n$ are the nominal values of the useful power on the motor shaft and its efficiency, respectively; $\omega_1$ and $\omega_{1n}$ are the current and nominal value of the angular frequency of rotation of the IM stator magnetic field, respectively (and, in the adopted relative system of units: $\omega_{1n}$ = 1 pu [10, 11]); $\omega$ and $\omega_n$ are the current and nominal value of the angular frequency of rotation (speed) of the rotor of the motor, respectively; $\Delta P_{ir.n}$, $\Delta P_{ad.n}$ and $\Delta P_{mech.n}$ are the nominal values (inherent in the nominal operating mode of the IM) of magnetic, additional and mechanical losses of the motor, respectively; $I_{1\Sigma y}$ is the component of the active projection $I_{sy}$ of the generalized stator current vector $\boldsymbol{I}_s$ (on the imaginary axis «$y$» of the rotational orthogonal coordinate system (ROCS) «$x$-$y$», connected by the real axis «$x$» to the generalized rotor flux linkage vector $\boldsymbol{\Psi}_r$ of the motor), which creates the electromagnetic torque and electromagnetic power in the air gap of the f-r IM; $\Psi_m$ and $\Psi_{mn}$ are, respectively, the current and nominal value of the module of the generalized vector of the magnetic (in the air gap) flux $\boldsymbol{\Phi}_m$ of the IM; $\lambda$ is the coefficient that takes into account the changes in magnetic losses $\Delta P_{ir}$ of the f-r IM from the angular frequency $\omega_1$ of the stator magnetic field (for general industrial IMs: $\lambda$ = 1.3 [10–12]); $T_{em}$ and $\beta$ are the current values of the electromagnetic torque (created by the motor in the air gap) and the absolute slip of the f-r IM, respectively.

The instantaneous value of the module $\Phi_m$ of the generalized magnetic flux vector is determined from known dependencies [10]:

$$\begin{cases} \Phi_m = \sqrt{\Phi_{mx}^2 + \Phi_{my}^2}, \\ \Phi_{mx} = k_r \cdot \left(\Psi_r + L_{\sigma r} I_{sx}\right), \;\; \Phi_{my} = k_r \cdot L_{\sigma r} I_{1\Sigma y} \end{cases}, \quad (2)$$

where $\Phi_{mx}$ and $\Phi_{my}$ are the projections of the generalized magnetic flux vector $\boldsymbol{\Phi}_m$ on the «$x$-$y$» axis of the ROCS, oriented with its real «$x$» axis along the generalized vector of the rotor flux linkage $\boldsymbol{\Psi}_r$; $\Psi_r$ is the current value of the module of the generalized flux linkage vector of the rotor $\boldsymbol{\Psi}_r$ of the motor; $L_{\sigma r}$ is the dissipation inductance of the IM rotor.

The energy balance of the f-r IM is characterized by the following known dependencies [12]:

$$\begin{cases} P_1 = P_{e.s} + \Delta P_{ir} + P_{em}; \\ P_{em} = \Delta P_{e.r} + P_{mech}; \\ P_{mech} = \Delta P_{mech} + \Delta P_{ad} + P_2, \\ \Delta P_{e.r} = P_{em} - P_{mech}, \end{cases} \quad (3)$$

where: $P_1$ and $P_{em}$ are the main consumed and electromagnetic (transmitted through the IM air gap) power of the motor, respectively; $P_{mech}$ and $P_2$ are the main mechanical and useful shaft power of the motor, respectively.

The relationship between energy values and torques created in the f-r IM is described by well-known relations [12]:

$$\begin{cases} P_{mech} = \omega \cdot T_{em} = \omega \cdot \left(T + \Delta T_{mech}\right) + \Delta P_{ad}; \\ P_2 = \omega \cdot T; \quad P_{em} = \omega_1 \cdot T_{em} = P_1 - \Delta P_{e.s} - \Delta P_{ir}; \\ \omega_1 T_{em} = \Delta P_{e.r} + \omega T_{em}; \;\; \Delta T_{mech.n} = \Delta P_{mech.n} / \omega_n; \\ \Delta T_{mech} = \Delta P_{mech} / \omega = \Delta T_{mech.n} \cdot \left(\omega / \omega_n\right), \end{cases} \quad (4)$$

where $T$ is the useful torque of the f-r IM transmitted to the working mechanism (equal to the static torque $T_s$ of the drive applied to the motor shaft in steady-state conditions); $\Delta T_{mech}$ and $\Delta T_{mech.n}$ are, respectively, the instantaneous and nominal value of the mechanical losses of the IM torque; ($T + \Delta T_{mech.n}$) is the motor torque.

Speed $n_1$[rpm] and angular frequency of rotation $\omega_1$[pu] of the magnetic field of the stator, speed $n$[rpm] and angular frequency of rotation $\omega$[pu] of the rotor, absolute slip $\beta$[pu] of the f-r IM are found from known expressions [12]:

$$\begin{cases} n_1[\text{rpm}] = \dfrac{60 \cdot f_1}{z_p}; \; n_{1n} = \dfrac{60 \cdot f_{1n}}{z_p}; \; \omega_1 = \dfrac{n_1}{n_{1n}}; \\ \omega = \dfrac{n}{n_{1n}}; \; \Delta n\,[\text{rpm}] = n_{1n} - n; \; \beta = \dfrac{\Delta n}{n_{1n}} = \omega_1 - \omega, \end{cases} \quad (5)$$

where $f_1$ and $f_{1n}$ = 50 Hz are, respectively, the instantaneous and nominal value of the frequency of the main harmonics of the phase stator voltages (or currents) of the f-r IM; $z_p$ is the number of motor pole pairs.

According to [1, 2], the instantaneous value of the torque ($T + \Delta T_{mech}$) of the idealized f-r IM (which does not take into account magnetic and additional power losses) is determined as:

$$T + \Delta T_{mech} = \Phi_m \cdot I_{1q} \;\text{ or }\; T + \Delta T_{mech} = k_r \Psi_r I_{1y}, \quad (6)$$

where $\Phi_m$ and $\Psi_r$ are the modules of the generalized vectors of the magnetic (in the air gap) flux $\boldsymbol{\Psi}_m$ and flux linkage of the motor rotor $\boldsymbol{\Psi}_r$, respectively; $I_{1q}$ is the active (torque-generating) projection of the generalized vector of the stator current $\boldsymbol{I}_1$ of the f-r IM on the imaginary axis «$q$» of the ROCS «$d$-$q$», which is oriented with the real axis «$d$» in the direction of the generalized vector of the magnetic flux $\boldsymbol{\Phi}_m$; $I_{1y}$ is the active (torque-generating) projection of the generalized vector of the stator current $\boldsymbol{I}_1$ of the idealized IM on the imaginary axis «$y$» of the ROCS «$x$-$y$», which is oriented with the real axis «$x$» in the direction of the generalized vector of the rotor flux linkage $\boldsymbol{\Psi}_r$.

At the *first stage* of research for a real (non-idealized) f-r IM through the magnetizing $I_{sx}$ and active projections $I_{sy}$, $I_{1\Sigma y}$, $I_{1y}$ of the generalized vectors $\boldsymbol{I}_s$, $\boldsymbol{I}_{1\Sigma}$, $\boldsymbol{I}_{1y}$ of the stator current (on the «$x$» and «$y$» axis of the ROCS « $x$-$y$», which is oriented with the real axis «$x$» along the generalized rotor flux linkage vector $\boldsymbol{\Psi}_r$) we give the calculation relationships for the modules $I_s$, $I_{1\Sigma}$, $I_{1y}$ of these vectors in the form:

$$\begin{cases} I_s = \sqrt{I_{sx}^2 + I_{sy}^2}; \quad I_{1\Sigma} = \sqrt{I_{sx}^2 + I_{1\Sigma y}^2}; \\ I_{1y} = \sqrt{I_{sx}^2 + I_{1y}^2}; \quad I_1 = I_{sx} + \mathrm{j} I_{1y}; \\ I_{1\Sigma} = I_{sx} + \mathrm{j} I_{1\Sigma y}; \quad I_s = I_{sx} + \mathrm{j} I_{sy}; \\ I_{sy} = I_{1\Sigma y} + \Delta I_{sy}; \quad I_{1\Sigma y} = I_{1y} + \Delta I_{1y}. \end{cases} \quad (7)$$

In (7), the generalized vectors $\boldsymbol{I}_{1\Sigma}$ and $\boldsymbol{I}_1$ are mathematically formed from the generalized vector $\boldsymbol{I}_s$ of the stator current of the real f-r IM, if the active projection $I_{sy}$ of this vector does not take into account the increment $\Delta I_{sy}$ or simultaneously two increments $\Delta I_{sy}$ and $\Delta I_{1y}$, respectively; the value of the active projection $I_{1y}$ is found from the second relation in (6) through the value of the torque $T + \Delta T_{mech}$ and the module $\Psi_r$ of the generalized flux linkage vector of the motor rotor. The indicated increments $\Delta I_{sy}$ and $\Delta I_{1y}$ are caused by the transportation (transmission) of magnetic $\Delta P_{ir}$ and additional $\Delta P_{ad}$ losses to the magnetization circuit or through the air gap of this motor, respectively. Figure 1 shows a diagram illustrating the distribution, according to the relations in (7), of the increments $\Delta I_{sy}$, $\Delta I_{1y}$ of the stator current between the active projections $I_{sy}$, $I_{1\Sigma y}$ and $\Delta I_{1y}$ of the generalized vectors $\boldsymbol{I}_s$, $\boldsymbol{I}_{1\Sigma}$ and $\boldsymbol{I}_1$ of the f-r IM, respectively.

