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Computational analysis method of the electromagnetic field propagation and deformation of conductive bodies

Introduction. *The electromagnetic field is an integral attribute of the operation of many technical and technological systems. The action of an electromagnetic field leads to deformation, a change in temperature, a change in the physical properties of the materials.* ***Problem.*** *High-intensity electromagnetic fields can cause such a strong deformation of conductive bodies that it can lead to irreversible shape change or destruction. This fact is used in a class of technological operations: electromagnetic forming. Here, both the workpiece and the equipment are subjected to intense force action. As a result, equipment elements may become inoperable.* ***Goal.*** *Creation of a computational analysis method of the electromagnetic field propagation in systems of conductive bodies and subsequent analysis of deformation. Application of this method to the study of processes in electromagnetic forming systems in order to determine rational operational parameters that provide the result of a technological operation.* ***Methodology.*** *A variational formulation of the problems of an electromagnetic field propagation and deformation of conductive bodies systems is used. Numerical modeling and analysis are performed using the finite element method.* ***Results.*** *In a general form, a system of resolving equations for the values of the vector magnetic potential and displacements is obtained. The influence of the electromagnetic field is taken into account by introducing electromagnetic forces. The results of calculations for a technological system designed for electromagnetic forming of curved thin-walled workpieces are presented.* ***Originality.*** *For the first time, a method of computational analysis is presented, which involves modeling within the framework of one design scheme both the process of electromagnetic field propagation and the process of deformation.* ***Practical significance.*** *The proposed method of computational analysis can be used for various technological systems of electromagnetic forming in order to determine the rational parameters that ensure both the operability of the equipment and the purpose of the technological operation - the necessary shaping of the workpiece.* References 18, table 1, figures 3.

Key words: **computational analysis, electromagnetic field, electromagnetic forming, deformation, finite element method.**

Вступ. *Електромагнітне поле є невід'ємним атрибутом роботи багатьох технічних і технологічних систем. Дія електромагнітного поля призводить до деформації, зміни температури, зміни фізичних властивостей матеріалів.* ***Проблема.*** *Електромагнітні поля високої інтенсивності можуть викликати настільки сильну деформацію провідних тіл, що це може призвести до незворотної зміни форми або руйнування. Цей факт використовується в класі технологічних операцій: електромагнітна обробка. Тут як заготовка, так і обладнання піддаються інтенсивній силовій дії. В результаті елементи обладнання можуть виходити з ладу.* ***Мета.*** *Створення методу розрахункового аналізу поширення електромагнітного поля в системах провідних тіл і подальшого аналізу деформування. Застосування цього методу для дослідження процесів в системах електромагнітної обробки з метою визначення раціональних робочих параметрів, що забезпечують результат технологічної операції.* ***Методологія.*** *Використано варіаційну постановку задач про поширення електромагнітного поля та деформування систем провідних тіл. Чисельне моделювання та аналіз виконано методом скінченних елементів.* ***Результати.*** *У загальному вигляді отримано систему визначальних рівнянь для значень векторного магнітного потенціалу та переміщень. Вплив електромагнітного поля враховується введенням електромагнітних сил. Наведено результати розрахунків для технологічної системи електромагнітної обробки вигнутих тонкостінних заготовок.* ***Оригінальність.*** *Вперше представлено метод розрахункового аналізу, який передбачає моделювання в рамках єдиної розрахункової схеми як процесу поширення електромагнітного поля, так і процесу деформування.* ***Практичне значення.*** *Запропонований метод розрахункового аналізу може бути використаний для різних технологічних систем електромагнітного формування з метою визначення раціональних параметрів, що забезпечують як працездатність обладнання, так і мету технологічної операції – необхідне формоутворення заготовки.* Бібл. 18, табл. 1, рис. 3.

