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Estimation of electrical resistivity of conductive materials of random shapes

Introduction. *Electrical resistivity is an important material characteristic in the field of electrical engineering and material science. There are several methods that can be used to measure resistance, like the 4-wire method which relates the resistance to a voltage drop at a given current flow, but to define the resistivity from the resistance value requires an analytical expression for the given system which requires a sufficient mathematical apparatus for describing complicated shapes. Therefore we use finite element method computations to compute the resistivity of a metal material. This approach has been already used for different materials like concrete and aluminum in the past. We then compare this method with an analytical expression that due to intuition could approximate the solution sufficiently. After that, the same material is used again to test the electrical isotropy of the sample.* ***Novelty.*** *A method is developed by combining the results of experimental studies and the results of mathematical modelling of the process of determining the electrical conductivity of metals. The* ***goal*** *is to describe and employ a method of measuring the electrical resistivity of metal objects of random shapes. Using this method, it is possible to measure the resistivity of materials without the need to manufacture them into wires or ribbons.* ***Methods.*** *The solution to the problem was carried out by the finite element method via the COMSOL Multiphysics 5.6 simulation program in a cartesian coordinate system and the resistance between two points of the metal sample was measured by the 4-wire method.* ***Results.*** *A similar resistance value was obtained when the measuring terminals were placed in different places. The difference between them was within 1,5 % and the obtained values were close to the values given by the literature for the electrical resistivity of electrical steels. Terminal size influences the measured conductivity and a max error of 5,2 % was estimated.* ***Practical value.*** *A method of estimating the resistivity of materials without the need to manufacture them into specific shapes, like wires or ribbons, for which analytical expressions between resistivity and resistance are easily derived.* References 18, tables 7, figures 12.

Key words: **electrical resistivity, finite element method, electrical resistivity measurements, numerical simulation.**

Вступ. *Питомий електричний опір є важливою характеристикою матеріалу в галузі електротехніки та матеріалознавства. Існує кілька методів, які можна використовувати для вимірювання опору, наприклад, 4-провідний метод, який пов'язує опір з падінням напруги при заданому струмі, але для визначення питомого опору за значенням опору потрібен аналітичний вираз для даної системи, який вимагає достатнього математичного апарату для опису складних форм. Тому ми використовуємо розрахунки методом скінченних елементів до розрахунку питомого опору металевого матеріалу. Цей підхід вже використовувався в минулому для різних матеріалів, таких як бетон та алюміній. Потім ми порівнюємо цей метод з аналітичним виразом, який завдяки інтуїції може достатньо апроксимувати рішення. Після цього матеріал знову використовується для перевірки електричної ізотропії зразка.* ***Новизна***. *Розроблено метод шляхом поєднання результатів експериментальних досліджень та результатів математичного моделювання процесу визначення електропровідності металів.* ***Мета*** *– описати та застосувати метод вимірювання питомого елективного опору металевих предметів довільної форми. Використовуючи цей метод, можна вимірювати питомий опір матеріалів без необхідності виготовлення дротів або стрічок.* ***Методи***. *Розв'язання задачі здійснювалося методом скінченних елементів за допомогою програми моделювання COMSOL Multiphysics 5.6 у декартової системі координат, а опір між двома точками металевого зразка вимірювався 4-провідним методом.* ***Результати***. *Отримано аналогічне значення опору під час розміщення вимірювальних клем у різних місцях. Різниця між ними знаходилася в межах 1,5% і отримані значення були близькими до наведених у літературі значень електричного опору електротехнічних сталей. Розмір клеми впливає на провідність, що вимірюється, максимальна похибка становить 5,2 %.* ***Практична цінність***. *Метод оцінки питомого опору матеріалів без необхідності надання їм певної форми, наприклад, дроту або стрічок, для якого легко отримати аналітичні вирази між питомим опором та опором.* Бібл. 18, табл. 7, рис. 12.

Ключові слова: **електричний опір, метод скінченних елементів, вимірювання питомого електричного опору, чисельне моделювання.**

Introduction. Electrical resistivity is an important material characteristic. The theory of its measurement is well established and commonly used measurement techniques like the 2-wire or 4-wire method are used in praxis [1]. Because metals are usually very good conductors the measurement of their resistivities can be difficult [2, 3]. A similar problem of measuring the material resistivity of samples with different shapes was worked on in the study [4].