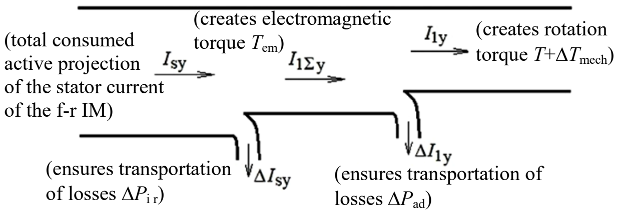



Fig. 1. Diagram of the distribution of increments $\Delta I_{sy}$ and $\Delta I_{1y}$ of the stator current between the active projections $I_{sy}$, $I_{1\Sigma y}$, $I_{1y}$ of the generalized vectors $\boldsymbol{I}_s$, $\boldsymbol{I}_{1\Sigma}$, $\boldsymbol{I}_1$ in the real f-r IM (the purpose of the projections and their increments is indicated in parentheses)

Based on the second and last relations in (1), we obtain, similarly to the second relation from (6), the calculation expressions (through the active projection $I_{1\Sigma y}$ of the generalized stator current vector $\boldsymbol{I}_{1\Sigma}$, the module $\Psi_r$ of the generalized rotor flux linkage vector $\boldsymbol{\Psi}_r$ and the active resistance $R_r$ of the rotor winding) to find the value of the electromagnetic torque $T_{em}$ created in the air gap of the f-r IM

$$T_{em} = k_r \Psi_r I_{1\Sigma\,y}, \quad (8)$$

as well as the absolute slip $\beta$ and the angular frequency of rotation $\omega_1$ of the stator magnetic field:

$$\beta = \Delta P_{e.r} / T_{em} = k_r R_r I_{1\Sigma y} \text{ and } \omega_1 = \omega + \beta . \quad (9)$$

At the *second stage*, we obtain an analytical dependence for determining the active projection $I_{1\Sigma y}$ of the generalized stator current vector $\boldsymbol{I}_{1\Sigma}$ of the f-r IM. Before that, we recall that the increment $\Delta I_{sy}$, which is part of the active projection $I_{sy}$ of the generalized stator current vector $\boldsymbol{I}_s$, is caused by the transport of magnetic power losses to the magnetization circuit of the motor (where these losses are dissipated in the stator core [4, 12]). As a result, the specified increment $\Delta I_{sy}$ of the stator current does not physically affect, according to (4), the electromagnetic power $P_{em}$ transmitted through the IM air gap (and, therefore, it also does not affect the mechanical power $P_{mech}$ and the additional losses $\Delta P_{ad}$ included in it).

Therefore, in order to increase the accuracy of the determination of the additional losses of the f-r IM, we will below calculate them (in contrast to the known calculation relationship from (1) for them [12, 13]) in a different way, excluding the mentioned increment $\Delta I_{sy}$ from the composition of the module $I_s$ of the stator current. Namely, it is proportional to the second degree of the module $I_{1\Sigma}$ of the generalized vector $\boldsymbol{I}_{1\Sigma}$ of the stator current from the following expressions:

$$\begin{cases} \Delta P_{ad} = R^*_{ad} \cdot I^2_{1\Sigma} = R^*_{ad} \cdot \left( I^2_{sx} + I^2_{1\Sigma\,y} \right); \\ R^*_{ad} = \dfrac{\Delta P_{ad.n}}{I^2_{sx.n} + I^2_{1\Sigma\,y.n}} = \dfrac{\Delta P_{ad.n}}{I^2_{1\Sigma.n}} = R_{ad} \cdot \left( \dfrac{I_{s.n}}{I_{1\Sigma.n}} \right)^2 \end{cases} \quad (10)$$

through the value of the specified module $I_{1\Sigma}$, magnetizing $I_{sx}$ and active $I_{1\Sigma y}$ projections of the generalized vector $\boldsymbol{I}_{1\Sigma}$ on the ROCS «*x-y*» axis. In (10), $R^*_{ad}$ is the refined value of the equivalent (intended for the calculation of additional losses) active resistance of the motor, which is calculated from the nominal (i.e., corresponding to the nominal IM mode) values of additional losses $\Delta P_{ad.n}$, as well as of the module $I_{1\Sigma.n}$, magnetizing $I_{sx.n}$ and active projection of the generalized $I_{1\Sigma y.n}$ vector of the stator current $\boldsymbol{I}_{1\Sigma.n}$ of the motor.

Substituting the expression for the electromagnetic torque $T_{em}$ from (8) and the expression (10) for additional losses into the first relation from (4), we convert this relation to the following dependence:

$$\omega \cdot \left( k_r \Psi_r I_{1\Sigma\,y} \right) = \omega \cdot \left( T + \Delta T_{mech} \right) + R^*_{ad} \cdot \left( I^2_{sx} + I^2_{1\Sigma\,y} \right), \quad (11)$$

which, in turn, we reduce by equivalent transformations to the form of an algebraic equation of the second order:

$$R^*_{ad} I^2_{1\Sigma\,y} - \omega k_r \Psi_r I_{1\Sigma\,y} + \omega \cdot \left( T + \Delta T_{mech} \right) + R^*_{ad} I^2_{sx} = 0 . \quad (12)$$

The solution of (12) is the following analytical dependence:

$$I_{1\Sigma\,y} = \left( k_r \Psi_r / 2R^*_{ad} \right) \cdot \left\{ \omega - \left( \omega^2 - \left( \frac{2R^*_{ad}}{k_r \Psi_r} \right)^2 \cdot \left[ \frac{\omega \cdot \left( T + \Delta T_{mech} \right)}{R^*_{ad}} + I^2_{sx} \right] \right)^{0.5} \right\}. \quad (13)$$

It allows to determine the value of the active projection $I_{1\Sigma y}$ of the generalized vector of the stator current of the f-r IM through the instantaneous values of the parameters of the motor mode: the speed $\omega$ of the rotor, the torque ($T+\Delta T_{mech}$), the module $\Psi_r$ of the generalized vector of the flux linkage of the rotor and the magnetizing projection $I_{sx}$ of the generalized stator current vector.

We determined from the second relation from (6) the active projection $I_{1y}$ of the stator current and based on the obtained dependence (13) and the last relation from (7), we find for the real IM the increment $\Delta I_{1y}$ as part of the active projection $\Delta I_{1\Sigma y}$ of the generalized stator current vector $\boldsymbol{I}_{1\Sigma}$ from the expression:

$$\Delta I_{1y} = I_{1\Sigma\,y} - I_{1y}, \quad (14)$$

caused by the transport of additional power losses $\Delta P_{ad}$ through the motor air gap.

Thus, a refined analytical calculation dependence (13) was obtained for determining the active projection $I_{1\Sigma y}$ of the generalized stator current vector $\boldsymbol{I}_{1\Sigma}$ of the f-r IM and expressions in the form of: formula (14) and the last relation from formula (7), – for finding the increment $\Delta I_1$ of the active projection $I_{1y}$ of the stator current. With the help of this increment, the effect of the electrical component of losses, which is caused by the transportation of additional power losses through the air gap of the motor, is taken into account.

At the *third stage*, we obtain an analytical calculation dependence for determining the increment $\Delta I_{sy}$, which is caused by the transport of magnetic losses $\Delta P_{ad}$ to the magnetization circuit of the motor and is part of the active projection $I_{sy}$ of the generalized vector of the stator current $\boldsymbol{I}_s$ of the real f-r IM.

From [4], the classical method of determining the specified increment $\Delta I_{sq}$ of the active projection $I_{sq}$ (on the «*q*» axis of the «*d-q*» ROCS, which is oriented with the real axis «*d*» along the generalized magnetic flux vector $\boldsymbol{\Phi}_m$ of the IM) is known, in which this projection is founded based on of the energy balance between the additional electric power supplied to the magnetizing circuit of the motor and equal to the product $E_m \cdot \Delta I_{sq}$, and the magnetic losses $\Delta P_{i\,r}$, which are dissipated in the stator core:

$$E_m \cdot \Delta I_{sq} = \Delta P_{i\,r}, \quad (15)$$

where $E_m$ is the instantaneous value of the module of the generalized magnetizing electromotive force vector $\boldsymbol{E}_m$.

From (15), the instantaneous increment $\Delta I_{sq}$ of the active projection $I_{sq}$ of the IM stator current is determined in the form:

$$\Delta I_{sq} = \Delta P_{i\,r} / E_m = \Delta P_{i\,r} / \omega_1 \Phi_m , \text{ where } E_m = \omega_1 \Phi_m . \quad (16)$$

In another (known from [11]) method, the aforementioned increment $\Delta I_{sq}$ of the stator current (caused by the transport of magnetic power losses to the magnetization circuit of the f-r IM) is proposed to be determined in the form of the increment $\Delta I_{sq}$ of the active projection $I_{sq}$ of the generalized motor stator current vector after being added to the motor shaft of the static fictitious torque $T_f$. According to the first relation in (6), the indicated increment $\Delta I_{sq}$ of the active projection of the stator current is founded in the form:

$$\Delta I_{sq} = T_f / \Phi_m. \quad (17)$$

After equating the increments $\Delta I_{sq}$ calculated from (16) and (17), we determine the required value of the fictitious torque $T_f$, taking into account the third relation in (1):

$$T_f = \Delta P_{i\,r} / \omega_1 = \Delta P_{i\,r.n} \cdot \left( \frac{\Phi_m}{\Phi_{m.n}} \right)^2 \cdot \frac{\omega_1^{\lambda-1}}{\omega_{1n}^{\lambda}}, \quad (18)$$

in which, in the second method, the value of the increment $\Delta I_{sq}$ of the active projection of the stator current is calculated, which is completely identical to the first (classical) method.

Let's pay attention to the well-known simplification of vector automatic control systems and calculation of energy modes of the f-r IMs when using for them ROCS «x-*y*», which is oriented with the real axis «*x*» along the generalized vector of flux linkage of the rotor [2, 3, 10]. Taking this into account, the second method (using the fictitious torque) is more appropriate in practice. With this method, increment $\Delta I_{sy}$ of the active projection $I_{sy}$ of the stator current (on the «*y*» axis of the «*x*-*y*» ROCS, which is connected by the real «*x*» axis to the direction of the generalized rotor flux linkage vector $\boldsymbol{\Psi}_r$) is determined after substituting expression (18) in the second relation from (6) in the form:

$$\Delta I_{sy} = \frac{T_f}{k_r \Psi_r} = \frac{\Delta P_{i\,r.n}}{k_r \Psi_r} \cdot \left( \frac{\Psi_m}{\Psi_{mn}} \right)^2 \cdot \frac{\omega_1^{\lambda-1}}{\omega_{1n}^{\lambda}}. \quad (19)$$

Thus, a theoretical justification of the refined analytical dependence (19) is given for determining the increment $\Delta I_{sy}$ caused by the transport of magnetic power losses to the motor magnetization circuit, which is part of the active projection $I_{sy}$ of the generalized stator current vector $\boldsymbol{I}_s$ of the f-r IM.