Ключові слова: **розрахунковий аналіз, електромагнітне поле, електромагнітна обробка, деформування, метод скінченних елементів.**

Introduction. A large number of technical and technological facilities are exploited under conditions of intense electromagnetic fields (EM-fields). Technologies that use EM-fields cause a variety of power, thermal effects on materials, influence on magnetic properties, and so on. The most important is the power effect that occurs when the EM-field acts on the conductive body and causes its motion or deformation. The above-stated facts indicate the necessity of using the computational methods analysis of EM-field propagation and deformation process of technological devices equipment elements of the electromagnetic forming (EMF) at the design and proofing stage. Thus, the scientific and applied problem, which consists in the creation of new computational methods for evaluating the EM-fields effect on elastic-plastic deformation of workpieces and equipment, taking into account the association of EM-field propagation and deformation processes, as well as computational investigations of the EM-field distribution and deformation processes under the conditions of specific technological operations is relevant, which determines the direction of this article.

The creation of computational methods for the analysis of any processes is based on an appropriate theoretical basis. Theoretical fundamentals describing models of continuum mechanics, which take into account the effect of the coupled fields of different physical nature (including electromagnetic) presents in classical works of Maugin, Nowacki, Eringen and others [1-4]. Within the framework of these models, the influence of an external EM-field on the thermomechanical state of the body is taken into account by introducing electromagnetic forces into the equilibrium equations. The presented model is based on Maxwell's equations, describing the nature of the electromagnetic field in vacuum and in moving deformed body, in accordance with its electromagnetic properties. For tasks in which the main objective is the analysis of the structural strength can be used the theory of magnetoelasticity. Fundamentals of the theory of magnetoelasticity with consideration of the coupling effects EM-field and mechanical stresses and strains in a moving conductive body (in the general case, the body is polarized or magnetized), were founded by Knopoff [5].

Force influence is used in the class of technological operations, called EMF. The technological equipment of EMF is deformed together with the workpieces under the influence of EM-field, which can lead to a reduction in durability and inoperability.

Modeling of forming and stamping workpieces processes dedicated [6, 7], in which, using the finite element method (FEM), highlights the solutions features of the coupled problems of magneto-thermo-elastic with regard to high-strain-rate deformation. The current state of issues related to the classification of EMF technological operations and descriptions of the corresponding equipment is comprehensively presented in review articles [8, 9]. It should be noted that non-traditional directions of the EMF are currently being developed. The basic questions of some modern trends in the development of EMF technologies are presented in articles [10-12].

The development of technological equipment for any EMF operation requires scrupulous computational studies. For example, works [13, 14] are devoted to these issues. An analysis of modern sources of information allows us to conclude that the most effective calculation tool in this case is the FEM. FEM allows in this case, within the framework of a single design scheme, to analyze the distribution of the main components of both the EM-field and the stress-strain state (SSS).

The goal of the paper is the theoretical substantiation and creation of a computational analysis method of the EM-field propagation and the process of conductive bodies deformation.

Mathematical formulation of the calculation analysis problem. For real technical and technological systems, which have a rather complex geometry and the deformation process is characterized by nonlinearities of various nature the solution process should be based on the use of appropriate numerical methods. FEM at the current stage of the computational mechanics development is the most suitable for solving the problems of the deformable body mechanics. Also, FEM has proven itself well for solving problems of various physical nature fields determine, including electromagnetic and thermal.

The construction of the FEM algorithm is based on weak formulations of the corresponding initial boundary value problems and is reduced to finding the stationary values of the corresponding functionals. Functionals can be obtained in various ways, for example, provided that the original differential equation is the Euler-Ostrogradsky equation for a certain functional, or the functional is constructed according to some general physical principle.

Certain difficulties arise when taking into account the nonlinearity of the process and the procedure of using functionals requires linearization of the original problem in one way or another, most often, an iterative process is built in which the original nonlinear problem is presented as a series of linearized problems.

The complete system of differential equations of the EM-field propagation initial-boundary problem and deformation of conductive bodies systems is presented in the articles [15, 16].

To construct functionals that correspond to the initial-boundary problem of EM-field propagation, we will consider the vector magnetic $\vec{A}$ and scalar electric φ potentials:

$$\vec{B}=\nabla\times\vec{A};\quad \nabla\cdot\vec{A}=0;\quad \vec{E}=-\frac{\partial\vec{A}}{\partial t}-\nabla\varphi\,, \tag{1}$$

here $\vec{B}$ is the magnetic induction vector; $\vec{E}$ is the vector of electric field intensity.

