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Acceleration of exit to steady-state mode when modeling semiconductor converters

*The **purpose** of the article is to develop a method and algorithm for the accelerated calculation of steady states of thyristor converters using computer models of converters based on the use of the theory of difference equations in the form of recurrent linear relationships for state variables on the boundaries of the converter periods. **Methodology.** The article is devoted to the solution of the problem of reducing the cost of computer time to achieve the steady state of the thyristor converter. For this, it is proposed to use difference equations, for which the values of the state variables at the limits of the periods of the converter's operation are taken as variables. These values are accumulated during the initial periods of the transient process of the converter, after which the coefficients of the difference equations are calculated, and the following limit values of the state variables are found using the defined difference equations. A program in the algorithmic language of the MATLAB system is presented, which implements the proposed method and algorithm compatible with the visual model of the converter. **Results**. The theoretical foundations of the proposed method and the area of its applicability are substantiated. Recommendations are presented for determining the number of periods of the flow process that must be calculated for further implementation of the method. An algorithm for forming matrix relations for determining the coefficients of difference equations with respect to the values of state variables at the boundaries of periods is shown. Matrix equations are given that allow calculating the parameters of the steady state. All stages of the algorithm are illustrated with numerical examples. **Originality.** The method rationally combines all the advantages of visual modeling based on the numerical integration of equations using the method of state variables for the periods of operation of the converter with the analytical solution of the recurrence relations obtained on this basis for the values of state variables at the boundaries of adjacent periods. **Practical value.** The proposed method makes it possible to reduce by several orders of magnitude the computer time spent on calculating the parameters of the steady-state mode of the converter and, at the same time, to significantly improve the accuracy of these calculations. The practical application of the method is very effective in research and design of thyristor converters of electrical energy parameters.* References 10, tables 2, figures 4.

Key words: **thyristor converter, state variables, difference equations, steady state, visual model**.

Стаття присвячена вирішенню проблеми зменшення витрат комп'ютерного часу для досягнення усталеного режиму тиристорного перетворювача. Для цього запропоновано використати різницеві рівняння, для яких в якості змінних приймаються значення змінних стану на межах періодів роботи перетворювача. Ці значення накопичуються на початкових періодах перехідного процесу перетворювача, після чого вираховуються коефіцієнти різницевих рівнянь, і наступні межові значення змінних стану знаходяться з використанням визначених різницевих рівнянь. Представлена програма на алгоритмічній мові системи MATLAB, яка реалізує запропоновані метод і алгоритм сумісно з візуальною моделлю перетворювача. Бібл. 10, табл. 2, рис. 4.

Ключові слова: **тиристорний перетворювач, змінні стану, різницеві рівняння, усталений режим, візуальна модель.**

Introduction. Problem definition. Research and design of thyristor converters at the current stage cannot be imagined without the use of computer models [1]. The functioning of such models is based on the piecewise linear approximation of the volt-ampere characteristics of the valve elements [2, 3]. Here, to simulate electromagnetic processes in the converter, the fitting method is used, in which the solution is stitched together from the links of solutions of linear differential equations describing the behavior of the converter in the interval of invariance of the state of the valve elements of the converter. Thus, the converter model spends computer time on analyzing the structure of the power part of the converter, forming graphs and topological matrices, finding the coefficients of linear differential equations by the method of, say, state variables, integrating the system of differential equations by a numerical method capable of overcoming the problem of system rigidity, and also for calculating valve switching moments and determining the next state of the valves [2]. Steady-state modes are usually in well-known programs for modeling converters by the setting method [4], which actually simulates the real start-up of the converter, usually from zero initial values for the state variables, which are capacitor voltages and inductance currents. To achieve a steady-state mode of the converter, it is necessary to calculate a large number of periods of the transient process. This process of reaching a steady-state mode, which in a real converter takes a certain real time and is considered fundamentally inevitable and necessary, can take a significant amount of computer time in computer models. The problem deepens when the process of exiting to a stable mode is slowed down. This happens when there are reactive elements in the converter circuit which slowly accumulate large amounts of electromagnetic energy, as well as in cases of weakly damped converter circuits [5]. Added to this is the increase in simulation time when trying to increase the accuracy of calculations by reducing the step of integration of systems of differential equations of the converter during the periods of the transient process of setting the mode. When modeling converters in the MATLAB/Simulink/SimPowerSystem computer system, the factor that this system uses the interpreter mode, when the conversion of operators into machine command code is carried out at each call to the operator, is also involved, which is especially sensitive when implementing cyclic algorithms, so characteristic for modeling converters. Therefore, when the converter circuit is complicated, the simulation time increases significantly, as was observed, for example, when modeling steady-state modes in three-phase thyristor reactive power compensators. And therefore, the solution to the problem of speeding up calculations of steady-state modes of converters, and even just electrical systems, in computer modeling does not lose its **relevance** even at the present time