It should be noted that the very form of the previously obtained active projection $I_{1\Sigma y}$ of the stator current of the calculation dependence (13) indicates that in the range of very low values of the motor speeds $\omega$ it is possible to reach a negative value in the radical expression of this dependence. In particular, as shown by the researches of the AT250L4U2 motor, negative values in the subdued expression of dependence (13) occur at speeds: $\omega \leq (0.02 - 0.03)\omega_n$. This situation is explained, obviously, by the impossibility of applying in practice the known mathematical expression from (1) or the one proposed from the relation from (10) for calculating additional losses $\Delta P_{ad}$ of the f-r IM in the range of very low motor speeds. Therefore, in order to find a refined calculation dependence for the active projection $I_{1\Sigma y}$ of the stator current in this low speed range of the f-r IM, it is necessary to preliminary obtain (on the basis of additional experimental studies or recommendations of electrical machine designers) a refined analytical calculation dependence $\Delta P_{ad} = f\,(I_{sx}, I_{1\Sigma y}, \omega)$ for additional power losses of the f-r IM in relation to its low speed range.

At the *fourth stage*, setting the values of the speed $\omega$ and the useful torque $T$, from the previously obtained calculation dependencies, let's calculate for the f-r IM AT250L4U2 (which is characterized by the nominal parameters and basic values presented in Table 1) the electromagnetic and energy processes of this motor in the motor (at $T > 0$) and generator (at $T < 0$) steady-state modes for the first (when $n \leq n_n$) and the second (when $n > n_n$) speed zones. The results of these calculations are shown in Table 2, 3.

Table 1

Nominal parameters of the f-r IM AT250L4U2 and its basic values for the relative system of units

| I. Nominal parameters, dimensions | Value |
|---|---|
| Useful power, kW | 120 |
| Active linear stator voltage, V | 400 |
| Active phase stator current, A | 202.5 |
| Stator voltage frequency, Hz | 50 |
| Number of pole pairs | 2 |
| Slip, % | 1.5 |
| Maximum speed, rpm | 4000 |
| Efficiency, % | 94 |
| Power factor | 0.91 |
| Multiplicity of the maximum torque | 3.5 |
| Multiplicity of the starting torque | 3.5 |
| Connection of phase stator windings | Y |
| Electrical power losses in the stator winding, kW | 2.625 |
| Electrical power losses in the rotor winding, kW | 1.849 |
| Magnetic power losses, kW | 1.800 |
| Additional power losses, kW | 0.638 |
| Mechanical power losses, kW | 0.748 |
| Module $\Phi_m$ of the generalized magnetic flux vector, pu | 0.9562 |
| Modulus $\Psi_r$ of the generalized rotor flux linkage vector, pu | 0.9574 |
| Useful torque $T_n$, pu | 0.8684 |
| II. Parameters of the «T»-shaped substitution circuit | |
| Active resistance of the phase stator winding, pu | 0.01871 |
| Equivalent resistance of the phase rotor winding, pu | 0.01569 |
| Active resistance of the magnetization circuit, pu | 71.3 |
| Magnetization inductance, pu | 2.6421 |
| Dissipation inductance of the stator winding, pu | 0.06850 |
| Dissipation inductance of the rotor winding, pu | 0.07633 |
| III. Base values for the relative system of units: | |
| - for voltage, V | 326.6 |
| - for current, A | 286.4 |
| - for active and inductive resistance, Ω | 1.1404 |
| - for power, kW | 140.296 |
| - for torque, N·m | 893.15 |
| - for magnetic flux and flux linkage, Wb | 1.0396 |
| - for inductances, mH | 3.630 |
| - for angular frequency of rotation, rad/s | 157.08 |
| - for time, ms | 3.183 |

The reliability of the performed calculations of electromagnetic processes and energy modes is confirmed by comparing their values with the corresponding values calculated in the MATLAB software package (the relative deviation does not exceed 0.2 %).

From the analysis of the computational data given in Table 2, 3, it was found that in the motor mode of operation of the f-r IM at equal speeds $n$ and the same (by absolute value) useful torques $T$ of the motor, the values of the modules of the stator current $I_s$ and stator voltage $U_s$ of the motor exceed their corresponding values in the generator mode of operation.

Table 2

Results of the refined calculation in the first speed zone at steady-state motor (at $T > 0$ ) and generator (at $T < 0$) modes of electromagnetic and energy quantities for the f-r IM AT250L4U2 depending on the speed and relative useful torque $T/T_n$

| $n$ | $\frac{T}{T_n}$ | $\frac{I_s}{I_{sn}}$ | $\Delta n$ | $\omega_1$ | $\frac{\Psi_m}{\Psi_{mn}}$ | $\Delta P_{e.s}$ | $\Delta P_{e.r}$ | $\Delta P_{i\,r}$ | $\Delta P_{ad}$ | $\Delta P_{mech}$ | $\Delta P^*_{em}$ | $\frac{P_1}{P_{1n}}$ | $\eta$ | $\cos\varphi$ | $\frac{U_s}{U_{sn}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| rpm | – | – | rpm | pu | – | kW | kW | kW | kW | kW | kW | – | % | – | – |
| 1477.5 | 0.5 | 0.566 | 11.32 | 0.993 | 0.998 | 0.842 | 0.467 | 1.775 | 0.205 | 0.748 | 3.289 | 0.502 | 93.70 | 0.825 | 0.977 |
| | 1 | 1 | 22.55 | 1 | 1 | 2.625 | 1.853 | 1.800 | 0.638 | 0.748 | 6.916 | 1.000 | 94.00 | 0.910 | 1 |
| | 1.5 | 1.459 | 33.83 | 1.008 | 1.003 | 5.585 | 4.170 | 1.829 | 1.358 | 0.748 | 12.94 | 1.517 | 92.93 | 0.921 | 1.028 |
| | 2 | 1.986 | 45.17 | 1.015 | 1.008 | 9.739 | 7.431 | 1.865 | 2.368 | 0.748 | 21.40 | 2.054 | 91.55 | 0.915 | 1.061 |
| | –0.5 | 0.531 | –10.98 | 0.978 | 0.998 | 0.739 | 0.439 | 1.740 | 0.180 | 0.748 | 3.098 | –0.440 | 93.59 | –0.798 | 0.945 |
| | –1 | 0.954 | –22.05 | 0.970 | 1 | 2.387 | 1.771 | 1.730 | 0.581 | 0.748 | 6.468 | –0.884 | 93.99 | –0.900 | 0.937 |
| | –1.5 | 1.400 | –33.07 | 0.963 | 1.003 | 5.147 | 3.984 | 1.724 | 1.252 | 0.748 | 12.11 | –1.309 | 92.86 | –0.912 | 0.937 |
| | –2 | 1.852 | –44.04 | 0.956 | 1.008 | 9.004 | 7.066 | 1.722 | 2.190 | 0.748 | 19.98 | –1.718 | 91.36 | –0.904 | 0.933 |
| 1000 | 0.5 | 0.563 | 11.27 | 0.674 | 0.998 | 0.832 | 0.462 | 1.073 | 0.202 | 0.342 | 2.570 | 0.340 | 93.55 | 0.825 | 0.666 |
| | 1 | 0.998 | 22.53 | 0.682 | 1 | 2.613 | 1.849 | 1.093 | 0.636 | 0.342 | 6.192 | 0.687 | 92.67 | 0.911 | 0.687 |
| | 1.5 | 1.459 | 33.88 | 0.689 | 1.003 | 5.588 | 4.180 | 1.117 | 1.359 | 0.342 | 12.24 | 1.052 | 90.71 | 0.923 | 0.711 |
| | 2 | 1.930 | 45.30 | 0.697 | 1.008 | 9.781 | 7.476 | 1.144 | 2.379 | 0.342 | 20.78 | 1.437 | 88.55 | 0.917 | 0.739 |
| | –0.5 | 0.534 | –11.03 | 0.659 | 0.998 | 0.749 | 0.443 | 1.043 | 0.182 | 0.342 | 2.417 | –0.297 | 93.48 | –0.798 | 0.635 |
| | –1 | 0.956 | –22.07 | 0.652 | 1 | 2.399 | 1.774 | 1.032 | 0.583 | 0.342 | 5.788 | –0.589 | 92.59 | –0.898 | 0.624 |
| | –1.5 | 1.400 | –33.03 | 0.645 | 1.003 | 5.148 | 3.975 | 1.023 | 1.252 | 0.342 | 11.40 | –0.863 | 90.45 | –0.910 | 0.617 |
| | –2 | 1.849 | –43.92 | 0.637 | 1.007 | 8.972 | 7.028 | 1.017 | 2.182 | 0.342 | 19.20 | –1.120 | 88.04 | –0.901 | 0.612 |
| 500 | 0.5 | 0.561 | 11.27 | 0.341 | 0.998 | 0.827 | 0.463 | 0.442 | 0.201 | 0.086 | 1.933 | 0.174 | 91.19 | 0.829 | 0.341 |
| | 1 | 1.001 | 22.66 | 0.348 | 1 | 2.629 | 1.870 | 0.457 | 0.639 | 0.086 | 5.595 | 0.362 | 87.84 | 0.916 | 0.360 |
| | 1.5 | 1.471 | 34.21 | 0.356 | 1.003 | 5.676 | 4.263 | 0.473 | 1.380 | 0.086 | 11.79 | 0.570 | 83.75 | 0.928 | 0.380 |
| | 2 | 1.954 | 46.93 | 0.364 | 1.008 | 10.03 | 7.686 | 0.492 | 2.438 | 0.086 | 20.64 | 0.798 | 79.71 | 0.923 | 0.402 |
| | –0.5 | 0.536 | –11.03 | 0.326 | 0.998 | 0.754 | 0.443 | 0.417 | 0.183 | 0.086 | 1.797 | –0.145 | 91.01 | –0.793 | 0.310 |
| | –1 | 0.957 | –21.96 | 0.319 | 1 | 2.388 | 1.756 | 0.407 | 0.581 | 0.086 | 5.131 | –0.278 | 87.29 | –0.892 | 0.297 |
| | –1.5 | 1.391 | –32.74 | 0.312 | 1.003 | 5.078 | 3.905 | 0.397 | 1.235 | 0.086 | 10.62 | –0.394 | 82.53 | –0.901 | 0.286 |
| | –2 | 1.829 | –43.39 | 0.304 | 1.007 | 8.783 | 9.859 | 0.389 | 2.136 | 0.086 | 18.17 | –0.494 | 77.60 | –0.889 | 0.276 |
| 150 | 0.5 | 0.568 | 11.52 | 0.108 | 0.998 | 0.846 | 0.483 | 0.099 | 0.206 | 0.008 | 1.634 | 0.061 | 78.85 | 0.851 | 0.114 |
| | 1 | 1.033 | 23.53 | 0.116 | 1 | 2.801 | 2.017 | 0.109 | 0.681 | 0.008 | 5.608 | 0.139 | 68.48 | 0.932 | 0.132 |
| | 1.5 | 1.551 | 36.23 | 0.124 | 1.004 | 6.313 | 4.782 | 0.121 | 1.535 | 0.008 | 12.75 | 0.243 | 58.90 | 0.944 | 0.151 |
| | 2 | 2.110 | 49.76 | 0.133 | 1.010 | 11.69 | 9.020 | 0.134 | 2.842 | 0.008 | 23.68 | 0.376 | 50.71 | 0.941 | 0.142 |
| | –0.5 | 0.531 | –10.81 | 0.093 | 0.998 | 0.739 | 0.426 | 0.082 | 0.180 | 0.008 | 1.426 | –0.037 | 76.57 | –0.757 | 0.083 |
| | –1 | 0.930 | –21.27 | 0.086 | 1 | 2.268 | 1.648 | 0.074 | 0.552 | 0.008 | 4.542 | –0.060 | 62.72 | –0.849 | 0.069 |
| | –1.5 | 1.337 | –31.34 | 0.079 | 1.003 | 4.689 | 3.577 | 0.067 | 1.140 | 0.008 | 9.474 | –0.069 | 48.16 | –0.831 | 0.056 |
| | –2 | 1.736 | –41.05 | 0.726 | 1.006 | 7.909 | 6.138 | 0.060 | 1.923 | 0.008 | 16.03 | –0.065 | 34.21 | –0.754 | 0.045 |