Resistivity defines the power losses of electrical conductors and in addition to parasitic capacitances and inductances, it can determine the transient behavior of circuits. It determines the skin depth of the magnetic and electric skin effect [5, 6]. The measurement of conductivity is also important in sensing the progress of concrete curing [7, 8] and also important in estimating its durability [9]. Not all metal materials can be measured this way and different techniques are used for porous materials [10]. Resistance measurements also yield structural information [11]. Most magnetic metals have a grain structure that experiences specific effects on resistivity [12]. Measuring the electrical resistance is done relative to two arbitrary points. In this work, it consists of connecting the points (terminals) to a power supply and measuring the current flowing from the power supply and the voltage difference between the two points. The resistance is then given by Ohm's law. However, calculating the resistivity based on resistance can be challenging especially when dealing with irregularly shaped objects. Then numerical methods can be employed to compute the electrical field distribution throughout the object. The current then flows in the direction of the electric field vectors (if we assume electric isotropy). The measurement of anisotropic materials has been done in the past, but in this work, the material is considered to be isotropic which will be tested [13].

After the resistance of the material is measured, the resistivity is computed from numerical analysis of the system by fitting the resistivity to fit the simulated voltage drop to the measured one.

The goal of the paper is to describe and test a method of electrical resistivity measurement of metal objects of non-standard shapes. Using it, it is possible to measure the resistivity of materials without the need to manufacture them into wires or ribbons.

The subject of investigations. This paper defines the used equations for the systems. The mathematics used is well-known in the field of electrical engineering.

After defining the problem and choosing a shape of interest, the numerical computation is done with the aid of the finite element method, which computes the discretized approximation of the system. After doing one simulation with a random resistivity value, the real value is computed which fits the simulation to the experiment. Because the chosen shape resembles a bus bar, the difference between an analytic expression and the simulation result of the conductivity is calculated. The terminal size influence is analyzed.

Theory and basic formulas. The equations governing electrostatics describe the electric field in a medium that arises due to static electrical charges. Via the material equations the relationship between the electric field $\boldsymbol{E}$ and current density $\boldsymbol{J}$ is established (1), which is the Ohm's law in differential form [14]. Throughout the paper we assume electrical isototropy of the medium so only scalar material characteristics are considered [15]:

$$\boldsymbol{J} = \sigma \boldsymbol{E}, \tag{1}$$

where σ is the conductivity of the material, which we want to estimate.

Electrical voltage is the potential difference between two points marked T1 and T2 in Fig. 1. The flowing current and voltage difference is expressed by (2) where the integration surface S is marked on the picture as well [16].

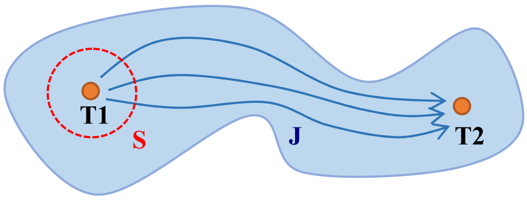

Fig. 1. A suitable integration surface for determining the current flowing through the body of the object. The integration surface S contains one terminal (T1) of the connected power supply

The current flowing through a medium is the flux of the current density vectors through a given surface. This surface should be suitably chosen like an enclosing sphere around one terminal of the object as it is shown in Fig. 2 [17].

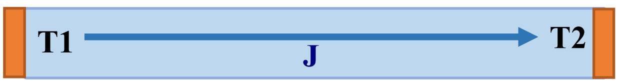

Fig. 2. A wire of uniform cross-section as a special case of the system

Equation (2) defines the resistance and so the relationship between the electric field and a corresponding current flow. The proportionality constant is the conductivity. By changing the conductivity at a given current we can fit the voltage drop from experiments and so the best-fit value will represent the conductivity of the material:

$$R = \frac{U}{I} = \frac{\int_{T1}^{T2} \boldsymbol{E} \mathrm{d}l}{\sigma \int_{S} \boldsymbol{E} \mathrm{d}S}. \tag{2}$$

Special case. One frequently used shape for which electrical resistance is computed is a long thin cylinder like it is in the case of an electrical wire.