Initial and boundary conditions are formulated for vector magnetic and scalar electric potentials:

$$\vec{A}(0)=0;\ \varphi(0)=0. \tag{2}$$

$$\vec{A}\Big|_{\infty}=0;\ \varphi\Big|_{\infty}=0. \tag{3}$$

$$\begin{aligned}&\left.\frac{\partial\varphi_i}{\partial x_i}\right|_{\Gamma}=-E_{\Gamma i},\quad i=1,2,3;\\&\left.\left(\frac{\partial A_i}{\partial x_j}-\frac{\partial A_j}{\partial x_i}\right)\right|_{\Gamma}=B_{\Gamma k},\quad i\neq j\neq k=1,2,3.\end{aligned} \tag{4}$$

Boundary conditions (3) are applied when the body is considered together with the surrounding environment and model the attenuation of the EM-field at a distance. In the case when the EM-field components are specified at some boundary of the body, then (in the quasi-stationary case) the boundary conditions (4) are applied. Here, the symbol Γ means that the corresponding quantity belongs to the boundary of the body.

In the case of elastic deformation of the conductive bodies system, the solution is sought from the condition of minimum total energy E_{TOT}:

$$\delta E_{TOT}=0,\quad E_{TOT}=U+W\,, \tag{5}$$

where U is the energy of elastic deformation; W is the EM-field energy. The energy of elastic deformation is determined as follows:

$$\begin{aligned}&U=\frac{1}{2}\int_V\hat{\varepsilon}\cdot\cdot{}^{(4)}\hat{C}\cdot\cdot\hat{\varepsilon}dV-\int_S\vec{p}\cdot\vec{u}dS;\\&{}^{(4)}\hat{C}=-\frac{\nu E}{(1+\nu)(1-2\nu)}\hat{I}\otimes\hat{I}+\\&+\frac{E}{2(1+\nu)}\left(e_k\otimes\hat{I}\otimes e^k+e_i\otimes e_k\otimes e^i\otimes e^k\right),\end{aligned} \tag{6}$$

where $\hat{\varepsilon}$ is the tensor of deformations; $\vec{p}$ is the vector of surface mechanical loads; $\vec{u}$ is the vector of displacements; ν is the Poisson's ratio; E is the Young's modulus; $\hat{I}$ is the unit tensor; V is the body volume; S is the body surface on which mechanical loads and displacements are known.

The EM-field energy is generally defined as follows:

$$W=\int_V\left(\int_0^B\vec{H}d\vec{B}\right)dV+\int_V\left(\int_0^D\vec{E}d\vec{D}\right)dV, \tag{7}$$

where $\vec{H}$ is the vector of magnetic field intensity; $\vec{D}$ is the induction vector of electric field.

In the case of a linear relationship between the vectors that characterize the EM-field (or in the case of a linearized problem), the expression for the EM-field energy is simplified to the form:

$$\begin{aligned}W&=\frac{1}{2}\int_V\vec{H}\cdot\vec{B}dV+\frac{1}{2}\int_V\vec{E}\cdot\vec{D}dV=\\&=\frac{1}{2}\int_V\frac{1}{\mu_c}\left(\vec{B}\right)^2dV+\frac{1}{2}\int_V\varepsilon_c\left(\vec{E}\right)^2dV,\end{aligned}$$

where μ_c and ε_c are the magnetic and electrical permeability of the material.

Let's carry out the substitution through the potentials and get the following expression:

$$W = \int_V \left(\frac{1}{2\mu_c} \left| \vec{\nabla} \times \vec{A} \right|^2 - \vec{j} \times \vec{A} \right) dV + \\ + \int_V \left(\frac{\varepsilon_c}{2} \left| \vec{\nabla} \varphi \right|^2 - \rho_e \varphi \right) dV. \tag{8}$$

Formula (8) takes into account the possibility of EM-field sources – currents $\vec{j}$ and electric charges ρ_e distributed over the volume.