Moreover, by using vector control [2, 3], constant values of the rotor flux linkage module $\Psi_r$ and the magnetizing projection $I_{sx}$ of the generalized stator current vector (equal to their nominal values $\Psi_{r.n} = 0.9574$ pu and $I_{sx.n} = 0.288$ pu) were set for the f-r IM in the first zone:

$$\Psi_r = \Psi_{r\,n} = \text{const}; \quad I_{sx} = I_{sx.n} = \text{const}, \qquad (20)$$

and in the second zone, the values of the module $\Psi_r$ the generalized rotor flux linkage vector and the magnetizing projection $I_{sx}$ of the generalized stator current vector changed inversely proportional to the angular frequency of rotation $\omega_1$ of the IM stator magnetic field [10]:

$$\Psi_r = \Psi_{r\,n} / \omega_1; \quad I_{sx} = I_{sx.n} / \omega_1. \qquad (21)$$

According to the results of the calculations for the steady-state motor mode (at $T > 0$), Fig. 2 presents constructed (relative to the considered IM AT250L4U2) graphical dependencies of the following energy quantities: $I_s/I_{sn}$, $U_s/U_{sn}$, $P_1/P_{1n}$, $\eta$, $\cos\varphi$, $\Delta P^*_{em}/\Delta P^*_{em.n}$ – functions of change in speed $n$ and relative torque $T/T_n$ of the f-r IM. Moreover, in the mentioned quantities, which are given in the form of fractions, the instantaneous value of the quantity is indicated in the numerator, and the value (indicated by the additional index «$n$») of this quantity corresponding to the nominal mode of operation of the motor is indicated in the denominator.

In steady-stable modes, the values of the efficiency $\eta$, the power factor $\cos\varphi$ and the module $U_s$ of the generalized vector of the stator voltage of the f-r IM were determined from known dependencies [1, 12]:

$$\begin{cases} U_{sx} = R_s \cdot I_{sx} - \omega_1 \cdot L_\sigma \cdot I_{sy}; \\ U_{sy} = R_s \cdot I_{sy} + \omega_1 \cdot L_\sigma \cdot I_{sx} + \omega_1 \cdot k_r \cdot \Psi_r; \\ U_s = \sqrt{U_{sx}^2 + U_{sy}^2}; \quad \cos\varphi = \cos(\Theta_U - \Theta_I); \\ \Theta_I = \text{sign}(I_{sx}) \cdot \arcsin\left(\frac{I_{sy}}{I_s}\right) + \frac{\pi}{2} \cdot [1 - \text{sign}(I_{sx})]; \\ \Theta_U = \text{sign}(U_{sx}) \cdot \arcsin\left(\frac{U_{sy}}{U_s}\right) + \frac{\pi}{2} \cdot [1 - \text{sign}(U_{sx})]; \\ L_\sigma = L_{\sigma s} + k_r \cdot L_{\sigma r}; \quad \eta = P_2 / P_1; \\ \text{sign}(I_{sx}) = \begin{cases} 1, & I_{sx} \ge 0; \\ -1, & I_{sx} < 0; \end{cases} \text{sign}(U_{sx}) = \begin{cases} 1, & U_{sx} \ge 0; \\ -1, & U_{sx} < 0, \end{cases} \end{cases} ,(22)$$

where $L_\sigma$ is the total dissipation inductance of the motor; sign( ) is the mathematical operation for determining the sign of an algebraic value shown in parentheses.

Based on the analysis of graphical dependencies in Fig. 2, let's compare the values shown on these graphs for the first and second speed control zones in relation to the f-r IM operating in motor mode:

Table 3

Results of the refined calculation in the second speed zone at steady-state motor (at $T > 0$ ) and generator (at $T < 0$) modes of electromagnetic and energy quantities for the f-r IM AT250L4U2 depending on the speed $n$ and relative useful torque $T/T_n$

| $n$ | $\frac{T}{T_n}$ | $\frac{I_s}{I_{sn}}$ | $\Delta n$ | $\omega_1$ | $\Psi_r$ | $\frac{\Psi_m}{\Psi_{mn}}$ | $\Delta P_{e.s}$ | $\Delta P_{e.r}$ | $\Delta P_{i\,r}$ | $\Delta P_{ad}$ | $\Delta P_{mech}$ | $\Delta P^*_{em}$ | $\frac{P_1}{P_{1n}}$ | $\eta$ | $\cos\varphi$ | $\frac{U_s}{U_{sn}}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| rpm | – | – | rpm | pu | pu | – | kW | kW | kW | kW | kW | kW | – | % | – | – |
| 2000 | 0.5 | 0.687 | 20.65 | 1.347 | 0.711 | 0.742 | 1.238 | 0.856 | 1.459 | 0.301 | 1.371 | 3.854 | 0.677 | 93.96 | 0.904 | 0.993 |
| | 1 | 1.318 | 41.95 | 1.361 | 0.703 | 0.739 | 4.560 | 3.459 | 1.469 | 1.109 | 1.371 | 10.60 | 1.366 | 93.14 | 0.916 | 1.030 |
| | 1.5 | 1.980 | 64.35 | 1.376 | 0.696 | 0.740 | 10.29 | 7.964 | 1.494 | 2.501 | 1.371 | 22.24 | 2.094 | 91.16 | 0.888 | 1.084 |
| | 2 | 2.665 | 87.96 | 1.392 | 0.688 | 0.745 | 18.64 | 14.55 | 1.537 | 4.533 | 1.371 | 39.26 | 2.863 | 88.88 | 0.845 | 1.156 |
| | –0.5 | 0.631 | –19.05 | 1.321 | 0.725 | 0.757 | 1.046 | 0.758 | 1.479 | 0.255 | 1.371 | 3.538 | –0.598 | 93.96 | –0.889 | 0.969 |
| | –1 | 1.212 | –37.62 | 1.308 | 0.732 | 0.768 | 3.857 | 3.012 | 1.505 | 0.938 | 1.371 | 9.311 | –1.189 | 93.42 | –0.912 | 0.979 |
| | –1.5 | 1.791 | –55.42 | 1.296 | 0.739 | 0.782 | 8.422 | 6.656 | 1.541 | 2.048 | 1.371 | 18.67 | –1.752 | 91.78 | –0.890 | 1.000 |
| | –2 | 2.360 | –72.51 | 1.285 | 0.745 | 0.798 | 14.62 | 11.60 | 1.586 | 3.556 | 1.371 | 31.36 | –2.289 | 89.92 | –0.855 | 1.031 |
| 2500 | 0.5 | 0.835 | 32.59 | 1.688 | 0.567 | 0.594 | 1.830 | 1.357 | 1.254 | 0.445 | 2.142 | 4.887 | 0.850 | 93.53 | 0.919 | 1.008 |
| | 1 | 1.646 | 66.51 | 1.711 | 0.560 | 0.596 | 7.108 | 5.505 | 1.287 | 1.729 | 2.142 | 15.63 | 1.730 | 91.95 | 0.883 | 1.084 |
| | 1.5 | 2.493 | 102.7 | 1.735 | 0.552 | 0.606 | 16.32 | 12.76 | 1.353 | 3.969 | 2.142 | 34.41 | 2.672 | 89.29 | 0.813 | 1.199 |
| | 2 | 3.378 | 141.5 | 1.761 | 0.544 | 0.624 | 29.95 | 23.52 | 1.461 | 7.284 | 2.142 | 62.21 | 3.685 | 86.32 | 0.734 | 1.353 |
| | –0.5 | 0.760 | –29.49 | 1.647 | 0.581 | 0.608 | 1.517 | 1.168 | 1.274 | 0.369 | 2.142 | 4.327 | –0.745 | 93.63 | –0.913 | 0.976 |
| | –1 | 1.493 | –58.05 | 1.628 | 0.588 | 0.624 | 5.850 | 4.632 | 1.318 | 1.423 | 2.142 | 13.22 | –1.470 | 92.43 | –0.887 | 1.011 |
| | –1.5 | 2.211 | –85.18 | 1.610 | 0.595 | 0.643 | 12.83 | 10.20 | 1.381 | 3.121 | 2.142 | 27.53 | –2.153 | 90.26 | –0.829 | 1.069 |
| | –2 | 2.911 | –111.0 | 1.593 | 0.601 | 0.666 | 22.25 | 17.69 | 1.462 | 5.411 | 2.142 | 46.82 | –2.798 | 87.94 | –0.764 | 1.145 |
| 3000 | 0.25 | 0.524 | 23.87 | 2.016 | 0.475 | 0.496 | 0.720 | 0.511 | 1.102 | 0.175 | 3.084 | 2.508 | 0.521 | 91.59 | 0.911 | 0.993 |
| | 0.5 | 0.996 | 47.41 | 2.032 | 0.471 | 0.497 | 2.602 | 1.984 | 1.116 | 0.633 | 3.084 | 6.334 | 1.028 | 92.82 | 0.909 | 1.033 |
| | 1 | 1.985 | 97.25 | 2.065 | 0.464 | 0.507 | 10.35 | 8.080 | 1.186 | 2.516 | 3.084 | 22.13 | 2.106 | 90.62 | 0.822 | 1.174 |
| | 1.5 | 3.026 | 151.2 | 2.101 | 0.456 | 0.529 | 24.03 | 18.87 | 1.323 | 5.843 | 3.084 | 50.06 | 3.279 | 87.31 | 0.711 | 1.388 |
| | –0.25 | 0.454 | –20.83 | 1.986 | 0.482 | 0.503 | 0.541 | 0.401 | 1.112 | 0.132 | 3.084 | 2.186 | –0.436 | 91.35 | –0.897 | 0.973 |
| | –0.5 | 0.896 | –42.07 | 1.972 | 0.486 | 0.511 | 2.108 | 1.658 | 1.134 | 0.513 | 3.084 | 5.413 | –0.888 | 93.03 | –0.910 | 0.991 |
| | –1 | 1.774 | –82.58 | 1.945 | 0.492 | 0.531 | 8.261 | 6.566 | 1.205 | 2.009 | 3.084 | 18.04 | –1.743 | 91.33 | –0.837 | 1.068 |
| | –1.5 | 2.626 | –120.7 | 1.920 | 0.499 | 0.558 | 18.10 | 14.40 | 1.310 | 4.403 | 3.084 | 38.22 | –2.540 | 88.70 | –0.742 | 1.185 |
| 4000 | 0.25 | 0.689 | 43.38 | 2.696 | 0.355 | 0.374 | 1.246 | 0.944 | 0.912 | 0.303 | 5.483 | 3.405 | 0.706 | 90.14 | 0.913 | 1.021 |
| | 0.5 | 1.334 | 86.10 | 2.724 | 0.352 | 0.380 | 4.674 | 3.639 | 0.957 | 1.137 | 5.483 | 10.41 | 1.397 | 91.09 | 0.843 | 1.130 |
| | 0.75 | 2.002 | 131.0 | 2.754 | 0.348 | 0.394 | 10.52 | 8.243 | 1.042 | 2.558 | 5.483 | 22.36 | 2.127 | 89.75 | 0.748 | 1.293 |
| | 1 | 2.691 | 178.4 | 2.786 | 0.344 | 0.414 | 19.01 | 14.94 | 1.170 | 4.623 | 5.483 | 39.74 | 2.899 | 87.78 | 0.653 | 1.502 |
| | –0.25 | 0.577 | –36.19 | 2.643 | 0.362 | 0.380 | 0.873 | 0.683 | 0.919 | 0.212 | 5.483 | 2.688 | –0.572 | 89.94 | –0.914 | 0.988 |
| | –0.5 | 1.171 | –73.39 | 2.618 | 0.366 | 0.392 | 3.602 | 2.864 | 0.965 | 0.876 | 5.483 | 8.306 | –1.165 | 91.51 | –0.859 | 1.053 |
| | –0.75 | 1.756 | –109.1 | 2.594 | 0.369 | 0.408 | 8.089 | 6.440 | 1.035 | 1.967 | 5.483 | 17.53 | –1.728 | 90.55 | –0.776 | 1.155 |
| | –1 | 2.327 | –143.3 | 2.571 | 0.372 | 0.428 | 14.22 | 11.32 | 1.126 | 3.458 | 5.483 | 30.12 | –2.266 | 89.04 | –0.692 | 1.280 |