The integrals from (2), because the wire is assumed to be uniform throughout its length becomes (3). Using Ohm's law we can obtain an analytic solution of (2) for simple wire-like objects (their length is the only significant dimension) [15]:

$$R = \frac{U}{I} = \frac{\boldsymbol{E} \cdot l}{I \cdot S} = \frac{1}{\sigma} \cdot \frac{l}{S}. \tag{3}$$

Material sample. A transformer steel strip was selected for the measurement (Fig. 3). The sheet's dimensions are 96 × 16,5 × 0,3 mm and two holes with a diameter of 5 mm are located near its ends. The holes were sanded, and two copper wires were soldered to their inner halves, which represent the two terminals of the object.

Fig. 3. The transformer sheet whose conductivity is the subject of determination

Based on this a 2D model was created in the Fusion 360 software that was extruded to 3D by giving the 2D plane a thickness of 0,3 mm (Fig. 4).

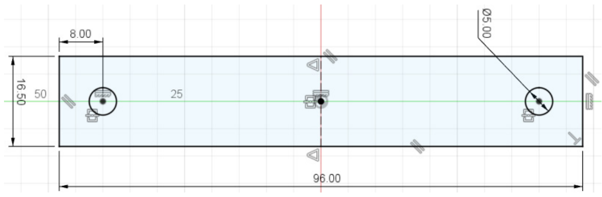

Fig. 4. Fusion 360 model of the steel strip

The model was imported into COMSOL Multiphysics modeling software as a .dxf file and an external domain representing an infinite air domain was added (Fig. 5). The air domain and the material were given a relative permittivity ε_r of 1. Air was given a conductivity σ of 10^{-10} S·m^{-1} (because 0 makes the model not converge) and the metal conductivity was set to $2 \cdot 10^6$ S·m^{-1}.

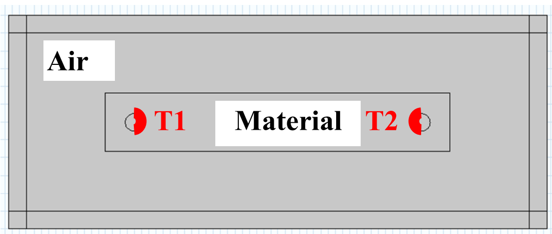

Fig. 5. COMSOL Multiphysics model of the system

Measurement. The measurement setup is shown in Fig. 6. It consisted of a constant current source (R&S HMP4040) and a voltmeter (RIGOL DM 3068), which measured the voltage difference between the terminals of the object.

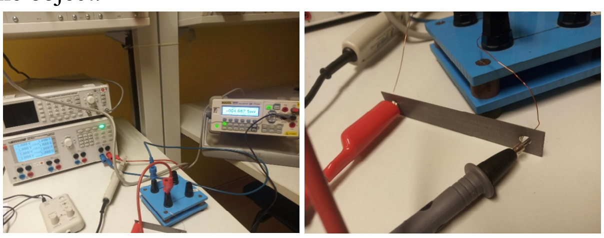

Fig. 6. Left – the measurement setup; right – placement of the voltage meter probes

The voltage measurement was done at currents in the range of 1 to 5 A with a step of 1 A. The obtained values are shown in Table 1. The measurement method used is the 4-wire resistance measurement method [1].

Table 1
Measured values of voltage at a given current

Current, A	Voltage, mV	Resistance of strip, mΩ
1	7,0798	7,0798
2	14,1352	7,0676
3	21,135	7,045
4	28,2366	7,05915
5	35,3046	7,06092

The average value of the resistance was taken using an arithmetic mean (4). So a resistance of 7,0625 mΩ was computed:

$$\overline{R} = \frac{1}{5} \cdot \sum_{i=1}^{5} R_i = 7{,}0625 \text{ m}\Omega . \qquad (4)$$

Simulation. The simulated system's terminals were connected to a 1 A constant current source and the output of the simulation was the voltage difference between the terminals that the current creates (Fig. 7, 8). The original guess of material conductivity σ = $2\cdot10^6$ S·m^{-1} was not correct, because the computed voltage drop was 9,5032 mV. The resistance of an object is inversely dependent on its conductivity, therefore linearly dependent on its resistivity ρ_R (ρ was used for volumetric charge density earlier). Two points in the resistivity/voltage drop graph define the linear relationship. At zero resistivity the voltage drop will be always zero so only one point is needed. The point coordinates are shown in Table 2.

