UDC 621.317.727.1 https://doi.org/10.20998/2074-272X.2023.3.08

V.O. Brzhezitsky, Y.O. Haran, Y.O. Trotsenko, O.R. Protsenko, A.O. Derzhuk, M.M. Dixit

Ultimate effect of non-identity of resistive elements of high-voltage arm on frequency characteristics of broadband voltage divider (analytical research)

Purpose. *Determination in the analytical form of the maximum limiting influence of the non-identity of the resistive elements of the high-voltage arm on the amplitude-frequency characteristic and phase-frequency characteristic of the voltage divider with parallel-series connection of R-, C-elements of the high-voltage arm.* ***Methodology****. Based on the previously developed theory of broadband voltage dividers with parallel-series connection of R-, C-elements, analytical expressions for amplitude-frequency and phase-frequency characteristics of the voltage divider are obtained and investigated taking into account the limit case of non-identical resistive elements of high-voltage arm.* ***Results****. The nature of the dependencies of the frequency characteristics of the broadband voltage divider on the value of the tolerance of the resistive elements of the high-voltage arm, the division factor of the voltage divider in a wide range of frequency changes are determined. Simplified approximating expressions for the maximum values of frequency characteristics of the voltage divider are proposed and their error is determined.* ***Originality****. For the first time in the analytical form the limiting influence of non-identity of resistive elements of a high-voltage arm of a voltage divider on its frequency characteristics is considered. A mathematical model of this influence is constructed and the limit values of frequency characteristics of the voltage divider are determined.* ***Practical value****. It is recommended to introduce into the normative documentation of broadband voltage dividers the corrected value of the division factor, which allows to significantly reduce the deviation of the actual value of the division factor of the voltage divider from the normalized value in a wide range of frequency changes.* References 16, tables 3, figures 3.

Key words: **high-voltage divider, frequency characteristics, analytical expressions, tolerance of resistive elements, parameters adjustment.**

На основі раніше розвинутої теорії широкосмугових подільників напруги з паралельно-послідовним з'єднанням R-, C-елементів вперше одержані аналітичні вирази для амплітудно-частотної та фазо-частотної характеристик подільника напруги з урахуванням граничного випадку неідентичності резистивних елементів високовольтного плеча. Визначений загальний характер залежностей частотних характеристик від значення допуску резистивних елементів, коефіцієнта ділення подільника напруги в широкому діапазоні зміни частоти. Запропоновані спрощені апроксимуючі вирази для максимальних значень частотних характеристик та визначено їх похибки. Рекомендовано уведення в нормативно-технічну документацію широкосмугових подільників напруги відкоригованого значення коефіцієнта ділення. Бібл. 16, табл. 3, рис. 3.

Ключові слова: **високовольтний подільник напруги, частотні характеристики, аналітичні вирази, допуск резистивних елементів, коригування параметрів.**

Introduction. For the normal functioning of electric power systems, information about the instantaneous values of high voltage in certain areas is very important. Traditionally, for more than 100 years, electromagnetic voltage transformers have been used and continue to be used for this purpose [1]. The advantage of electromagnetic voltage transformers is high load capacity, which allows to complete various secondary circuits based on them, including of relay protection and control. There are even «DC voltage» transformers. This term refers to a converter consisting of a high-voltage DC resistor, a magnetic amplifier controlled by the DC of this resistor, and a rectifier for the output voltage of the magnetic amplifier. As a result, the output DC voltage of such a converter is proportional to its input voltage, and the converter is characterized by high load capacity. However, a significant disadvantage of voltage transformers is inertia. In this regard, they are not actually used to register fast-moving processes, when, on the contrary, a quick response of control systems is required. The situation improves significantly with the transition to the «digital substation» concept, when secondary circuits can be built on the basis of computer systems with minimal energy consumption. Here, high-voltage transformers can be replaced by broadband voltage dividers, which can be used to obtain information about instantaneous values of high voltage. This will allow, on the one hand, to significantly improve the management of power systems and, on the other hand, to obtain complete information about the quality of electricity online.

The goal of the work is to continue previous research [2] and to study the ultimate impact of the non-identity of non-capacitive but resistive elements of the high-voltage arm on the amplitude-frequency and phase-frequency characteristics of the voltage divider.

It should be noted that in [2] the influence of the non-identity of only the capacitive elements of the high-voltage arm on the characteristics of the voltage divider was considered.

General information about broadband voltage dividers. It should be noted that the corresponding development of research on high-voltage broadband voltage dividers was realized mainly in the last 50 years. The processes that take place in high-voltage dividers are much more complicated than in voltage transformers. This is due to the variety of types of voltage dividers, ranges of their parameters and modes of use.

In research on high-voltage dividers in recent years [3–15], considerable attention is paid to increasing the accuracy of their mathematical models (up to the level of several ppm), stability of parameters, taking into account various factors, features of metrological calibration and normalization of characteristics. The considered substitution circuits of various types of high-voltage dividers are built on the use of shielded parallel-series connections of resistive and capacitive elements of the high-voltage arm, formed, as a rule, from the same (identical) elements. As a rule, the selection of the nominal values of resistive and capacitive elements is performed based on approximately the same conductivity of the corresponding branches of the electric circuit at the main frequency of the voltage divider. Here, during high-frequency transients in the electric circuit of the voltage divider, the capacitive components of the branches of this circuit are more conductive (for example, the conductivity of the capacitive branch between two nodes of the electric circuit of the high-voltage arm of the voltage divider is 4-5 orders of magnitude greater than the parasitic capacitive conductivity between these nodes), therefore, they practically shunt the parasitic capacitive leakage of currents from the connection nodes of lumped circuit elements to grounded surfaces and circuit elements that are under a different potential. As a result, the design of a voltage divider with a series-parallel connection of resistive and capacitive lumped elements is the most effective in the development of broadband voltage dividers. In different operating modes of the voltage divider, the conductances of the resistive and capacitive branches of its substitution circuit change, so the influence on the error of the scale transformation coefficient of the voltage divider is a complex function of the dependence on the values of the resistances and capacities of the concentrated elements, as well as the current frequency. However, in reality, the used R_{RV}, C_{RV} elements have a tolerance:

$$R_N(1-\beta) \le R_{RV} \le R_N(1+\beta),$$

$$C_N(1-\Delta) \le C_{RV} \le C_N(1+\Delta),$$

where R_N, C_N are the nominal values of resistive and capacitive elements; β, Δ are the values of tolerances in relative units determined by the manufacturer. The influence of tolerances depends on their value, as well as the type of distribution of parameters within the tolerance. The latter is usually not normalized. Therefore, it is justified to consider (for the first time) the limit variant, when the capacitive elements of the high-voltage arm have equally probable values:

$$C'_{RV} = C_N(1-\Delta), \quad C''_V = C_N(1+\Delta).$$

This case was considered in a previous work [2]. This article considers (for the first time) another limit variant, when the resistive elements have the value:

$$R''_{RV} = R_N(1-\beta), \quad R''_{RV} = R_N(1+\beta).$$

Mathematical model of the voltage divider and study of the amplitude-frequency characteristic (frequency response). According to [1], broadband voltage dividers consist of a large number of resistive and capacitive elements connected in parallel-series (see Fig. 1).

In Fig. 1: U_{in} – the input high voltage; U_{out} – the output low voltage; R_i and C_i – the elements of the high-voltage arm; r and c – the elements of the low-voltage arm.

The values of resistance and capacity, respectively, of resistors and capacitors included in the voltage divider can change under the influence of external conditions over time (temperature, humidity, etc.). In this regard, there is a need to study the frequency characteristics of the voltage divider in view of the non-identity of its components.

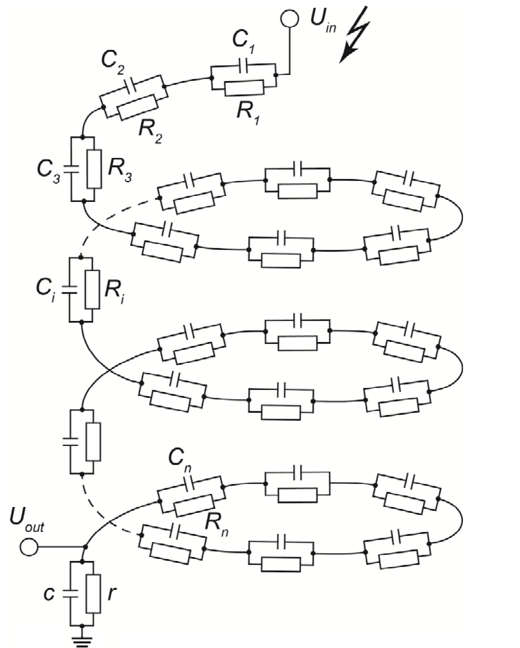

Fig. 1. Schematic diagram of a capacitive-ohmic voltage divider [1]

The schematic diagram of the voltage divider shown in Fig. 1 does not show parasitic capacitive branches, because, as was shown above, the influence of leakage of parasitic currents of a capacitive nature is significantly smaller (by 2-3 orders of magnitude) compared to the deviations of the actual parameters of the lumped elements from the nominal values.

The non-identity of the resistive and capacitive elements of the voltage divider affects the stability of its frequency characteristics, especially the pronounced nature of this effect on frequency response. Reducing frequency response changes in the operating frequency range is an important factor in improving the transmission characteristics of the measuring device. According to [1], the frequency response of the voltage divider is determined by (1), (2):

$$A = \frac{1}{K} A^* ; \qquad (1)$$

$$A^* = \sqrt{\frac{1+\gamma^2}{\left(1+\frac{K-1}{K} f\right)^2 + \gamma^2 \left(1+\frac{K-1}{K}\delta\right)^2}} , \qquad (2)$$

where A is the frequency response; A^* is the reduced frequency response; K is the nominal value of the division coefficient of the broadband voltage divider; f and δ are the averaged parameters that take into account the non-identity of the elements of the parallel-series connection of resistive R_i and capacitive C_i elements of the high-voltage arm of the capacitive-ohmic voltage divider.

The dimensionless parameter γ depends on the angular frequency ω and is defined as:

$$\gamma = \omega R_0 C_0 ; \qquad (3)$$

$$R_0 = \frac{1}{n}\sum_{i=1}^{n} R_i ; \qquad (4)$$

$$C_0 = \frac{1}{n}\sum_{i=1}^{n} C_i , \qquad (5)$$

where R_0 and C_0 are the average values of the elements of the high-voltage arm; n is the number of elements of the high-voltage arm.