If we use formal mathematical approaches, then in the variational equation (5) we have three independent variables – scalar electric potential, vector magnetic potential and displacement, therefore the equality of zero of the total energy variation leads to three equalities:

$$\delta E_{TOT} = 0 \Rightarrow \begin{cases} \dfrac{\partial E_{TOT}}{\partial \varphi} = 0; \\ \dfrac{\partial E_{TOT}}{\partial \vec{A}} = 0; \\ \dfrac{\partial E_{TOT}}{\partial \vec{u}} = 0. \end{cases} \tag{9}$$

If we present the expressions for the elastic deformation energy and the EM-field energy in matrix-vector form:

$$U = \frac{1}{2} \int_V \{u\}^{\mathrm{T}} [K] \{u\} dV - \int_S \{p\} \{u\} dS; \tag{10}$$

$$W = \frac{1}{2} \int_V \{A\}^{\mathrm{T}} [M] \{A\} dV - \int_V \{J\} \{A\} dV + \\ + \frac{1}{2} \int_V \{\varphi\}^{\mathrm{T}} [\Sigma] \{\varphi\} dV - \int_V \{\rho_e\} \{\varphi\} dV, \tag{11}$$

where $[K]$ is the stiffness matrix; $[M]$ is the «magnetic» matrix; $[\Sigma]$ is the «dielectric» matrix; $\{u\}$, $\{p\}$, $\{A\}$, $\{\varphi\}$, $\{J\}$, $\{\rho_e\}$ are the column vectors of displacements, surface distributed forces, vector magnetic potential, electric potential, specified current densities and electric charge.

Then condition (9) leads to such a system of algebraic equations:

$$\begin{cases} [\Sigma]\{\varphi\} + [\Sigma_m]\{A\} + [\Sigma_k]\{u\} = \{\rho_e\}; \\ [M]\{A\} + [M_e]\{\varphi\} + [M_k]\{u\} = \{J\}; \\ [K]\{u\} + [K_\varphi]\{\varphi\} + [K_m]\{A\} = \{p\}, \end{cases} \tag{12}$$

the additional matrices that arose after the variation are defined as follows:

$$[\Sigma_m] = \frac{1}{2}\{A\}^{\mathrm{T}} \frac{\partial [M]}{\partial \{\varphi\}}, \quad [\Sigma_k] = \frac{1}{2}\{u\}^{\mathrm{T}} \frac{\partial [K]}{\partial \{\varphi\}},$$

$$[M_e] = \frac{1}{2}\{\varphi\}^{\mathrm{T}} \frac{\partial [\Sigma]}{\partial A};$$

$$[M_k] = \frac{1}{2}\{u\}^{\mathrm{T}} \frac{\partial [K]}{\partial \{A\}}, \quad [K_\varphi] = \frac{1}{2}\{\varphi\}^{\mathrm{T}} \frac{\partial [\Sigma]}{\partial u},$$

$$[K_m] = \frac{1}{2}\{A\}^{\mathrm{T}} \frac{\partial [M]}{\partial u},$$

where the matrices $[\Sigma_m]$, $[M_e]$ characterize the changes in the magnetic field due to the presence of an electric one, and vice versa, the matrices $[\Sigma_k]$, $[M_k]$ characterize the changes in the electric and magnetic fields due to deformation (i.e, piezo effects). When considering traditional structural materials, such changes are either absent or insignificant and these components can be neglected.

In order to find out the nature of the second and third components from the third equation, we will use the principle of virtual work, with the help of which we will determine the forces by which the EM-field acts on a conductive body. At the same time, we believe that the EM-field energy is completely spent on body deformation. In the general case of dependence between EM-field vectors, we obtain the following expression:

$$\vec{f}_{\mathrm{em}} = -\frac{\partial W}{\partial \vec{u}} = -\frac{\partial}{\partial \vec{u}} \left(\int_V \left(\int_0^B \vec{H} \mathrm{d}\vec{B} \right) \mathrm{d}V \right) - \\ -\frac{\partial}{\partial \vec{u}} \left(\int_V \left(\int_0^B \vec{E} \mathrm{d}\vec{D} \right) \mathrm{d}V \right) = -\int_V \vec{H} \frac{\partial \vec{B}}{\partial \vec{u}} \mathrm{d}V - \\ - \int_V \left(\int_0^B \frac{\partial \vec{H}}{\partial \vec{u}} \mathrm{d}\vec{B} \right) \mathrm{d}V - \int_V \vec{E} \frac{\partial \vec{D}}{\partial \vec{u}} \mathrm{d}V - \\ - \int_V \left(\int_0^B \frac{\partial \vec{E}}{\partial \vec{u}} \mathrm{d}\vec{D} \right) \mathrm{d}V. \tag{13}$$