Table 2
Two points from the resistivity/voltage drop space

Resistivity, Ω·m	Voltage, V
0	0
$5\cdot10^{-7}$	0,016107

From them, we can define the voltage drop U as a function of resistivity. The expression is:

$$U = 16107\cdot\rho_R. \qquad (5)$$

The desired voltage drop at 1 A is 7,0625 mV, so the material resistivity is fit as the value $4{,}38\cdot10^{-7}$ Ω·m. When this value is set as the material resistivity in the simulation program, the computed voltage drop has the same value as the experimental one.

Comparison with the special case equation. The current density (Fig. 7–9) seems to be uniformly distributed through the middle part of the sheet. This may suggest that the analytical approach from the chapter «Special case» could be used with enough precision because the sheet has a long uniform middle section. The sheet cross-section in the middle is a rectangle that has dimensions of 0,3×16,5 mm. For the «wire length» we take the shortest path between the terminals, which is the strip axis between the holes (75 mm). Inserting these values into (3) returns a conductivity of $2{,}145\cdot10^6$ S·m^{-1}.

When computing the relative error between these two approaches formula (6) yields a relative error of approx. 6 % which is to be decided by the application if it is tolerable:

$$\delta_\sigma = \left| \frac{\sigma_{eq} - \sigma_{sim}}{\sigma_{sim}} \right| \cdot 100\% = 5{,}932\ \% . \qquad (6)$$

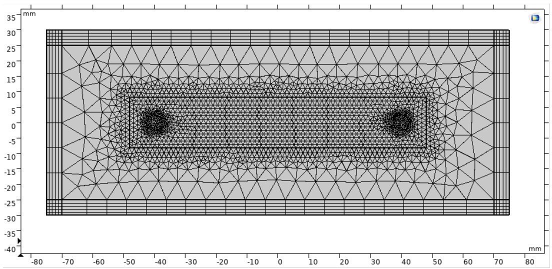

Fig. 7. The mesh generated for the given system

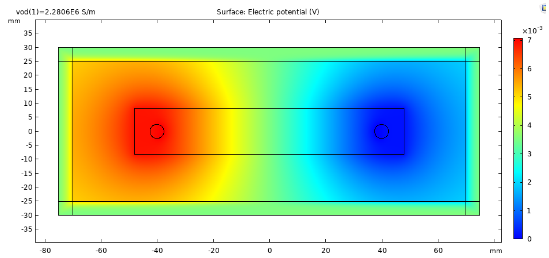

Fig. 8. The electrical potential at points of the system

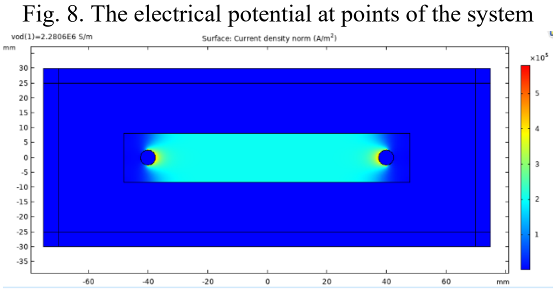

Fig. 9. The current density norm at points of the system

Measurement and simulation of resistivity at different terminals. The measurement and simulation were executed again at different points of the same sheet to test the obtained results. The tested terminal placements are shown in Fig. 10, 11. The lengths of the terminals were 5 mm in case *a*) and 2 mm in case *b*).