The values of the parameters of the low-voltage arm are usually determined as follows:

$$r = \frac{nR_0}{K-1}, \quad c = \frac{C_0}{n}(K-1).$$

From a generalized consideration of the frequency characteristics of a broadband voltage divider with a parallel-serial connection of R-, C-elements of the high-voltage arm [1] for this case we have: $R_0 = R_N$; $C_1 = C_2 = \ldots = C_N = C$; $\Delta = 0$. Parameters f, Δ are the non-identity functions of resistive elements $\beta' = -\beta$ and $\beta'' = +\beta$, which are defined as:

$$f = \frac{1}{2}D(\beta') + \frac{1}{2}D(\beta''); \quad (6)$$

$$\delta = \frac{1}{2}G(\beta') + \frac{1}{2}G(\beta''), \quad (7)$$

where

$$D(\beta) = \frac{\gamma^2\left(-3\beta^2 - \beta^3 + \gamma^2\left(\beta^2 + \beta^3\right)\right)}{\left(1+\gamma^2\right)\left(1+\gamma^2(1+\beta)^2\right)}; \quad (8)$$

$$G(\beta) = \frac{\beta^2 - \gamma^2\left(3\beta^2 + 2\beta^3\right)}{\left(1+\gamma^2\right)\left(1+\gamma^2(1+\beta)^2\right)}. \quad (9)$$

As a result, we obtain:

$$2f = \frac{\gamma^2\left(-3\beta^2 + \beta^3 + \gamma^2\left(\beta^2 - \beta^3\right)\right)}{\left(1+\gamma^2\right)\left(1+\gamma^2(1-\beta)^2\right)} + \frac{\gamma^2\left(-3\beta^2 - \beta^3 + \gamma^2\left(\beta^2 + \beta^3\right)\right)}{\left(1+\gamma^2\right)\left(1+\gamma^2(1+\beta)^2\right)}; \quad (10)$$

$$2\delta = \frac{\beta^2 - \gamma^2\left(3\beta^2 - 2\beta^3\right)}{\left(1+\gamma^2\right)\left(1+\gamma^2(1-\beta)^2\right)} + \frac{\beta^2 - \gamma^2\left(3\beta^2 + 2\beta^3\right)}{\left(1+\gamma^2\right)\left(1+\gamma^2(1+\beta)^2\right)}. \quad (11)$$

Below are the test checks of the resulting ratios that were performed:

1) if $\beta = 0$, then $A^* \equiv 1$ for any values of γ, K;

2) if $\gamma = 0$, then also $A^* \equiv 1$ for any values of β, K; if $\gamma \to \infty$, similarly, $A^* \equiv 1$ for any values of β, K.

The results of the conducted tests confirm the adequacy of the used mathematical model to the physical object under study.

To study the dependence, similarly to [2], we apply the approach when it is possible to find the limiting expressions under the conditions $\gamma \to 0$ and $\gamma \to \infty$. Substituting $\gamma \to 0$ into expressions (10), (11) gives the dependencies:

$$f_{\gamma\to 0} = -3\beta^2\gamma^2; \quad \delta_{\gamma\to 0} = \beta^2 - \gamma^2\left(5\beta^2 + \beta^4\right). \quad (12)$$

In turn, using (12) under the condition $\gamma \to 0$ allows (2) to obtain the limiting expression:

$$A^*_{\gamma\to 0} = 1 + \frac{K-1}{K}\beta^2\gamma^2\left(2 - \frac{K-1}{2K}\beta^2\right). \quad (13)$$

That is, A^* grows from γ in a parabolic dependence with the coefficient $\frac{K-1}{K}$ and β^2. The expression in parentheses (13) is a small variable value and in the range $0 \le \beta \le 0.2$ is 2...1.98 (for $K \to \infty$).

Substituting $\gamma \to \infty$ into expressions (10), (11) provides:

$$f_{\gamma\to\infty} = \frac{\beta^2 - \beta^4}{\left(1-\beta^2\right)^2}; \quad \delta_{\gamma\to\infty} = \frac{1}{\gamma^2}\frac{3\beta^2 - \beta^4}{\left(1-\beta^2\right)^2},$$

and, finally:

$$A^*_{\gamma\to\infty} = 1 + \frac{K-1}{K}\beta^2\frac{1}{\gamma^2}\left(\frac{3-\beta^2}{2\left(1-\beta^2\right)^2}\right). \quad (14)$$

The expression in parentheses (14) in the range $0 \le \beta \le 0.2$ is 1.5...1.6, i.e. it is a slightly variable quantity.

The results obtained in (13), (14) allow a purposeful approach to further research of the frequency response of the voltage divider based on computerized calculations.

Further calculations were made of the dependencies of $A^*(\gamma)$ for different values of β and K. In Fig. 2 the resulting graphs of $A^*(\gamma)$ at $\beta = 0.05$ and $\beta = 0.2$ for $K = 10$ and $K = 10^6$ in the range of γ change from 0.001 to 1000 are plotted. Dependencies of $A^*(\gamma)$ have a typical maximum in the region $\gamma \approx 1$. The influence of the maximum in the regions of $\lg\gamma \le -1.5$ and $\lg\gamma \ge +1.5$ is negligible.

Figure 2 shows: curve 1 – the dependence $A^*(\gamma)$ at $\beta = 0.05$ and $K = 10$; curve 2 – the dependence $A^*(\gamma)$ at $\beta = 0.05$ and $K = 10^6$; curve 3 – the dependence $A^*(\gamma)$ at $\beta = 0.2$ and $K = 10$; curve 4 – the dependence $A^*(\gamma)$ at $\beta = 0.2$ and $K = 10^6$.

To find the maximum $A^*_{\max}$, it is necessary to equate the derivative $\frac{dA^*}{d\gamma}$ to zero and to determine the value of $\gamma_{\max}$ from this condition. By substituting this value in (2), using (10), (11), it is possible to obtain the desired value of $A^*_{\max}$. In connection with the complex dependence of A^* on the initial values, which practically makes it impossible to carry out these operations in an analytical form, software tools were used to find $A^*_{\max}(\gamma_{\max})$.