In the case of considering a linear relationship between the vectors characterizing the EM-field distribution, we obtain the following expression for electromagnetic forces:

$$\{f_{\mathrm{em}}\} = -\frac{\partial W}{\partial \{u\}} = -\frac{\partial}{\partial \{u\}} \left(\frac{1}{2}\{A\}^{\mathrm{T}} [M] \{A\} \right) - \\ -\frac{\partial}{\partial \{u\}} \left(\frac{1}{2}\{\varphi\}^{\mathrm{T}} [\Sigma] \{\varphi\} \right) = \\ = -\frac{1}{2}\{A\}^{\mathrm{T}} \frac{\partial [M]}{\partial \{u\}} \{A\} - \frac{1}{2}\{\varphi\}^{\mathrm{T}} \frac{\partial [\Sigma]}{\partial \{u\}} \{\varphi\} = \\ = -[K_m]\{A\} - [K_\varphi]\{\varphi\}. \tag{14}$$

So, we see that the expression for electromagnetic forces is exactly the same as the sum of the second and third components of (12), i.e. in conditions of a conductive body elastic deformation under the action of EM-field; its influence is limited to electromagnetic forces distributed over the volume of the body. Elastic deformation, in turn, for the selected model does not affect the distribution of EM-field, therefore the analysis of the distribution of EM-field and the analysis of SSS taking into account electromagnetic forces in the case of quasi-stationary approximation can be carried out separately.

The first and second equations in the system (12) become independent (based on the results of their solution, we obtain the EM-field distribution), and on this basis, it is possible to solve the third equation taking into account the electromagnetic forces (12) for the purpose of SSS analysis. Thus, the system of defining equations of the problem takes the form:

$$\begin{cases}[\Sigma]\{\varphi\}=\{\rho_e\};\\ [M]\{A\}=\{J\};\\ [K]\{u\}=\{p\}+\{f_{em}\},\end{cases} \qquad (15)$$

$$\{f_{em}\}=-\frac{1}{2}\{A\}^{\mathrm{T}}\frac{\partial[M]}{\partial u}\{A\}-\frac{1}{2}\{\varphi\}^{\mathrm{T}}\frac{\partial[\Sigma]}{\partial u}\{\varphi\}.$$

In the resulting equations (15), electromagnetic forces are a consequence of the both magnetic and electrostatic fields' action; however, it is known that the force effect of an electrostatic field is many times smaller than that of a magnetic field. Based on this, the contribution of the electrostatic field can be neglected for the analysis of systems deformation that takes place in technologies based on strong magnetic fields. Moreover, in the quasi-stationary setting, the electrostatic field does not affect the magnetic field (independence of the first and second equations in (13)).

So, to analyze the deformation of conductive bodies systems under the action of large magnetic fields in the quasi-stationary approximation, the defining system of equations takes the form:

$$\begin{cases}[M]\{A\}=\{J\};\\ [K]\{u\}=\{p\}+\{f_{em}\},\end{cases} \qquad (16)$$

$$\{f_{em}\}=-\frac{1}{2}\{A\}^{\mathrm{T}}\frac{\partial[M]}{\partial u}\{A\}.$$

Calculation example. Let us consider the application of the proposed approach to the analysis of EM-field distribution and subsequent deformation for the case of EMF of thin-walled curved workpieces. In many cases, curved thin-walled metal workpieces are the basis for the manufacture of structural elements for various purposes.

Usually, the necessary curved structural elements are manufactured in two stages: in the first stage, they reach the required general (overall) dimensions and shape, in the second stage they achieve the required quality directly in the corner zone.

Part of the EMF technological operations is aimed at creating conditions for the occurrence of residual deformations in curved thin-walled metal workpieces directly in the bending zone. This group of technological operations was named technological operations of «filling corners». This term is known from the field of «traditional» pressure metal processing, and, in practice, it means the reduction of rounding radii to acceptable values in the bending zones of thin-walled workpieces. From the point of view of the technological operation conditions, it is necessary to exert the maximum force around the corner.