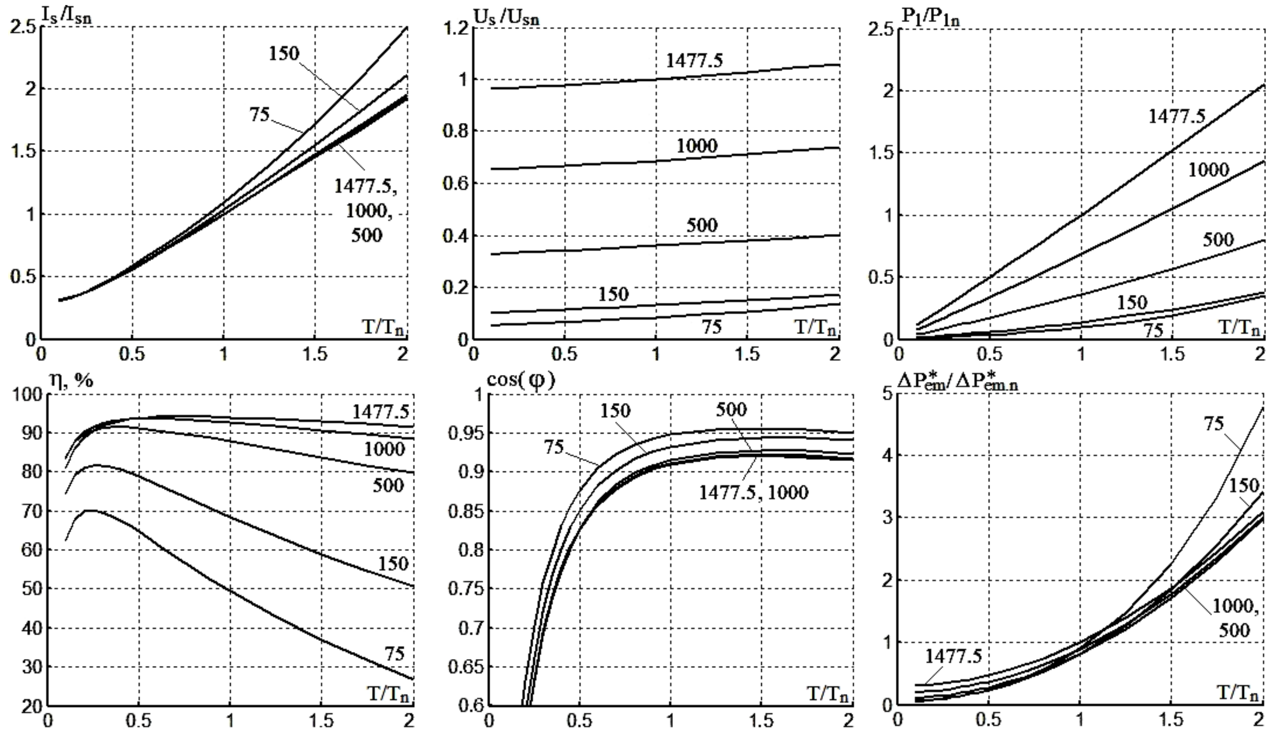
- with the same values of the torque $T$, the corresponding values of the module $I_s$ of the generalized stator current vector and the electromagnetic power losses $\Delta P^*_{em}$ in the first speed zone (when $n \le n_n$) are significantly smaller than in the second zone (when $n > n_n$);
- extreme (maximum) values are characteristic of graphic dependencies for the efficiency $\eta$ and the power factor $\cos\varphi$ of the f-r IM in the first and second zones;
- values of motor efficiency $\eta$ with increasing speed: in the first zone – increase, while in the second zone – decrease;
- the optimal value of the motor torque (corresponding to the maximum value of the efficiency) with increasing speed: in the first zone – it increases, while in the second zone – on the contrary, it decreases.

At the *fifth stage*, we quantitatively estimate the possible errors regarding the main (caused by the first harmonic components of the phase stator voltages and currents) electromagnetic losses $\Delta P_{em}$, if additional and magnetic power losses, as well as components of electrical losses caused by the transport of these power losses through the air gap or to the magnetization circuit of the motor were not taken into account (as, for example, in well-known publications [1–11]).

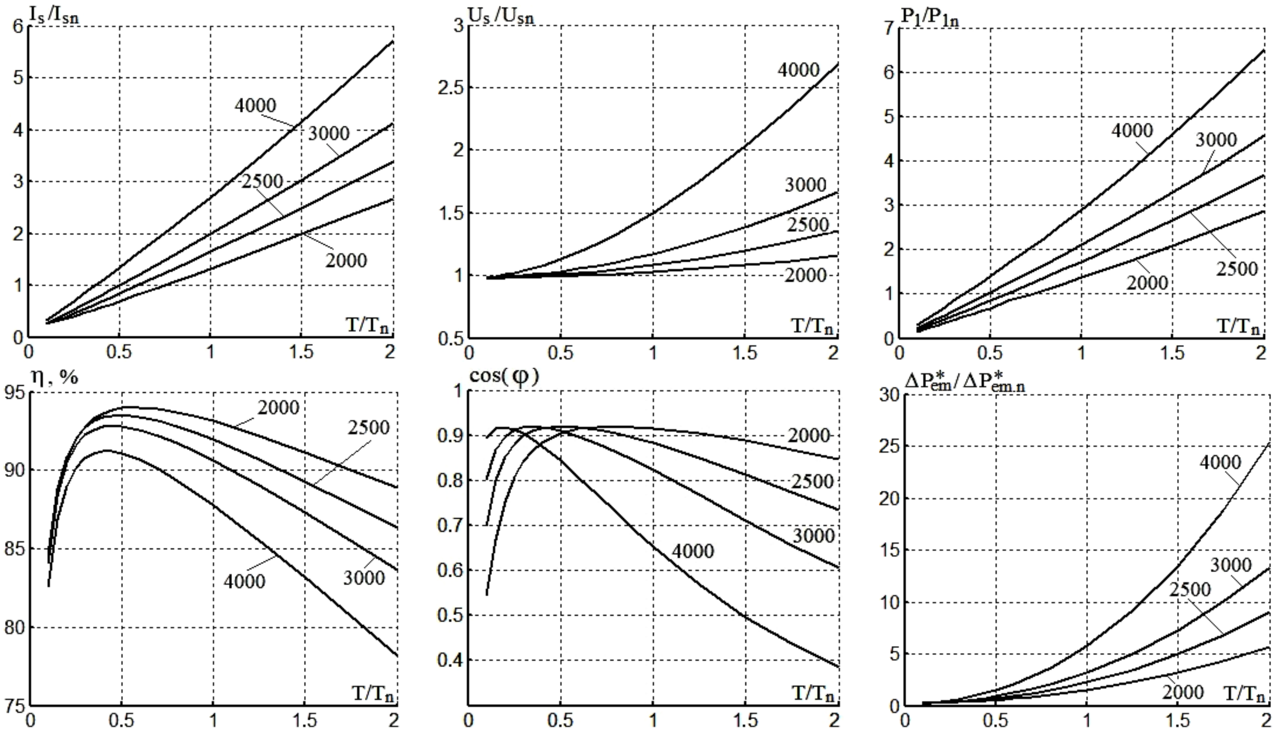
The calculated analytical dependence for the main electromagnetic power losses (MEPL) $\Delta P^*_{em}$ of the short-circuited f-r IM (which simultaneously takes into account: all types of power losses present in the motor, as well as the above-mentioned electrical components of losses associated with the mentioned transportation of additional and magnetic power losses) obtained based on the diagram in Fig. 1 and formulas (1), (10), (13), (14), (19) is refined and has the following form:

$$\Delta P^*_{em} = R_s \cdot \left[ I_{sx}^2 + \left( I_{1y} + \Delta I_{1y} + \Delta I_{sy} \right)^2 \right] + \Delta P_{i\,r} + k_r^2 R_r \cdot \left( I_{1y} + \Delta I_{1y} \right)^2 + R^*_{ad} \cdot \left[ I_{sx}^2 + \left( I_{1y} + \Delta I_{1y} \right)^2 \right]. \quad (23)$$