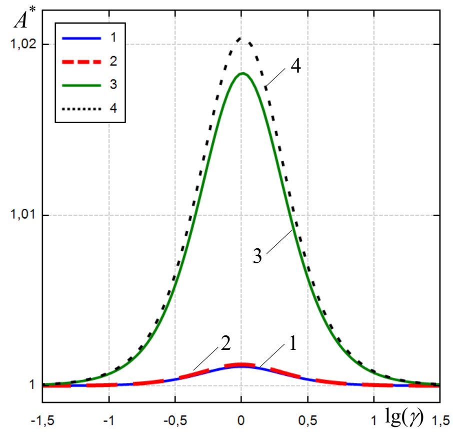

Fig. 2. Graph of the reduced frequency response depending on the dimensionless frequency parameter γ in a semi-logarithmic scale

In the program package SMath Solver [16], the functional dependence $A^*(\gamma)$ was deduced, after which, with the help of mathematical modules of this program package, $\gamma_{\max}$ was found for the extremum point and the value of the extremum $A^*_{\max}$ of this function at different β and K (through iterative calculations in the program cycle). Data arrays of various combinations of parameters were obtained.

Table 1 shows examples of the obtained results of the calculation of $A^*_{\max}$, $\gamma_{\max}$ for values of β = 0.01; 0.02; ... 0.2 and the value of $K = 10^1; 10^2; 10^3; 10^4; 10^6$. The analysis of the obtained data is given in the next section.

Table 1

Results of calculations of $A^*_{\max}(\gamma_{\max})$ for $K = 10^1; 10^2; 10^3; 10^4; 10^6$

K	10		100		1000		10000		1000000	
β	$\gamma_{\max}$	$A_{\max}$	$\gamma_{\max}$	$A_{\max}$	$\gamma_{\max}$	$A_{\max}$	$\gamma_{\max}$	$A_{\max}$	$\gamma_{\max}$	$A_{\max}$
0,01	1,000068779	1,000045002	1,000068779	1,000049502	1,000068779	1,000049952	1,000068779	1,000049997	1,000068779	1,000050002
0,02	1,000210615	1,000180032	1,000210615	1,000198039	1,000210615	1,00019984	1,000210615	1,00020002	1,000210615	1,00020004
0,03	1,000473922	1,000405164	1,000473923	1,000445699	1,000473923	1,000449752	1,000473923	1,000450158	1,000473923	1,000450202
0,04	1,000785403	1,000720519	1,000785401	1,000792628	1,000785401	1,000799839	1,000785401	1,00080056	1,000785401	1,00080064
0,05	1,001228359	1,001126267	1,001228355	1,001239033	1,001228355	1,001250311	1,001228355	1,001251439	1,001228355	1,001251563
0,06	1,00181747	1,001622629	1,001817473	1,001785181	1,001817474	1,001801439	1,001817474	1,001803065	1,001817474	1,001803244
0,07	1,002475774	1,002209873	1,00247578	1,002431397	1,002475781	1,002453555	1,002475781	1,002455771	1,002475781	1,002456015
0,08	1,003236693	1,002888318	1,003236703	1,003178068	1,003236705	1,003207052	1,003236705	1,003209951	1,003236705	1,00321027
0,09	1,00410082	1,003658335	1,004100836	1,004025641	1,004100838	1,004062386	1,004100838	1,004066061	1,004100838	1,004066465
0,1	1,005026594	1,004520342	1,005026583	1,004974624	1,005026582	1,005020075	1,005026582	1,005024621	1,005026582	1,005025121
0,11	1,006092522	1,00547481	1,006092507	1,00602559	1,006092506	1,006080701	1,006092506	1,006086213	1,006092506	1,006086819
0,12	1,007264098	1,006522264	1,007264079	1,007179173	1,007264077	1,007244911	1,007264076	1,007251485	1,007264076	1,007252209
0,13	1,008542457	1,007663279	1,008542432	1,008436072	1,00854243	1,008513416	1,008542429	1,008521152	1,008542429	1,008522002
0,14	1,00992884	1,008898485	1,009928808	1,009797051	1,009928805	1,009886996	1,009928805	1,009895991	1,009928805	1,009896981
0,15	1,011424591	1,010228564	1,011424552	1,011262941	1,011424548	1,011366495	1,011424548	1,011376852	1,011424548	1,011377991
0,16	1,013031165	1,011654257	1,013031119	1,012834641	1,013031115	1,01295283	1,013031114	1,012964651	1,013031114	1,012965951
0,17	1,014750131	1,013176359	1,014750077	1,014513117	1,014750072	1,014646987	1,014750071	1,014660376	1,014750071	1,014661849
0,18	1,01658317	1,014795722	1,016583109	1,01629941	1,016583103	1,016450024	1,016583102	1,016465088	1,016583102	1,016466745
0,19	1,018532086	1,016513258	1,018532017	1,018194629	1,01853201	1,018363072	1,01853201	1,018379919	1,01853201	1,018381773
0,2	1,020598807	1,018329939	1,020598732	1,020199959	1,020598724	1,020387339	1,020598723	1,020406081	1,020598723	1,020408142

Analysis of frequency response results. Processing of the obtained data array allows us to propose a simplified expression for $A^*_{\max}$ in the form:

$$A^*_{\max} = 1 + 0{,}505113\frac{K-1}{K}\beta^2 . \qquad (15)$$

Formula (15) is applicable for any values $\beta \leq 0.2$ and $K \geq 10$. Here, the error of only the additional term in the right-hand side of (15) in relation to the exact data does not exceed ±1 % in absolute value, which can be considered quite acceptable.

Analyzing the obtained data, it can be noted that the non-identity of the resistive elements of the high-voltage arm of the voltage divider can lead to a significant increase in its error (up to 2 % or more). It is possible to halve this error value by using a corrected frequency response value:

$$A^*_{cor} = 1 + 0{,}25256\frac{K-1}{K}\beta^2 . \qquad (16)$$

Expression (16) can be entered in the technical documentation (passport) of the voltage divider.