In works [5, 6], the determination of steady-state parameters of a single-phase rectifier with a third-order smoothing filter based on Newton method is considered. Work [7] was carried out in the same direction. In these works, finding a solution is connected with the calculation of derivatives and carrying out an iterative process. In [8], the replacement of the integration of state equations by difference equations is considered, but not a DC converter is analyzed, but only a substitute circuit without semiconductor switches, which are necessarily included in the circuits of power parameter converters. It is also worth noting that the application of this method requires calculations using rather cumbersome analytical expressions. Steady-state processes in active converter systems are discussed in [9, 10]. The methods proposed in them are not of a general nature, but take into account the peculiarities of pulse width modulation, which is used in power converters of this class only.

The goal of the article is to develop a method and algorithm for accelerated calculation of steady states of thyristor converters using computer models of converters based on the use of the theory of difference equations in the form of recurrent linear relationships for state variables on the boundaries of converter periods.

The main part of the study.

1. Study of the transient process of starting the inverter. We will consider the circuit of a single-phase autonomous current inverter on thyristors, which is used in practice for systems of high-frequency induction heating of metal [1, 2]. The structure of the circuit is clear from the converter model in the SimPowerSystem system [3], which is shown in Fig. 1.

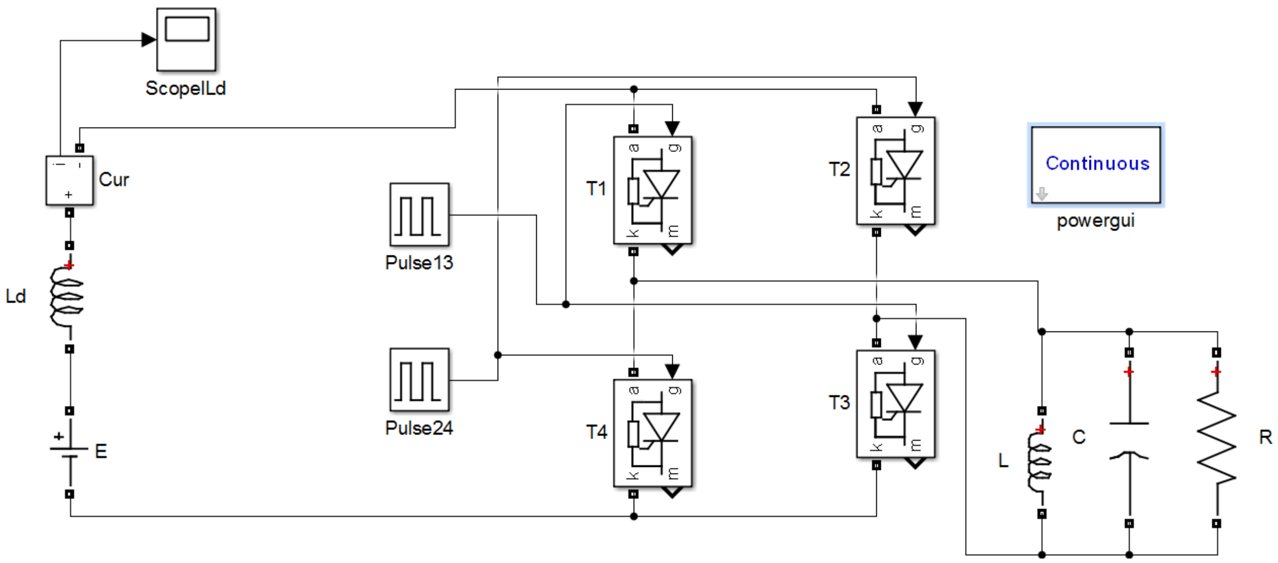

Fig. 1. The investigated model of the converter in SimPowerSystem

The inverter is fed from a constant voltage source E through a choke Ld with a large inductance. The voltage from the source is applied to the vertical diagonal of the semiconductor bridge consisting of thyristors T1–T4. An inverter load is connected to the horizontal diagonal, consisting of a switching capacitor C and active-inductive complex resistance R and L. Normalized circuit parameters: E = 100 V, Ld = 40 H, L = 1 H, C = 0.111 F, R = 50 Ω. The control period of thyristors is taken as 2 s, it is specified in the property windows of the corresponding virtual thyristor control pulse generators. With zero initial conditions, the inverter start-up process was simulated. The simulation results are presented in the form of time diagrams in Fig. 2, namely: *a* – the voltage on the capacitor, *b* – the current in the load inductance, *c* – the current in the input chokes Ld.

From Fig. 2, it is especially clear that the start-up process is weakly damped, which entails the need to run a large number of periods to achieve a steady-state of the inverter.

Difference equations for periods. We will proceed from the fact that on the intervals of invariance of the state of the thyristors, the substitute circuits of the inverter are linear and are described by systems of linear differential equations. This, in turn, determines the linear dependencies between the values of the variables of the state at the boundaries of the periods. We will use the following designations of state variables in the future: voltage on the switching capacitor $v_C = x_1$; current in the load inductance $i_{Ld} = x_3$; input choke current $i_{Ld} = x_3$. Then for the adjacent k-th and (k+1)-th boundaries of the periods, the following difference equations can be drawn up:

$$\begin{aligned} x_1^{k+1} &= a_{11}x_1^k + a_{12}x_2^k + a_{13}x_3^k + b_1E; \\ x_2^{k+1} &= a_{21}x_1^k + a_{22}x_2^k + a_{23}x_3^k + b_2E; \\ x_3^{k+1} &= a_{31}x_1^k + a_{32}x_2^k + a_{33}x_3^k + b_3E. \end{aligned} \quad (1)$$

In these equations, the superscripts mean the numbers of adjacent boundaries, on which the values of the inverter state variables are fixed. To determine the unknown coefficients of these equations, it is enough to have information about the values of the state variables at the boundaries of several initial periods of the starting transition process. The number of periods that must be calculated using the model should be equal to the sum of the number of reactive elements and power sources of the converter. For the inverter under consideration, taking k = 0, 1, 2, 3 consecutively and using only the first equation of the system (1), we obtain the following system of equations:

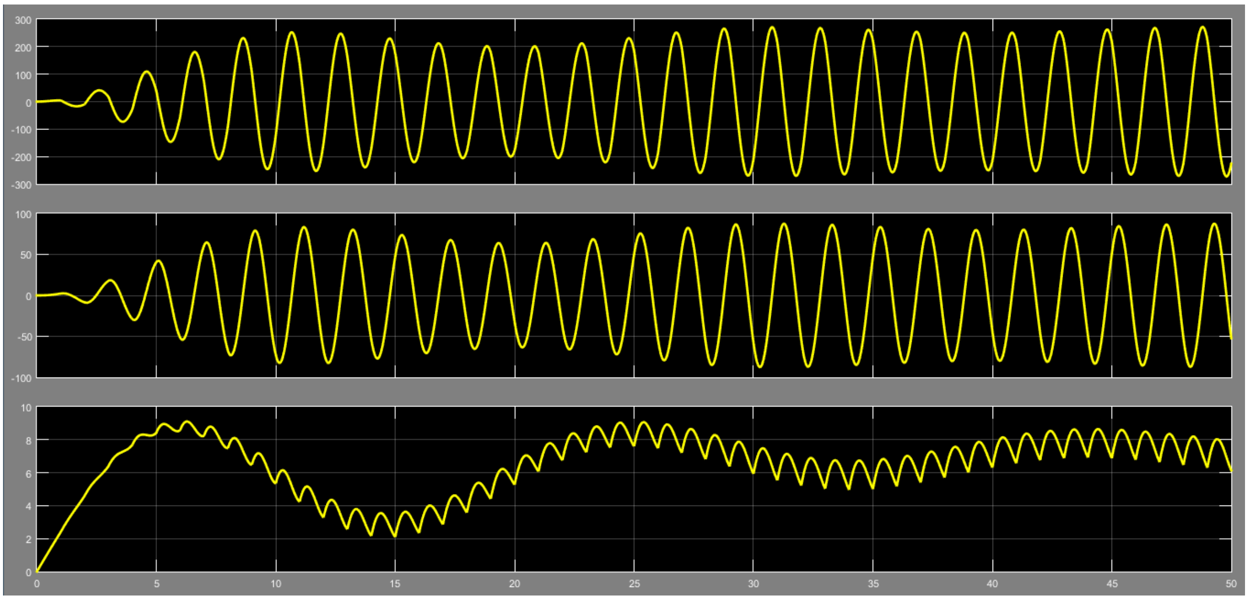

Fig. 2. Time diagrams:
a – the voltage on the capacitor, *b* – the current in the load inductance, *c* – the current in the input choke Ld

$$\begin{aligned} x_1^1 &= a_{11}x_1^0 + a_{12}x_2^0 + a_{13}x_3^0 + b_1 E; \\ x_1^2 &= a_{11}x_1^1 + a_{12}x_2^1 + a_{13}x_3^1 + b_1 E; \\ x_1^3 &= a_{11}x_1^2 + a_{12}x_2^2 + a_{13}x_3^2 + b_1 E; \\ x_1^4 &= a_{11}x_1^3 + a_{12}x_2^3 + a_{13}x_3^3 + b_1 E. \end{aligned} \quad (2)$$

Considering the coefficients a_{11}, a_{12}, a_{13}, b_1 as unknown values, we rewrite the system of equations (2) in the following matrix form:

$$\begin{bmatrix} x_1^0 & x_2^0 & x_3^0 & E \\ x_1^1 & x_2^1 & x_3^1 & E \\ x_1^2 & x_2^2 & x_3^2 & E \\ x_1^3 & x_2^3 & x_3^3 & E \end{bmatrix} \times \begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ b_1 \end{bmatrix} = \begin{bmatrix} x_1^1 \\ x_1^2 \\ x_1^3 \\ x_1^4 \end{bmatrix}. \quad (3)$$

To solve the resulting system of linear algebraic equations, the inverse matrix method can be used, and then the solution with respect to unknown coefficients can be written in the form:

$$\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ b_1 \end{bmatrix} = \begin{bmatrix} x_1^0 & x_2^0 & x_3^0 & E \\ x_1^1 & x_2^1 & x_3^1 & E \\ x_1^2 & x_2^2 & x_3^2 & E \\ x_1^3 & x_2^3 & x_3^3 & E \end{bmatrix}^{-1} \times \begin{bmatrix} x_1^1 \\ x_1^2 \\ x_1^3 \\ x_1^4 \end{bmatrix}. \quad (4)$$

Similarly, the coefficients of the remaining equations of the system (1) are found. It is worth noting that in this case the inverse square matrix does not change, and only the values of the elements of the column matrices in the left and right parts of the last matrix relation change.

After determining the coefficients, the system of equations (1) can be written in expanded matrix form:

$$\begin{bmatrix} x_1^{k+1} \\ x_2^{k+1} \\ x_3^{k+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \times \begin{bmatrix} x_1^k \\ x_2^k \\ x_3^k \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \times E. \quad (5)$$

In the shortened matrix form, the last system is written as follows:

$$\boldsymbol{X}_{k+1} = \boldsymbol{A} \times \boldsymbol{X}_k + \boldsymbol{B} \times \boldsymbol{E}. \quad (6)$$

This matrix recurrent equation allows, by determining the vector $\boldsymbol{X}^0$ of the initial values of the state variables, to calculate the next values of the state variables on the boundaries of the periods until reaching the steady-state, when these values on the adjacent boundaries will be repeated within the limits of the permissible error. Evidently, the expenses of computer time during such a steady-state mode will be several orders of magnitude smaller compared to the integration of differential equations with a fairly small step during the entire time the inverter model reaches steady-state modes. If it is still necessary to investigate the process during a certain period, it is enough to use the values of the state variables at the beginning of this period.