In this dependence: the first term is the calculation relation for the main electrical power losses in the stator winding (which take into account, according to (23), the magnetizing $I_{sx}$ and active $I_{1y}$ projections of the generalized stator current vector $\boldsymbol{I}_1$, as well as the increments $\Delta I_{sy}$ and $\Delta I_{1y}$ of the stator current, caused by the transportation of magnetic and additional losses; the second term is the magnetic losses $\Delta P_{i\,r}$, which are calculated from (1); the third term is the calculation relation for the main electrical losses in the rotor winding (calculated through the active projection $I_{1y}$ of the stator current and the increment $\Delta I_{1y}$ pf the stator current, which is calculated from (14) and is caused by the transport of additional power losses through the air gap of the motor), and the fourth one is the additional motor power losses $\Delta P_{ad}$, calculated from (10).




*a*) with $n \le n_n$




*b*) with $n > n_n$

Fig. 2. Graphical dependencies illustrating the change in the motor mode of operation in the first (*a*) and second (*b*) speed zones of: the modules of the generalized vectors of the stator current $I_s/I_{sn}$ and voltage $U_s/U_{sn}$, the efficiency $\eta$ and the power factor $\cos\varphi$, the main electromagnetic power losses $\Delta P^*_{em}/\Delta P^*_{em.n}$ and the main consumed active power $P_1/P_{1n}$, – for the f-r IM AT250L4U2 depending on the useful torque $T/T_n$ (at speeds $n$ equal to 75, 150, 500, 1000 and 1477.5 rpm for the first zone, or 2000, 2500, 3000 and 4000 rpm for the second zone)

The importance of obtaining for practice the proposed dependence (23) for the refined calculation of MEPL $\Delta P^*_{em}$ is explained by the direct effect of these power losses on motor heating (as a result, the use of this dependence can warn the designers of f-r induction electric drives about motor overheating during operation).

The values of MEPL, which are calculated from inaccurate dependencies (in which there is no taking into account of at least one type of motor power losses, or at least one of the electrical components spent on transporting additional or magnetic losses), are denoted without an asterisk from above in the form: $\Delta P_{em.1}$, $\Delta P_{em.2}$, $\Delta P_{em.3}$, $\Delta P_{em.4}$ or $\Delta P_{em.5}$. Based on (23) and the accepted notation, we obtain calculation dependencies for determining errors in the calculation of the MEPL of the f-r IM:

$$\Delta P_{\delta 1}=\Delta P^{*}_{em}-\Delta P_{em.1}=R_s\cdot\left[\left(I_{1y}+\Delta I_{1y}+\Delta I_{sy}\right)^2-\right. \\ \left.-\left(I_{1y}+\Delta I_{sy}\right)^2\right]+k_r^2 R_r\cdot\left[\left(I_{1y}+\Delta I_{1y}\right)^2-I_{1y}^2\right], \quad (24)$$

– in relation to the dependence for $\Delta P_{em.1}$, which is obtained from (23) when $\Delta I_{1y}$ = 0 and does not take into account the electrical component of losses caused by the transportation of additional power losses through the air gap of the motor (according to [11]);

$$\Delta P_{\delta 2}=\Delta P^{*}_{em}-\Delta P_{em.2}=R_s\cdot\left[\left(I_{1y}+\Delta I_{1y}+\Delta I_{sy}\right)^2-I_{1y}^2\right]+ \\ +k_r^2 R_r\cdot\left[\left(I_{1y}+\Delta I_{1y}\right)^2-I_{1y}^2\right], \quad (25)$$

– in relation to the dependence for $\Delta P_{em.2}$, which is obtained from (23) when $\Delta I_{1y}$ = 0, $\Delta I_{sy}$ = 0 and does not take into account the components of electrical losses due to the transportation of additional losses through the air gap and of magnetic power losses to the motor magnetization circuit (according to [10]);

$$\Delta P_{\delta 3}=\Delta P^{*}_{em}-\Delta P_{em.3}=\Delta P_{\delta 1}+R^{*}_{ad}\cdot\left(I_{sx}^2+I_{1\Sigma y}^2\right)= \\ =R_s\cdot\left[\left(I_{1y}+\Delta I_{1y}+\Delta I_{sy}\right)^2-\left(I_{1y}+\Delta I_{sy}\right)^2\right]+ \\ +k_r^2 R_r\cdot\left[\left(I_{1y}+\Delta I_{1y}\right)^2-I_{1y}^2\right]+R^{*}_{ad}\cdot\left(I_{sx}^2+I_{1\Sigma y}^2\right), \quad (26)$$

– in relation to the dependence for $\Delta P_{em.3}$, which is obtained from (23) when $\Delta I_{1y}$ = 0, $R^{*}_{ad}$ = 0 and does not take into account additional losses and the electrical component of losses caused by their transportation through the air gap of the motor (according to [3–9]);

$$\Delta P_{\delta 4}=\Delta P^{*}_{em}-\Delta P_{em.4}=R_s\cdot\left[\left(I_{1y}+\Delta I_{1y}+\Delta I_{sy}\right)^2-I_{1y}^2\right]+ \\ +k_r^2 R_r\cdot\left[\left(I_{1y}+\Delta I_{1y}\right)^2-I_{1y}^2\right]+R^{*}_{ad}\left(I_{sx}^2+I_{1\Sigma y}^2\right), \quad (27)$$

– in relation to the dependence for $\Delta P_{em.4}$, which is obtained from (23) when $I_{sy}$ = 0, $\Delta I_{1y}$ = 0, $R^{*}_{ad}$ = 0 and does not take into account additional losses, as well as components of electrical losses due to the transportation of magnetic and additional power losses;

$$\Delta P_{\delta 5}=\Delta P^{*}_{em}-\Delta P_{em.5}=\Delta P_{\delta 4}+\Delta P_{i\,r}= \\ =R_s\cdot\left[\left(I_{1y}+\Delta I_{1y}+\Delta I_{sy}\right)^2-I_{1y}^2\right]+\Delta P_{i\,r}+ \\ +k_r^2 R_r\cdot\left[\left(I_{1y}+\Delta I_{1y}\right)^2-I_{1y}^2\right]+R^{*}_{ad}\left(I_{sx}^2+I_{1\Sigma y}^2\right) \quad (28)$$

– in relation to the dependence for $\Delta P_{em.5}$, which is obtained from (23) when $\Delta I_{sy}$ = 0, $\Delta I_{1y}$ = 0, $R^{*}_{ad}$ = 0, $\Delta P_{i.r}$ = 0 and does not take into account magnetic and additional losses, as well as components of electrical losses for their transportation to the magnetization circuit or through the air gap of the motor, respectively (which, as is known, is characteristic of the idealized representation of the IM [1, 2]).

Results of calculations carried out by (24) – (28) of absolute values of errors $\Delta P_{\delta 1}$, $\Delta P_{\delta 2}$, $\Delta P_{\delta 3}$, $\Delta P_{\delta 4}$, $\Delta P_{\delta 5}$, as well as relative values of these errors $\Delta P_{\delta 1}/\Delta P^{*}_{em}$, $\Delta P_{\delta 2}/\Delta P^{*}_{em}$, $\Delta P_{\delta 3}/\Delta P^{*}_{em}$, $\Delta P_{\delta 4}/\Delta P^{*}_{em}$ i $\Delta P_{\delta 5}/\Delta P^{*}_{em}$ are given in Table 4, 5, from the analysis of which it follows that:

Table 4

Calculated ratios of electromagnetic and energy quantities, as well as absolute values of errors $\Delta P_{\delta 1}$, $\Delta P_{\delta 2}$, $\Delta P_{\delta 3}$, $\Delta P_{\delta 4}$, $\Delta P_{\delta 5}$ for the f-r IM AT250L4U2 in the first speed zone (when $n \le n_n$)