The development of the use of high-voltage broadband voltage dividers, including commercial implementation, requires the possibility of «quick assessment» of the quality of their frequency characteristics based on the initial data on the elemental «base», which can be determined using formula (15).

Just as for capacitive elements [2], the influence of the non-identity of resistive elements (15) is proportional to the multiplier $\frac{K-1}{K}$, thus, it is maximal for high-voltage dividers.

For values $1 < K < 10$, additional research is required.

The considered theory of voltage dividers with a parallel-series connection of resistive and capacitive elements can be successfully applied to the study of the so-called «capacitor» high-voltage insulation, when each layer of insulation can be represented by a parallel connection of resistive and capacitive elements. As a rule, for such insulation, the condition $C_1 = C_2 = \ldots = C_i = \ldots = C_n = C$ is used, while the non-identity of R-elements may be related to the wetting of individual layers of insulation or the deterioration of their properties over time. With regard to this option of using the considered theory, it should be emphasized that expressions (1)–(11) do not assume a small value of the parameter β, that is, they can be applied in the general case when β, for example, reaches values of 0.9; 0.99, etc., and any layer of «capacitor» insulation can be considered as the low-voltage arm of the voltage divider.

Study of the phase-frequency characteristic (PFC). According to [1], PFC of a voltage divider with a parallel-serial connection of R-, C-elements of the high-voltage arm is described by the expression:

$$\varphi = \mathrm{arctg}\left(\frac{(\delta - f)\gamma}{f + \frac{K}{K-1} + \gamma^2\left(\delta + \frac{K}{K-1}\right)}\right), \qquad (17)$$

where f, δ, γ, K have the same values as in (3) – (11).

Similarly (12) – (14), we can use the approach of determining the limit values in the approximations $\gamma \to 0$ and $\gamma \to \infty$. Here, we obtain:

$$\varphi_{\gamma \to 0} = \frac{K-1}{K}\beta^2\gamma , \qquad (18)$$

$$\varphi_{\gamma\to\infty} = -\frac{K-1}{K}\beta^2\frac{1}{\gamma}\left(1-\beta^2\right)^{-1}. \qquad (19)$$

Expression (19) has a factor $\left(1-\beta^2\right)^{-1}$, which under the conditions $0 \le \beta \le 0.2$ is an invariant value of 1...1.04.

Since the deviations from zero under the conditions $\gamma\to 0$ and $\gamma\to\infty$ are multipolar, it will be useful to determine φ at an intermediate point, for example, at $\gamma = 1$. The corresponding transformations according to (17) give the expression:

$$\varphi_{\gamma=1} = \operatorname{arctg}\left(\frac{2\beta^4(K-1)}{K\left(8+2\beta^4-4\beta^2\right)+4\beta^2}\right). \qquad (20)$$

Estimation of the right-hand side of (20) at $\beta = 0.2$ and $K\to\infty$ gives $\varphi_{\gamma=1}$ = arctg(0.000407), which corresponds to φ = 1.4$'$. Thus, all investigated dependencies of $\varphi(\gamma)$ will, in fact, pass at $\gamma = 1$ in the range 0...1.4$'$.

Figure 3 shows the curves of changes in the frequency response of the voltage divider φ (in arc minutes) on the dimensionless frequency parameter γ (when it changes from 0.001 to 1000) for values of $\beta = 0.05$; $\beta = 0.2$ and $K = 10$; $K = 10^6$. For clarity, the scale on the abscissa is shown on a logarithmic scale (from lgγ = –3 to lgγ = +3). The deviation of PFC from the zero value is negligible for lg$\gamma \le -2.5$ and lg$\gamma \ge 2.5$.

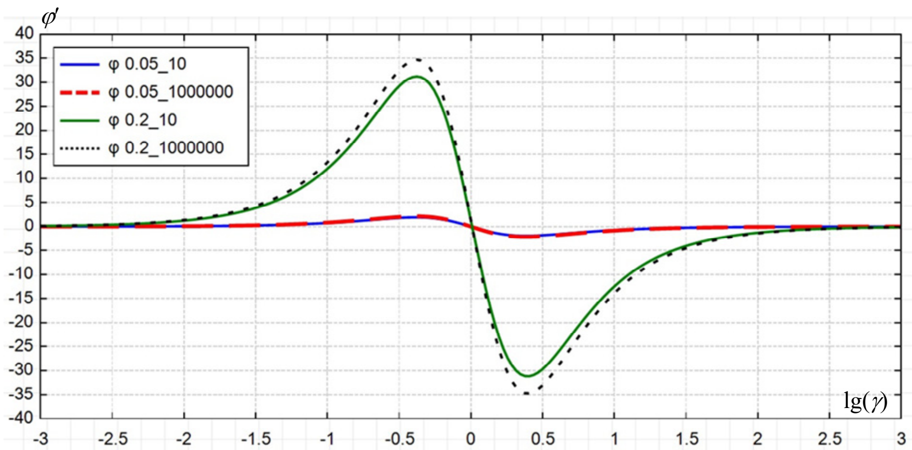

Fig. 3. Graph of PFC depending on the dimensionless frequency parameter γ in the semi-logarithmic scale

Next, similarly to the previous section, computerized calculations were performed for the two extrema of the dependence $\varphi(\gamma)$, and for the region $\gamma < 1$ it was data $\varphi_{max}(\gamma'_{max})$, and for the region $\gamma > 1$, respectively, $\varphi_{min}(\gamma'_{min})$.