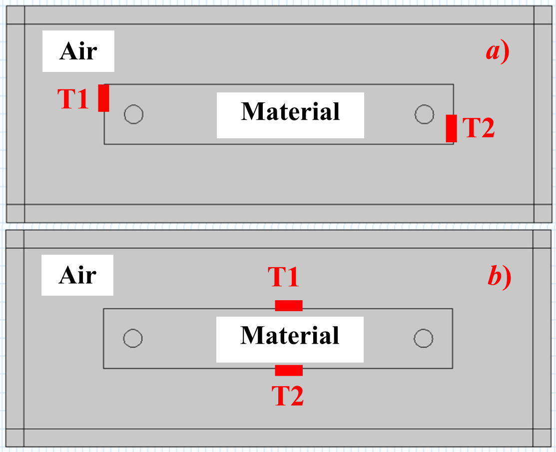

Fig. 10. Simulated samples of sheets

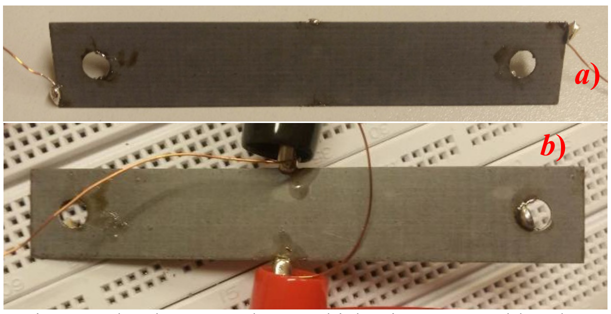

Fig. 11. The sheet samples to which wires were soldered as terminals as it was shown in Fig. 10

After the same process of measurement was repeated for sheets *a*) and *b*), the obtained results are shown below. After the resistance values were averaged from 5 current values (Table 3), the relative errors between the voltage drops if the same resistance value is used as the estimated one are shown in Table 4. The relative error was computed in the same way as in (6), but the *eq* index was replaced by *measurement*.

The resistances that would be obtained from such measurements are shown in Table 5. The error is again computed similarly as in (6) and relative to the original estimated sheet conductivity. The sheet seems to be electrically isotropic as it was expected.

Table 3

Measured values of sheet resistance

Sheet	Resistance of strip, mΩ
a)	10,5784
b)	2,7606

Table 4

Measured and simulated values of voltage drop at the given current

Sheet	Voltage drop at 1 A – measurement, mV	Voltage drop at 1 A – simulation, mV	Relative error, %
a)	10,5784	10,525	0,51
b)	2,7606	2,7857	0,9

Table 5

Measured and simulated values of voltage drop at the given current

Sheet	Resistivity, Ω·m	Error compared to the original sheet, %
Original	$4{,}385\cdot10^{-7}$	0
a)	$4{,}452\cdot10^{-7}$	1,52
b)	$4{,}429\cdot10^{-7}$	1,01

The effect of contact properties of probes to the surface of the sheet. The contacts that the probes make with the sheet affect the measured voltage drop. The transformer sheet was covered in an electrically isolating warnish that was removed in the places of probe connection and the sanded length was measured to correspond with the simulations. A simple ruler was used for the measurement with a resolution of 1 mm. To estimate how this affects the computed values of conductivity due to terminal size uncertainty the simulation was recomputed. Multiple terminal lengths from the interval of ±1 mm centered around the desired value were used. It is evident (Fig. 12) that the voltage drop monotonically decreases with terminal size. When computing the resistivity it can be seen that it increases monotonically with terminal size.

The main reason behind such large differences between the error values (Table 6) is the proximity of the measuring terminals, which was substantially lower in the case of *b*). The closer they are, the larger measuring uncertainty of resistivity can be expected, because at small distances it affects the electric field distribution the most. Also the solder connection resistance was not controlled and therefore also affects the measurement to some extent, since the solder conductivitiy is comparable to the sheet's conducivitity. The best method to suppress the effect of the connections' resistances is to place the terminals as far apart as possible in order to make the electric field lines between terminals as long as possible. This will render the resistance of the terminal connections as small as possible compared to the resistance of the sheet between the terminals and so the measured voltage drop will be mostly due to the sheet's resistance between the two terminals.