| $n$ | $T/T_n$ | $I_s/I_{1\Sigma}$ | $I_s/I_1$ | $I_{1\Sigma}/I_1$ | $\Delta I_{sy}/I_{sn}$ | $\Delta I_{1y}/I_{sn}$ | $\Delta P_{\delta.1}/\Delta P^{*}_{em}$ | $\Delta P_{\delta.2}/\Delta P^{*}_{em}$ | $\Delta P_{\delta.3}/\Delta P^{*}_{em}$ | $\Delta P_{\delta.4}/\Delta P^{*}_{em}$ | $\Delta P_{\delta.5}/\Delta P^{*}_{em}$ | $\Delta P_{\delta 1}$ | $\Delta P_{\delta 2}$ | $\Delta P_{\delta 3}$ | $\Delta P_{\delta 4}$ | $\Delta P_{\delta 5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| rpm | – | – | – | – | % | % | % | % | % | % | % | kW | kW | kW | kW | kW |
| 1477.5 | 0.5 | 1.021 | 1.024 | 1.002 | 1.370 | 0.153 | 0.210 | 1.258 | 6.341 | 7.389 | 60.31 | 0.007 | 0.041 | 0.209 | 0.243 | 1.983 |
| | 1 | 1.013 | 1.018 | 1.005 | 1.378 | 0.484 | 0.624 | 1.614 | 9.855 | 10.84 | 35.88 | 0.043 | 0.112 | 0.682 | 0.750 | 2.481 |
| | 1.5 | 1.009 | 1.017 | 1.007 | 1.391 | 1.037 | 1.069 | 1.866 | 11.64 | 12.44 | 25.78 | 0.138 | 0.242 | 1.507 | 1.610 | 3.336 |
| | 2 | 1.007 | 1.017 | 1.010 | 1.407 | 1.815 | 1.508 | 2.156 | 12.71 | 13.35 | 21.42 | 0.323 | 0.462 | 2.720 | 2.858 | 4.584 |
| | –0.5 | 0.979 | 0.977 | 0.998 | 1.363 | 0.146 | –0.201 | –1.250 | 6.019 | 4.970 | 62.19 | –0.006 | –0.039 | 0.187 | 0.154 | 1.927 |
| | –1 | 0.987 | 0.982 | 0.995 | 1.366 | 0.464 | –0.619 | –1.640 | 8.850 | 7.830 | 35.59 | –0.040 | –0.106 | 0.573 | 0.507 | 2.302 |
| | –1.5 | 0.991 | 0.984 | 0.993 | 1.371 | 0.992 | –1.065 | –1.890 | 9.756 | 8.931 | 23.99 | –0.129 | –0.229 | 1.181 | 1.081 | 2.905 |
| | –2 | 0.993 | 0.984 | 0.991 | 1.380 | 1.728 | –1.501 | –2.173 | 9.918 | 9.246 | 18.54 | –0.300 | –0.434 | 1.982 | 1.848 | 3.704 |
| 1000 | 0.5 | 1.019 | 1.022 | 1.004 | 1.220 | 0.224 | 0.391 | 1.576 | 8.177 | 9.362 | 49.95 | 0.010 | 0.041 | 0.210 | 0.241 | 1.284 |
| | 1 | 1.012 | 1.019 | 1.007 | 1.229 | 0.713 | 1.026 | 2.007 | 11.32 | 12.30 | 28.98 | 0.064 | 0.124 | 0.701 | 0.762 | 1.794 |
| | 1.5 | 1.008 | 1.019 | 1.011 | 1.241 | 1.535 | 1.672 | 2.422 | 12.88 | 13.63 | 22.00 | 0.205 | 0.297 | 1.577 | 1.669 | 2.694 |
| | 2 | 1.007 | 1.021 | 1.014 | 1.257 | 2.698 | 2.308 | 2.904 | 13.91 | 14.51 | 19.42 | 0.480 | 0.603 | 2.891 | 3.014 | 4.034 |
| | –0.5 | 0.981 | 0.978 | 0.997 | 1.212 | 0.217 | –0.386 | –1.591 | 7.648 | 6.443 | 50.79 | –0.009 | –0.038 | 0.185 | 0.156 | 1.228 |
| | –1 | 0.988 | 0.981 | 0.993 | 1.212 | 0.687 | –1.027 | –2.044 | 9.574 | 8.558 | 27.40 | –0.060 | –0.118 | 0.554 | 0.495 | 1.586 |
| | –1.5 | 0.992 | 0.982 | 0.990 | 1.216 | 1.463 | –1.670 | –2.449 | 9.800 | 9.021 | 18.78 | –0.190 | –0.279 | 1.117 | 1.028 | 2.140 |
| | –2 | 0.994 | 0.980 | 0.987 | 1.222 | 2.540 | –2.296 | –2.916 | 9.527 | 8.906 | 14.82 | –0.441 | –0.560 | 1.829 | 1.710 | 2.846 |
| 500 | 0.5 | 1.015 | 1.023 | 1.007 | 0.994 | 0.448 | 1.036 | 2.311 | 11.40 | 12.67 | 34.28 | 0.020 | 0.045 | 0.220 | 0.245 | 0.663 |
| | 1 | 1.010 | 1.024 | 1.014 | 1.005 | 1.441 | 2.294 | 3.179 | 13.80 | 14.69 | 21.97 | 0.128 | 0.178 | 0.772 | 0.822 | 1.229 |
| | 1.5 | 1.007 | 1.028 | 1.021 | 1.018 | 3.129 | 3.550 | 4.187 | 15.41 | 16.04 | 19.42 | 0.419 | 0.494 | 1.817 | 1.892 | 2.290 |
| | 2 | 1.005 | 1.034 | 1.029 | 1.035 | 5.544 | 4.803 | 5.296 | 16.80 | 17.30 | 19.19 | 0.991 | 1.093 | 3.469 | 3.570 | 3.960 |
| | –0.5 | 0.985 | 0.978 | 0.993 | 0.981 | 0.434 | –1.042 | –2.363 | 9.755 | 8.434 | 32.98 | –0.019 | –0.043 | 0.175 | 0.152 | 0.593 |
| | –1 | 0.990 | 0.977 | 0.987 | 0.978 | 1.361 | –2.296 | –3.224 | 9.552 | 8.624 | 17.48 | –0.118 | –0.165 | 0.490 | 0.443 | 0.897 |
| | –1.5 | 0.993 | 0.974 | 0.980 | 0.977 | 2.877 | –3.517 | –4.191 | 8.592 | 7.918 | 12.34 | –0.373 | –0.445 | 0.912 | 0.841 | 1.309 |
| | –2 | 0.995 | 0.969 | 0.974 | 0.979 | 4.961 | –4.816 | –5.242 | 7.485 | 6.959 | 9.626 | –0.857 | –0.952 | 1.360 | 1.264 | 1.749 |
| 150 | 0.5 | 1.011 | 1.035 | 1.024 | 0.704 | 1.542 | 4.248 | 5.311 | 16.90 | 17.96 | 22.96 | 0.069 | 0.087 | 0.276 | 0.294 | 0.375 |
| | 1 | 1.007 | 1.058 | 1.051 | 0.722 | 5.148 | 8.318 | 8.951 | 20.62 | 21.26 | 22.57 | 0.466 | 0.502 | 1.157 | 1.192 | 1.266 |
| | 1.5 | 1.005 | 1.085 | 1.080 | 0.743 | 11.65 | 12.57 | 13.00 | 24.82 | 25.25 | 25.76 | 1.603 | 1.658 | 3.165 | 3.219 | 3.285 |
| | 2 | 1.004 | 1.117 | 1.113 | 0.769 | 21.62 | 16.99 | 17.31 | 29.23 | 29.55 | 29.79 | 4.025 | 4.100 | 6.922 | 6.998 | 7.056 |
| | –0.5 | 0.989 | 0.968 | 0.978 | 0.673 | 1.407 | –4.229 | –5.376 | 8.991 | 7.845 | 14.70 | –0.060 | –0.077 | 0.128 | 0.112 | 0.210 |
| | –1 | 0.993 | 0.952 | 0.958 | 0.659 | 4.282 | –8.055 | –8.764 | 4.583 | 3.874 | 6.210 | –0.366 | –0.398 | 0.208 | 0.176 | 0.282 |
| | –1.5 | 0.995 | 0.935 | 0.940 | 0.647 | 8.818 | –11.84 | –12.34 | 0.636 | 0.135 | 1.342 | –1.122 | –1.169 | 0.060 | 0.013 | 0.127 |
| | –2 | 0.996 | 0.919 | 0.922 | 0.635 | 14.84 | –15.58 | –15.97 | –3.170 | –3.558 | –2.794 | –2.497 | –2.559 | –0.508 | –0.570 | –0.448 |

Table 5

Calculated ratios of electromagnetic and energy quantities, as well as absolute values of errors $\Delta P_{\delta 1}$, $\Delta P_{\delta 2}$, $\Delta P_{\delta 3}$, $\Delta P_{\delta 4}$, $\Delta P_{\delta 5}$ for the f-r IM AT250L4U2 in the second speed zone (when $n > n_n$)