Table 2 shows the results of the performed calculations of $\varphi_{max}(\gamma'_{max})$ for the values β = 0.01; 0.02; ... 0.20 and $K = 10^1; 10^2; 10^3; 10^4; 10^6$.

Table 3 shows the results of the performed calculations of $\varphi_{min}(\gamma'_{min})$ for the values β = 0.01; 0.02; ... 0.20 and $K = 10^1; 10^2; 10^3; 10^4; 10^6$.

Table 2

Results of calculations of $\varphi_{max}(\gamma'_{max})$ for $K = 10^1; 10^2; 10^3; 10^4; 10^6$

K	10		100		1000		10000		1000000	
β	γ'_{max}	φ_{max}, '	γ'_{max}	φ_{max}, '	γ'_{max}	φ_{max}, '	γ'_{max}	φ_{max}, '	γ'_{max}	φ_{max}, '
0,01	0,414210916	0,077351043	0,414211575	0,085086338	0,414211641	0,08585987	0,414211648	0,085937223	0,414211648	0,085945732
0,02	0,414248876	0,309425058	0,414251515	0,340370627	0,414251779	0,343465215	0,414251805	0,343774674	0,414251808	0,343808714
0,03	0,414312168	0,696284723	0,414318108	0,765928709	0,414318702	0,772893263	0,414318761	0,77358972	0,414318768	0,77366633
0,04	0,414400826	1,238034555	0,414411392	1,361887062	0,414412449	1,374272804	0,414412555	1,375511383	0,414412566	1,375647627
0,05	0,414514901	1,934820995	0,414531425	2,128422909	0,414533077	2,147784299	0,414533243	2,149720451	0,414533261	2,149933427
0,06	0,414654456	2,78683252	0,414678276	3,065764371	0,414680658	3,093660044	0,414680896	3,096449636	0,414680922	3,096756492
0,07	0,414819573	3,794299795	0,414852033	4,174190665	0,414855279	4,212184367	0,414855604	4,215983784	0,414855639	4,21640172
0,08	0,415010343	4,957495853	0,415052799	5,454032349	0,415057045	5,503693882	0,41505747	5,508660114	0,415057517	5,509206401
0,09	0,415226875	6,276736314	0,415280694	6,905671609	0,415286077	6,968577785	0,415286615	6,974868529	0,415286675	6,975560512
0,1	0,415469294	7,752379633	0,415535855	8,529542593	0,415542513	8,607278195	0,415543178	8,615051949	0,415543252	8,615907064
0,11	0,415737737	9,384827387	0,415818434	10,32613179	0,415826506	10,42029055	0,415827313	10,42970671	0,415827402	10,43074249
0,12	0,416032358	11,1745246	0,416128601	12,29597846	0,416138229	12,40816403	0,416139192	12,41938299	0,416139298	12,42061708
0,13	0,416353328	13,12196007	0,416466545	14,43967512	0,416477872	14,57150209	0,416479004	14,58468534	0,416479129	14,5861355
0,14	0,416700831	15,22766682	0,416832469	16,75786802	0,41684564	16,91096292	0,416846957	16,92627315	0,416847102	16,92795729
0,15	0,41707507	17,49222248	0,417226597	19,2512578	0,417241759	19,42726011	0,417243275	19,44486133	0,417243442	19,44679748
0,16	0,417476262	19,91624974	0,417649169	21,92060003	0,417666472	22,12116327	0,417668203	22,14122088	0,417668393	22,14342723
0,17	0,417904643	22,50041691	0,418100447	24,76670594	0,418120043	24,9934987	0,418122003	25,01617962	0,418122218	25,01867454
0,18	0,418360464	25,24543842	0,418580708	27,79044313	0,418602752	28,04515017	0,418604957	28,07062294	0,4186052	28,07342497
0,19	0,418843996	28,15207543	0,419090252	30,99273635	0,419114902	31,27705969	0,419117367	31,3054946	0,419117639	31,30862247
0,2	0,419355527	31,22113644	0,419629397	34,37456834	0,419656814	34,69022842	0,419659556	34,7217976	0,419659858	34,72527025

Table 3

Results of calculations of $\varphi_{min}(\gamma'_{min})$ for $K = 10^1$; 10^2; 10^3; 10^4; 10^6