It is possible to speed up the acquisition of steady-state parameters, assuming that after endless use of (6) for $k \to \infty$ we assume that $X^k = X^{k+1} = X^\infty$, and then the last matrix equation takes the form:

$$\boldsymbol{X}^\infty = \boldsymbol{A} \times \boldsymbol{X}^\infty + \boldsymbol{B} \times \boldsymbol{E}. \quad (7)$$

Solving this matrix equation with respect to the vector $\boldsymbol{X}^\infty$, we obtain the following matrix expression for finding the values of the state variables at the beginning of the steady-state period:

$$\boldsymbol{X}^\infty = (1 - \boldsymbol{A})^{-1} \times \boldsymbol{B} \times \boldsymbol{E}. \quad (8)$$

The use of equation (8) makes it possible to speed up the calculation of the steady-state parameters of the converter even more.

Results of numerical analysis. With the specified parameters of the inverter, the visual model of the inverter (Fig. 1) was run during the first four periods of the start-up process. Here, the values of the state variables were fixed at the boundaries of the periods with their recording in the MATLAB workspace. The results obtained in this way are copied from the workspace and presented in Table 1.

Table 1
Values of state variables at the boundaries of the start-up process

k	$x_1^k = v_c^k$	$x_2^k = i_L^k$	$x_3^k = i_{Ld}^k$
0	0	0	0
1	–10,050	–8,4836	4,595
2	–27,2585	–28,851	7,694
3	–54,555	–51,443	8,581
4	–88,909	–67,490	7,492

Now, to find the coefficients of the first equation of the system (1), we use the matrix relationship (4), in which we substitute the specific numerical values of the state variables of the converter on the boundaries of the periods, borrowed directly from the table:

$$\begin{bmatrix} a_{11} \\ a_{12} \\ a_{13} \\ b_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 100 \\ -10{,}050 & -8{,}4836 & 4{,}595 & 100 \\ -27{,}2585 & -28{,}851 & 7{,}694 & 100 \\ -54{,}555 & -51{,}443 & 8{,}581 & 100 \end{bmatrix}^{-1} \times$$

$$\times \begin{bmatrix} -10{,}050 \\ -27{,}2585 \\ -54{,}555 \\ -88{,}909 \end{bmatrix} = \begin{bmatrix} 0{,}82234 \\ 0{,}48584 \\ -1{,}04908 \\ -0{,}10050 \end{bmatrix}.$$

All coefficients of system equations (1) are calculated in a similar way. Taking into account these calculations, the matrix expression (8) takes the form:

$$\begin{bmatrix} v_C^{\infty} \\ i_L^{\infty} \\ i_{Ld}^{\infty} \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0{,}82234 & 0{,}48584 & -1{,}04908 \\ -0{,}053928 & 0{,}64337 & -3{,}36204 \\ -0{,}002911 & 0{,}084051 & 0{,}82288 \end{bmatrix} \right) \times \begin{bmatrix} -0{,}10050 \\ -0{,}084836 \\ 0{,}045959 \end{bmatrix} \times 100 = \begin{bmatrix} -228{,}673 \\ -49{,}19248 \\ 6{,}36275 \end{bmatrix}.$$

The found values of the state variables were further used as the initial values of the capacitor voltage and inductance currents. In this case, the steady-state mode is immediately established in the inverter. This is evidenced by the time diagrams presented in Fig. 3, 4. Figure 3 shows the time diagrams of the voltage on the capacitor and the currents of the load inductances and the input choke during the first four periods of the start-up process. Figure 4 shows the corresponding diagrams obtained as a result of the simulation of the steady-state process, obtained after running the model with the initial values of the state variables found using the proposed method.