| $n$ | $\frac{T}{T_n}$ | $\frac{I_s}{I_{1\Sigma}}$ | $\frac{I_s}{I_1}$ | $\frac{I_{1\Sigma}}{I_1}$ | $\frac{\Delta I_{sy}}{I_{sn}}$ | $\frac{\Delta I_{1y}}{I_{sn}}$ | $\frac{\Delta P_{\delta.1}}{\Delta P^*_{em}}$ | $\frac{\Delta P_{\delta.2}}{\Delta P^*_{em}}$ | $\frac{\Delta P_{\delta.3}}{\Delta P^*_{em}}$ | $\frac{\Delta P_{\delta.4}}{\Delta P^*_{em}}$ | $\frac{\Delta P_{\delta.5}}{\Delta P^*_{em}}$ | $\Delta P_{\delta 1}$ | $\Delta P_{\delta 2}$ | $\Delta P_{\delta 3}$ | $\Delta P_{\delta 4}$ | $\Delta P_{\delta 5}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| rpm | – | – | – | – | % | % | % | % | % | % | % | kW | kW | kW | kW | kW |
| 2000 | 0.5 | 1.016 | 1.019 | 1.003 | 1.118 | 0.226 | 0.357 | 1.338 | 8.132 | 9.114 | 46.00 | 0.014 | 0.052 | 0.313 | 0.352 | 1.773 |
| | 1 | 1.009 | 1.015 | 1.007 | 1.125 | 0.853 | 0.978 | 1.695 | 11.54 | 12.26 | 25.41 | 0.104 | 0.180 | 1.223 | 1.299 | 2.692 |
| | 1.5 | 1.006 | 1.016 | 1.010 | 1.145 | 1.955 | 1.615 | 2.140 | 13.03 | 13.55 | 19.75 | 0.359 | 0.476 | 2.898 | 3.015 | 4.393 |
| | 2 | 1.004 | 1.018 | 1.014 | 1.177 | 3.593 | 2.268 | 2.680 | 14.02 | 14.43 | 17.94 | 0.890 | 1.052 | 5.504 | 5.666 | 7.041 |
| | –0.5 | 0.983 | 0.981 | 0.997 | 1.133 | 0.200 | –0.318 | –1.327 | 7.319 | 6.310 | 49.12 | –0.011 | –0.047 | 0.259 | 0.223 | 1.738 |
| | –1 | 0.991 | 0.985 | 0.994 | 1.153 | 0.718 | –0.871 | –1.655 | 9.666 | 8.882 | 25.83 | –0.081 | –0.154 | 0.900 | 0.827 | 2.405 |
| | –1.5 | 0.994 | 0.985 | 0.992 | 1.181 | 1.545 | –1.395 | –1.992 | 10.02 | 9.422 | 18.28 | –0.260 | –0.372 | 1.870 | 1.759 | 3.411 |
| | –2 | 0.995 | 0.984 | 0.989 | 1.215 | 2.652 | –1.884 | –2.369 | 9.878 | 9.394 | 14.93 | –0.591 | –0.743 | 3.098 | 2.946 | 4.684 |
| 2500 | 0.5 | 1.011 | 1.016 | 1.004 | 0.961 | 0.338 | 0.528 | 1.363 | 9.671 | 10.51 | 35.34 | 0.026 | 0.067 | 0.473 | 0.513 | 1.727 |
| | 1 | 1.006 | 1.014 | 1.008 | 0.986 | 1.343 | 1.315 | 1.851 | 12.54 | 13.07 | 20.77 | 0.206 | 0.289 | 1.959 | 2.043 | 3.246 |
| | 1.5 | 1.004 | 1.017 | 1.013 | 1.037 | 3.139 | 2.118 | 2.506 | 13.87 | 14.25 | 17.80 | 0.729 | 0.862 | 4.770 | 4.904 | 6.124 |
| | 2 | 1.003 | 1.021 | 1.018 | 1.119 | 5.857 | 2.958 | 3.271 | 14.90 | 15.21 | 17.25 | 1.840 | 2.035 | 9.271 | 9.465 | 10.73 |
| | –0.5 | 0.988 | 0.984 | 0.996 | 0.976 | 0.286 | –0.464 | –1.349 | 8.508 | 7.623 | 37.95 | –0.020 | –0.058 | 0.368 | 0.330 | 1.642 |
| | –1 | 0.993 | 0.986 | 0.993 | 1.010 | 1.079 | –1.146 | –1.746 | 10.05 | 9.451 | 20.02 | –0.152 | –0.231 | 1.329 | 1.250 | 2.647 |
| | –1.5 | 0.995 | 0.985 | 0.990 | 1.058 | 2.331 | –1.769 | –2.219 | 9.982 | 9.531 | 15.00 | –0.487 | –0.611 | 2.748 | 2.624 | 4.130 |
| | –2 | 0.996 | 0.983 | 0.987 | 1.120 | 3.991 | –2.351 | –2.721 | 9.608 | 9.238 | 12.73 | –1.101 | –1.274 | 4.498 | 4.325 | 5.960 |
| 3000 | 0.25 | 1.016 | 1.018 | 1.002 | 0.844 | 0.131 | 0.246 | 1.127 | 7.195 | 8.076 | 51.14 | 0.006 | 0.028 | 0.181 | 0.203 | 1.283 |
| | 0.5 | 1.009 | 1.014 | 1.005 | 0.855 | 0.484 | 0.704 | 1.395 | 10.79 | 11.48 | 28.40 | 0.045 | 0.088 | 0.683 | 0.727 | 1.799 |
| | 1 | 1.005 | 1.015 | 1.010 | 0.908 | 1.971 | 1.649 | 2.070 | 13.22 | 13.64 | 18.58 | 0.365 | 0.458 | 2.924 | 3.018 | 4.110 |
| | 1.5 | 1.003 | 1.019 | 1.016 | 1.014 | 4.670 | 2.628 | 2.944 | 14.53 | 14.85 | 17.18 | 1.316 | 1.474 | 7.275 | 7.434 | 8.599 |
| | –0.25 | 0.983 | 0.980 | 0.998 | 0.852 | 0.104 | –0.194 | –1.086 | 6.213 | 5.321 | 57.10 | –0.004 | –0.024 | 0.136 | 0.116 | 1.248 |
| | –0.5 | 0.991 | 0.986 | 0.996 | 0.868 | 0.395 | –0.611 | –1.362 | 9.303 | 8.552 | 30.25 | –0.033 | –0.074 | 0.504 | 0.463 | 1.637 |
| | –1 | 0.995 | 0.987 | 0.992 | 0.923 | 1.512 | –1.403 | –1.883 | 10.15 | 9.670 | 16.83 | –0.253 | –0.340 | 1.831 | 1.744 | 3.036 |
| | –1.5 | 0.996 | 0.984 | 0.988 | 1.003 | 3.262 | –2.122 | –2.488 | 9.797 | 9.430 | 13.22 | –0.811 | –0.951 | 3.744 | 3.604 | 5.054 |
| 4000 | 0.25 | 1.010 | 1.014 | 1.003 | 0.699 | 0.230 | 0.430 | 1.157 | 9.386 | 10.11 | 36.17 | 0.015 | 0.039 | 0.320 | 0.344 | 1.232 |
| | 0.5 | 1.006 | 1.012 | 1.007 | 0.733 | 0.880 | 1.052 | 1.539 | 12.14 | 12.63 | 21.34 | 0.110 | 0.160 | 1.264 | 1.315 | 2.221 |
| | 0.75 | 1.004 | 1.014 | 1.010 | 0.798 | 2.007 | 1.677 | 2.047 | 13.33 | 13.70 | 17.99 | 0.375 | 0.458 | 2.981 | 3.063 | 4.022 |
| | 1 | 1.003 | 1.017 | 1.014 | 0.896 | 3.675 | 2.320 | 2.634 | 14.19 | 14.50 | 17.13 | 0.922 | 1.047 | 5.638 | 5.762 | 6.807 |
| | –0.25 | 0.988 | 0.985 | 0.997 | 0.704 | 0.165 | –0.329 | –1.115 | 7.976 | 7.190 | 42.18 | –0.009 | –0.030 | 0.214 | 0.193 | 1.134 |
| | –0.5 | 0.994 | 0.988 | 0.994 | 0.739 | 0.667 | –0.886 | –1.436 | 10.08 | 9.528 | 21.70 | –0.074 | –0.119 | 0.837 | 0.791 | 1.802 |
| | –0.75 | 0.996 | 0.987 | 0.992 | 0.792 | 1.479 | –1.399 | –1.820 | 10.23 | 9.807 | 16.13 | –0.245 | –0.319 | 1.793 | 1.719 | 2.828 |
| | –1 | 0.996 | 0.986 | 0.989 | 0.863 | 2.573 | –1.882 | –2.236 | 9.993 | 9.639 | 13.73 | –0.567 | –0.673 | 3.010 | 2.904 | 4.136 |

- due to taking into account the mentioned increment $\Delta I_{1y}$, the accuracy of determining the module $I_{1\Sigma}$ of the stator current in the motor mode of the f-r IM (in comparison with the module $I_1$ of the stator current, in which this increment is not taken into account) increases, according to the relations $I_{1\Sigma}/I_1$, by (0.2 – 11.3) %, where the largest values of these relations refer to low speeds and increased values of the useful torque $T$, and smaller values to increased speeds and reduced values of this torque $T$;

- according to the calculated values of the relationships $I_s/I_{1\Sigma}$, as a result of taking into account the mentioned increment $\Delta I_{sy}$, the accuracy of the calculation of the module $I_s$ of the generalized vector of the stator current $\boldsymbol{I}_s$ in the motor mode of the f-r IM increases by (0.3 – 2.1) %;

- in the motor mode, in the first speed zone, the value of increment $\Delta I_s/I_{sn}$ of the active projection of the stator current of the f-r IM increases with an increase in speed $n$, and in the second zone, on the contrary, they decrease;

- the relative errors $\Delta P_{\delta 3}/\Delta P^*_{em}$, $\Delta P_{\delta 4}/\Delta P^*_{em}$ and $\Delta P_{\delta 5}/\Delta P^*_{em}$ always have only positive values which indicates the underestimated values of MEPL from (26) – (28): $\Delta P_{em.3}<\Delta P^*_{em}$, $\Delta P_{em.4}<\Delta P^*_{em}$ i $\Delta P_{em.5}<\Delta P^*_{em}$ – in comparison with their refined value $\Delta P^*_{em}$;

- at the same speed and equal (in absolute value) values of the useful torque of the motor, the lowest values in absolute value among all considered types of errors are inherent, according to Table 4, 5, relative errors $\Delta P_{\delta 1}/\Delta P^*_{em}$ and $\Delta P_{\delta 2}/\Delta P^*_{em}$ (the algebraic signs of which are positive in motor mode and negative in generator mode); as a result of the latter, in the generator mode, the values of the calculated power losses $\Delta P_{em.1}$ and $\Delta P_{em.2}$ exceed the refined values of $\Delta P^*_{em}$ MEPL calculated from (23);

- in the first and second speed zones in motor and generator modes, the absolute value of all relative errors: $\Delta P_{\delta 1}/\Delta P^*_{em}$, $\Delta P_{\delta 2}/\Delta P^*_{em}$, $\Delta P_{\delta 3}/\Delta P^*_{em}$, $\Delta P_{\delta 4}/\Delta P^*_{em}$, $\Delta P_{\delta 5}/\Delta P^*_{em}$ – with a decrease in speed and an increase (by absolute value) of the useful torque, of the f-r IM increases, which is caused by the influence of the electrical loss component, which is due to the transportation of additional power losses through motor air gap.

We note that (despite the obtained identical type of calculation dependencies for electromagnetic processes and energy parameters of the real f-r IM mode in relation to the motor and generator modes of its operation) with the same absolute value, but with the opposite algebraic sign of the values of the useful torque $T$, it is observed (according to the calculated data from Tables 2 – 5) the difference in motor and generator operating modes in the calculated values: module $I_s$ of the stator current, electromagnetic torque $|T_{em}|$, refined MEPL $\Delta P^*_{em}$, calculation errors $\Delta P_{\delta 1}$, $\Delta P_{\delta 2}$, $\Delta P_{\delta 3}$,

$\Delta P_{\delta 4}$, $\Delta P_{\delta 5}$ and other motor mode parameters. This is explained by the fact that: firstly, in the motor mode, the directions of action of the useful torque $T$ and the mechanical losses of the torque $\Delta T_{mech}$ are opposite to each other (and in the generator mode, they coincide); secondly, in the motor mode, the algebraic sign of the component of the active projection I $I_{1y}$ of the generalized stator current vector coincides with the signs of the increments $\Delta I_{1y}$, $\Delta I_{sy}$ of the stator current caused by the transportation of additional and magnetic power losses (while in the generator mode, the algebraic sign of the projection $I_{1y}$ is opposite to the signs of the specified increments $\Delta I_{1y}$ and $\Delta I_{sy}$).

The results of the research carried out in the article are recommended to scientists and engineers who are engaged in the research of electromagnetic and energy processes of the f-r IMs and the design of energy-efficient control (which ensures the minimization of electromagnetic power losses or consumed active electric power) for these motors.

**Conclusions.** Analytical calculation dependencies are obtained to determine the increments of the active projection of the generalized stator current vector of the frequency-regulated induction motor, caused by the transportation of additional and magnetic power losses through the air gap or to the magnetization circuit of the motor, respectively. With the use of these dependencies, refined calculations of the main electromagnetic power losses, the main consumed active power, the efficiency and the power factor of the mentioned motor in steady-state motor and generator modes were performed.

**Conflict of interest.** The authors declare no conflict of interest.

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