K	10		100		1000		10000		1000000	
β	γ'_{min}	φ_{min}, '	γ'_{min}	φ_{min}, '	γ'_{min}	φ_{min}, '	γ'_{min}	φ_{min}, '	γ'_{min}	φ_{min}, '
0,01	2,414387533	-0,077351043	2,414383691	-0,085086338	2,414383307	-0,08585987	2,414383268	-0,085937223	2,414383264	-0,085945732
0,02	2,414890851	-0,309425058	2,414875476	-0,340370628	2,414873938	-0,343465215	2,414873784	-0,343774674	2,414873768	-0,343808715
0,03	2,415730325	-0,696284723	2,41569571	-0,76592871	2,415692248	-0,772893264	2,415691902	-0,773589721	2,415691864	-0,773666331
0,04	2,416906878	-1,238034557	2,416845284	-1,361887064	2,416839125	-1,374272806	2,416838509	-1,375511385	2,416838441	-1,375647628
0,05	2,418421802	-1,934820997	2,418325453	-2,128422912	2,418315818	-2,147784302	2,418314854	-2,149720453	2,418314748	-2,14993343
0,06	2,420276774	-2,786832524	2,420137838	-3,065764375	2,420123944	-3,093660049	2,420122554	-3,096449641	2,420122401	-3,096756496
0,07	2,422473853	-3,7942998	2,422284433	-4,17419067	2,42226549	-4,212184373	2,422263596	-4,215983789	2,422263387	-4,216401726
0,08	2,425015494	-4,95749586	2,424767616	-5,454032356	2,424742827	-5,50369389	2,424740348	-5,508660122	2,424740075	-5,509206409
0,09	2,427904558	-6,276736323	2,427590154	-6,905671619	2,427558711	-6,968577795	2,427555567	-6,974868539	2,427555221	-6,975560522
0,1	2,431144318	-7,752379644	2,430755217	-8,529542605	2,430716303	-8,607278208	2,430712412	-8,615051961	2,430711984	-8,615907076
0,11	2,434738475	-9,384827401	2,434266386	-10,32613181	2,434219171	-10,42029056	2,43421445	-10,42970672	2,43421393	-10,4307425
0,12	2,438691168	-11,17452461	2,438127666	-12,29597848	2,438071308	-12,40816405	2,438065672	-12,41938301	2,438065052	-12,4206171
0,13	2,443006988	-13,12196009	2,4423435	-14,43967514	2,44227714	-14,57150211	2,442270504	-14,58468536	2,442269774	-14,58613552
0,14	2,447690993	-15,22766685	2,446918783	-16,75786805	2,446841547	-16,91096294	2,446833823	-16,92627318	2,446832973	-16,92795731
0,15	2,452748722	-17,4922225	2,451858873	-19,25125783	2,451769868	-19,42726014	2,451760968	-19,44486136	2,451759989	-19,44679751
0,16	2,458186212	-19,91624977	2,457169613	-21,92060007	2,457067928	-22,12116331	2,457057759	-22,14122091	2,45705664	-22,14342727
0,17	2,464010018	-22,50041694	2,462857344	-24,76670598	2,462742044	-24,99349874	2,462730514	-25,01617966	2,462729245	-25,01867458
0,18	2,470227226	-25,24543846	2,468928924	-27,79044317	2,468799052	-28,04515021	2,468786065	-28,07062298	2,468784636	-28,07342501
0,19	2,476845481	-28,15207548	2,475391749	-30,9927364	2,475246324	-31,27705974	2,475231781	-31,30549465	2,475230181	-31,30862251
0,2	2,483873006	-31,22113649	2,482253776	-34,3745684	2,482091788	-34,69022848	2,482075589	-34,72179766	2,482073807	-34,7252703

Analysis of PFC results. The «impressive» factor is the practical coincidence of the absolute values of φ_{max} and φ_{min} (up to 8 significant figures, that is, up to the error of the calculations) at the same values of β and K.

The region of PFC deviation from zero is more «stretched» in γ ($-2.5 < \lg\gamma < 2.5$) compared to frequency response ($-1.5 < \lg\gamma < 1.5$), which is explained by the degree of dependence on γ in the corresponding expressions (18), (19), compared to (13), (14).

Processing of the received array of calculation data given in Table 2, 3, allows us to propose the following simplified expressions:

$$\varphi_{max}^{/} = 863{,}8\frac{K-1}{K}\beta^2\text{, arc minutes,} \tag{21}$$

$$\varphi_{min}^{/} = -863{,}8\frac{K-1}{K}\beta^2\text{, arc minutes.} \tag{22}$$

Formulas (21), (22) are applicable for any values of $\beta \leq 0.2$ and $K \geq 10$. Here, the error (21), (22) in relation to the exact calculated values according to (17) does not exceed ± 0.5 %, which is quite acceptable.

If the ranges $\gamma \leq 1$ or, conversely, $\gamma \geq 1$ are used in certain studies, correction values for φ can be introduced, which are 50 % of the values given in (21), (22).

A comparison of the obtained results with the data of publications [2–15] shows that the effect of the non-identity of the resistive elements of the high-voltage arm of the voltage divider is significant compared to such factors of parameter instability as temperature change, shielding imperfection, the influence of external electric fields, frequency change and should be taken into account in the theory and the practice of voltage dividers, especially for reference measurements.

Conclusions. The ultimate influence of the non-identity of the resistive elements of the high-voltage arm of the voltage divider on its amplitude-frequency and phase-frequency characteristics is quantitatively determined.

It is shown that this influence is proportional to the factor $\frac{K-1}{K}$, where K is the nominal value of the division coefficient of the voltage divider.

It is proposed to introduce the corrected value of the amplitude-frequency characteristic into the technical documentation of the voltage dividers, which makes it possible to reduce their error by a factor of two.

The carried out development of the theory of voltage dividers can be successfully applied to the study of processes in «capacitor» high-voltage insulation.

The materials of the article can be used for an express assessment of the quality of broadband high-voltage dividers, based on data on their element base.

A comparison of the obtained results with materials [2] shows that the ultimate effect of the non-identity of the resistive elements of the high-voltage arm gives fundamentally different results compared to the ultimate effect of the non-identity of the capacitive elements of the high-voltage arm of the voltage divider.

Conflict of interest. The authors of the article declare that there is no conflict of interest.

REFERENCES

1. Anokhin Y.L., Brzhezytskyi V.O., Haran Ya.O., Masliuchenko I.M., Protsenko O.P., Trotsenko Ye.O. Application of high voltage dividers for power quality indices measurement. *Electrical Engineering & Electromechanics*, 2017, no. 6, pp. 53-59. doi: https://doi.org/10.20998/2074-272x.2017.6.08.

2. Brzhezitsky V.O., Haran Y.O., Derzhuk A.O., Protsenko O.R., Trotsenko Y.O., Dixit M.M. Ultimate effect of non-identity of capacitive elements of high-voltage arm on frequency characteristics of voltage divider (analytical research). *Electrical Engineering & Electromechanics*, 2021, no. 4, pp. 46-52. doi: https://doi.org/10.20998/2074-272X.2021.4.06.