In works [17, 18], it is proposed to use an inductor with two turns, which have one common current line directed along the bend, to «fill the corners» on thin-walled curved workpieces, each of the turns is a plane that makes an angle of up to 15° with the wall of the workpiece.

Consider the results of EM-field calculations and deformation analysis for the design diagram shown in Fig. 1.

An electric current evenly distributed over the cross-section of the current conductor turns was considered as a source of EM-field. The magnitude of the non-zero component of the current density vector varied over time according to the law:

$$j(t)=j_m e^{-\delta\omega t}\sin(2\pi\nu t), \qquad (17)$$

where $j_m=\dfrac{4I_m}{\pi d^2}$ is the current density amplitude; I_m = 40 kA is the amplitude of the current in the pulse; ν = 2 kHz is the current frequency in the pulse; ω = $2\pi\nu$ is the cyclic frequency; δ = 0.3 is the attenuation coefficient; d is the diameter of the coil of the current conductor.

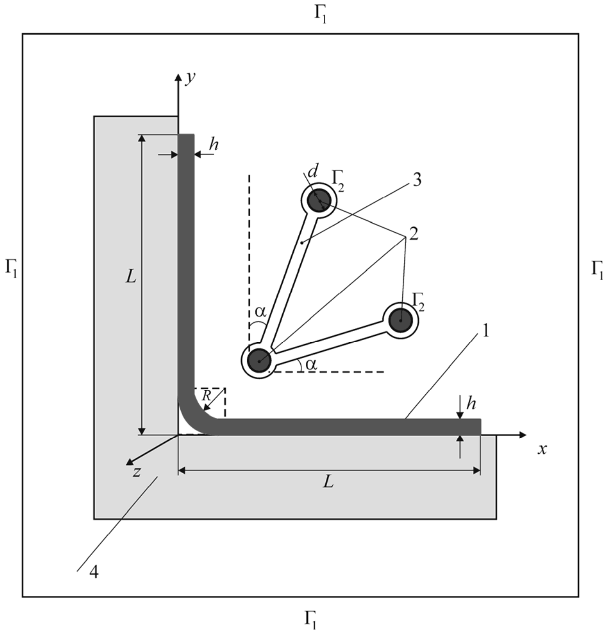

Fig. 1. Design diagram of a curved workpiece together with a two-turn inductor and a dielectric mold: 1 – workpiece; 2 – coils of the current conductor of the inductor; 3 – inductor insulation; 4 – dielectric mold

The solution was performed for zero initial conditions for one current pulse, in the time range from 0 to 3 ms, which guaranteed complete decay of the current in the pulse.

During calculations, the following values of geometric dimensions were considered: d = 10 mm, L = 100 mm, h = 2 mm, α = 15°. Finite element modeling was carried out using three nodal finite elements with a linear approximation of the corresponding (z) component of the vector magnetic potential and displacements (Fig. 2).

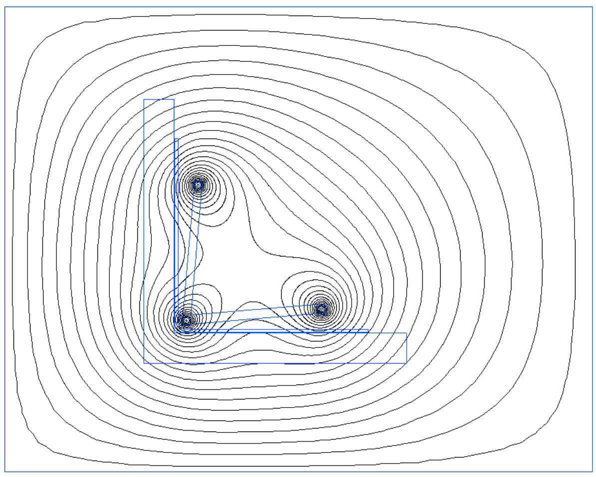

Fig. 2. Permanent lines of vector magnetic potential

The physical and mechanical parameters of the system elements, which were used in all subsequent calculations, are given in Table 1 (where σ_y is the yield strength of the material, σ_B^+ is the tensile strength limit, σ_B^- is the compressive strength limit.)