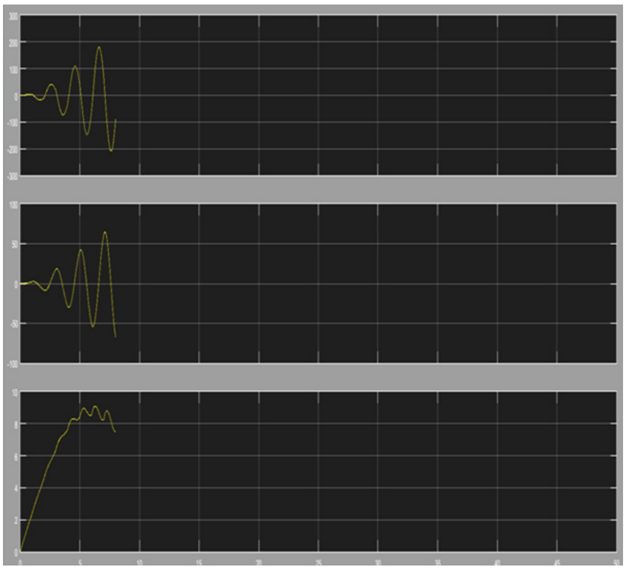

Fig. 3. Initial start-up periods of the inverter

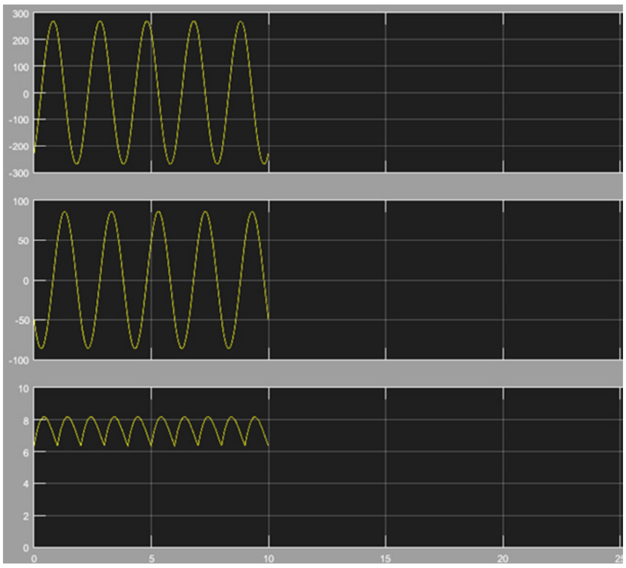

Fig. 4. Steady-state inverter mode

Table 2 allows to quantitatively assess the accuracy of determining the initial values of the inverter state variables for the steady-state using the proposed method.

Table 2
Values of state variables at the boundaries of the steady-state

k	$x_1 = v_c$	$x_2 = i_L$	$x_3 = i_{Ld}$
0	–228,673701	–49,192482	6,362751
1	–228,673719	–49,192489	6,362753
2	–228,673740	–49,192499	6,362754
3	–228,673765	–49,192509	6,362755
4	–228,673789	–49,192516	6,362754
5	–228,673812	–49,192518	6,362753

Here are the results of calculations of five periods of the steady-state of the inverter, represented by the values of the state variables at the boundaries of the periods of operation of the converter. As can be seen from Table 2, numerical values change from period to period only in 5-7 significant digits of the obtained results, which proves the high efficiency and accuracy of the proposed method. According to the described algorithm, a program was compiled in the algorithmic language of the MATLAB system. This program interacts with the converter visual model and the system workspace using built-in functions to implement matrix operations. The use of this program makes it possible to quickly determine the parameters of steady-state e modes in the circuits of other converters with regular alternating states of semiconductor power devices.

Conclusions. A method of determining the steady-state mode parameters of semiconductor converters based on the use of visual models of converters and the transition to recurrent formulas linking the values of state variables at the boundaries of periods is proposed. The method avoids the need to run the model for tens or hundreds of periods of the transient process before establishing a steady state. To implement the method, it is enough to calculate several periods of the transient process, which allows to find the coefficients of recurrence relationships using standard matrix functions. The use of these relationships makes it possible to run the process without integrating the differential equations by the method of state variables during each period, as well as to immediately find the values of the state variables at the beginning of the steady-state period. Numerical calculations carried out using the proposed method demonstrated high efficiency and accuracy of the results. Based on this algorithm, a MATLAB language program was compiled, which generalizes the proposed method for its application in the calculation of steady-state modes of converters with different circuit topology.

Conflict of interest. The authors of the article declare that there is no conflict of interest.

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