3. Li D., Liu K., Lei M., Zhou F., Yue C., Yu J. Study on the ratio change measurement of 1000 kV HVDC divider based on improved DC voltage summation method. *High Voltage*, 2020, vol. 5, no. 2, pp. 202-208. doi: https://doi.org/10.1049/hve.2019.0127.

4. Havunen J., Hällström J. Reference switching impulse voltage measuring system based on correcting the voltage divider response with software. *IEEE Transactions on Instrumentation and Measurement*, 2021, vol. 70, pp. 1-8. art. no. 1006008. doi: https://doi.org/10.1109/tim.2021.3063753.
5. Hrbac R., Kolar V., Bartlomiejczyk M., Mlcak T., Orsag P., Vanc J. A development of a capacitive voltage divider for high voltage measurement as part of a combined current and voltage sensor. *Elektronika ir Elektrotechnika*, 2020, vol. 26, no. 4, pp. 25-31. doi: https://doi.org/10.5755/j01.eie.26.4.25888.
6. Alf-Peter E., Hällström J., Bergman A. Optimization of the design of a wideband 1000 kV resistive reference divider. *XVII International Symposium on High Voltage Engineering*, Hannover, Germany, August 22-26, 2011.
7. Kovacevic U.D., Stankovic K.D., Kartalovic N.M., Loncar B.B. Design of capacitive voltage divider for measuring ultrafast voltages. *International Journal of Electrical Power & Energy Systems*, 2018, vol. 99, pp. 426-433. doi: https://doi.org/10.1016/j.ijepes.2018.01.030.
8. Thümmler T., Marx R., Weinheimer C. Precision high voltage divider for the KATRIN experiment. *New Journal of Physics*, 2009, vol. 11, no. 10, art. no. 103007. doi: https://doi.org/10.1088/1367-2630/11/10/103007.
9. Panko V., Banas S., Burton R., Ptacek K., Divin J., Dobes J. Enhanced Model of Nonlinear Spiral High Voltage Divider. *Radioengineering*, 2015, vol. 24, no. 1, pp. 130-136. doi: https://doi.org/10.13164/re.2015.0130.
10. Abdel Mageed H.M., El-Rifaie A.M., Aladdin O.M. Traceability of DC high voltage measurements using the Josephson voltage standard. *Measurement*, 2014, vol. 58, pp. 269-273. doi: https://doi.org/10.1016/j.measurement.2014.08.029.
11. Abdel Mageed H.M., Salah Eldeen R.S. Adapted Technique for Calibrating Voltage Dividers of AC High-Voltage Measuring Systems. *MAPAN*, 2020, vol. 35, no. 1, pp. 11-17. doi: https://doi.org/10.1007/s12647-019-00334-8.
12. Lee S.H., Yu K.M., Choi J.Y., Jang S.M. Low-Uncertainty Equality Between the Voltage-Dividing and Resistance Ratio of a DC Resistive High Voltage Divider. *Journal of Electrical Engineering & Technology*, 2019, vol. 14, no. 4, pp. 1789-1795. doi: https://doi.org/10.1007/s42835-019-00157-2.
13. Li Q., Wang L., Zhang S., Tang Y., Xu Y. Method to Determine the Ratio Error of DC High-Voltage Dividers. *IEEE Transactions on Instrumentation and Measurement*, 2012, vol. 61, no. 4, pp. 1072-1078. doi: https://doi.org/10.1109/TIM.2011.2178672.
14. Pan Feng, Xiao Yong, Lin Guoying, Xiao Xia, Shuai Hang. Analysis of the influencing factors for the 500 kV DC voltage reference divider used for on-site calibration. *2015 12th IEEE International Conference on Electronic Measurement & Instruments (ICEMI)*, 2015, pp. 25-29. doi: https://doi.org/10.1109/ICEMI.2015.7494180.
15. Boyko M.I., Syomkin S.O. Investigation of amplitude-temporal characteristics of a high-voltage resistive voltage divider. *Electrical Engineering & Electromechanics*, 2019, no. 4, pp. 59-68. doi: https://doi.org/10.20998/2074-272X.2019.4.09.
16. *MathStudio Manual*. Available at: http://mathstud.io/manual/ (accessed 22 May 2022).

Received 28.07.2022
Accepted 22.10.2022
Published 06.05.2023

V.O. Brzhezitsky[1], *Doctor of Technical Science, Professor,*
Y.O. Haran[1], *PhD,*
Y.O. Trotsenko[1], *PhD, Associate Professor,*
O.R. Protsenko[1], *PhD, Associate Professor,*
A.O. Derzhuk[1], *Postgraduate Student,*
M.M. Dixit [2], *Postgraduate Student,*
[1] National Technical University of Ukraine «Igor Sikorsky Kyiv Polytechnic Institute»,
37, Prospect Peremohy, Kyiv-56, 03056, Ukraine,
e-mail: y.garan@kpi.ua (Corresponding Author)
[2] Vishwaniketan Institute of Management Entrepreneurship and Engineering Technology,
Survey No. 52, Kumbhivali, Tal, Khalapur, Maharashtra, 410202, India,
e-mail: mandardixit78@gmail.com

How to cite this article:
Brzhezitsky V.O., Haran Y.O., Trotsenko Y.O., Protsenko O.R., Derzhuk A.O., Dixit M.M. Ultimate effect of non-identity of resistive elements of high-voltage arm on frequency characteristics of broadband voltage divider (analytical research). *Electrical Engineering & Electromechanics*, 2023, no. 3, pp. 52-58. doi: https://doi.org/10.20998/2074-272X.2023.3.08