Table 1

Physic-mechanical parameters of system elements

	Current conductor, copper	Workpiece, aluminum alloy	Insulation, kaprolon	Dielectric mold, fiberglass
μ_r	1	1	1	1
γ, $(\Omega \cdot m)^{-1}$	$7 \cdot 10^7$	$4.6 \cdot 10^7$	0	0
E, GPa	120	71	2.5	200
ν	0.33	0.29	0,3	0.27
σ_y, MPa	380	190	–	–
σ_B^+, MPa	–	–	70	100
σ_B^-, MPa	–	–	90	120

For the considered value of the current in the inductor, the maximum value of the stress intensity in the workpiece, which is observed on the workpiece surface, is 227 MPa (Fig. 3), which is greater than the yield strength of the aluminum alloy, thus it can be stated that from the point of view of the plastic deformations possibility in the workpiece, the technological operation is efficient.

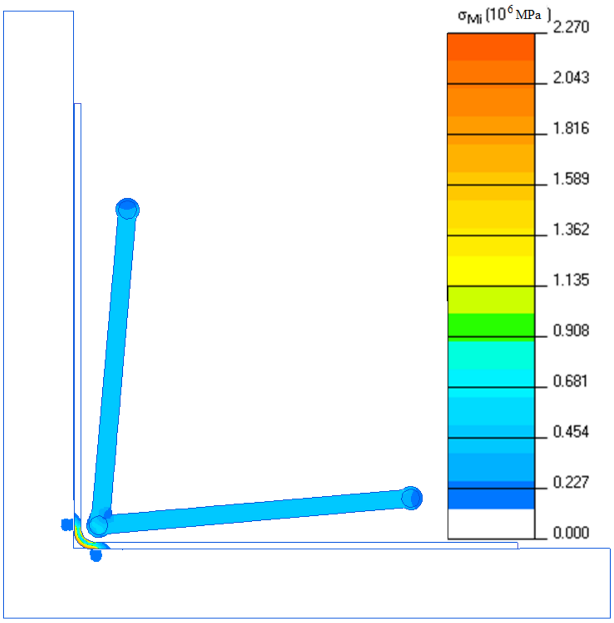

Fig. 3. Distribution of stress intensity

Note that the maximum values of normal stresses also occur on the workpiece surface. The largest displacement values are observed in the middle part of the workpiece rounding, i.e. directly opposite the current conductor of the inductor. Their maximum value is 5.4 mm, that is, under the operating conditions considered, the initial rounding of the workpiece is reduced by approximately 50 %. Note that the maximum intensity of stress in the current conductor of the inductor is approximately 60 MPa, which does not exceed the yield strength of the material, the maximum value of the equivalent stress according to Mohr's criterion in the insulation of the inductor is 52 MPa, which also does not exceed the limit of the tensile strength of the material. So, it can be concluded that in this case the inductor remains operational.

Conclusions. The prerequisites are considered and the necessity of creating computational methods for analyzing the propagation of an electromagnetic field and the further process of technological systems elements deformation of electromagnetic forming is substantiated.

To create an appropriate method of computational analysis, the main variational relations based on the principle of minimum total energy of the system are given. For a correct and convenient description of the electromagnetic field propagation processes, the concepts of scalar electric and vector magnetic potentials are introduced. Formulas for the energy of the electromagnetic field and the energy of elastic deformation are presented. For the case of elastic deformation of conductive bodies subjected to the action of an electromagnetic field, a system of resolving algebraic equations for the values of the vector magnetic potential and displacements is obtained in general form. The influence of the electromagnetic field is taken into account by introducing electromagnetic forces, the expression for which is also obtained.

As an illustration of the computational analysis proposed method application, the computational analyzing of a technological system for electromagnetic forming of thin-walled curved workpieces is considered.

The further development of this work consists in extending the proposed method of computational analysis to the cases of various nature nonlinearities and carrying out calculations for complex technological systems of electromagnetic forming.

Conflict of interest. The authors of the article declare that there is no conflict of interest.

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