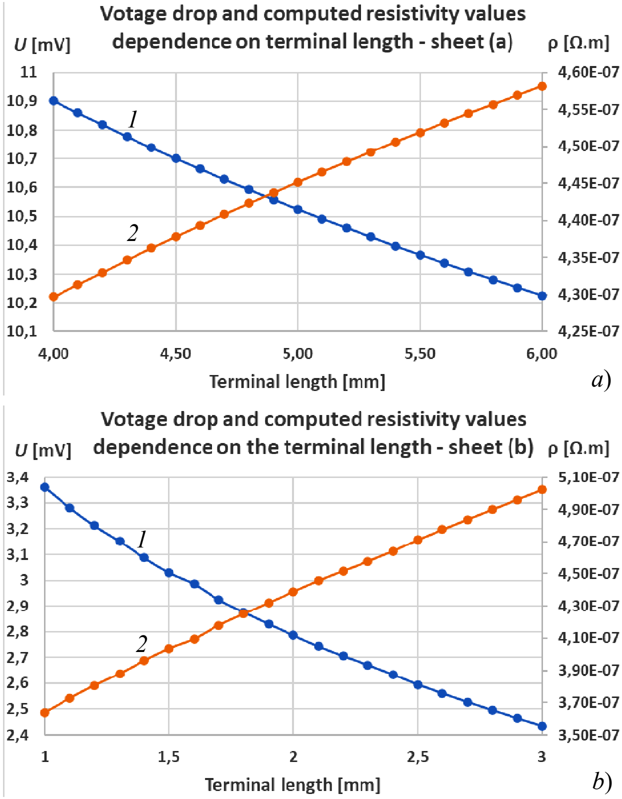

Fig. 12. Voltage drop dependence on terminal length when a conductivity of $2.25\cdot10^{6}$ S/m was used for the simulation on sheets (*a*) and (*b*) – blue graph *1* and the corresponding computed resistivity value to match experiments – orange graph *2*

Table 6

Calculated conducitivty interval due to a 1 mm terminal measuring error

Sheet	Lowest *ρ*, Ω·m	Largest *ρ*, Ω·m	Max error compared to Table 5, %
a)	$4{,}298\cdot10^{-7}$	$4{,}582\cdot10^{-7}$	3,47
b)	$3{,}638\cdot10^{-7}$	$5{,}022\cdot10^{-7}$	17,85

Uncertainty of the resistance measurement. The uncertainty values of measurements were calculated from the datasheet values of the used devices. The voltage drop was measured by the RIGOL DM 3068 multimeter which on the smallest 200 mV range has a 0,002 % error of reading and a 0,002 % error of range. The R&S HMP4040 current source has a regulation error consisting of a 0,01 % error of regulation and a 250 µA offset error. When setting a DC current value of 1 A, the current accuracy is 1 ± 0,00035 A. The voltage accuracies can be seen in Table 7.

Because the current and voltage were measured by two separate instruments, they are uncorrelated and because resistance is computed by division of these values, the resistance uncertainty is given as [18]:

$$u_R = \left(\frac{u_u}{U} + \frac{u_i}{I}\right)\cdot R\,, \tag{7}$$

where u_R is the resistance uncertainty, u_u and u_i are the voltage and current uncertainties, *U* and *I* are the measured voltage and current values and *R* is the computed resistance.

The resistance is then estimated as $R \pm u_R$ and is shown in Table 7. The relative resistance measurement uncertainty is 0,075 % for sheet *a*) and 0,182 % for sheet *b*) [18].

Table 7
Absolute uncertainty of the resistance measurement

Sheet	Absolute current uncertainty, A	Absolute voltage uncertainty, V	Absolute resistance uncertainty, Ω
a)	$3{,}5 \cdot 10^{-4}$	$4{,}212 \cdot 10^{-6}$	$7{,}914 \cdot 10^{-6}$
b)	$3{,}5 \cdot 10^{-4}$	$4{,}055 \cdot 10^{-6}$	$5{,}021 \cdot 10^{-6}$

When computing the resistance error due to terminal size uncertainty from Fig. 12 we obtain a relative uncertainty for sheet *a*) of 3,33 % and for sheet *b*) of 21,74 %. So the uncertainty in the connection parameters of the probes to the sheet strongly dominate the uncertainty of resistance measurement. The further apart the two terminals are, the smaller the error becomes.

Conclusions. The paper described a method of estimating the resistance of highly conductive materials of non-standard shapes. The method can estimate the conductivity of metals even when placed in different places of the object with a relative difference of approximately 1,5 %. However, because there is uncertainty in the terminal connection sizes and resistances, putting them too close together can yield very large uncertainty values. Terminals should be placed on opposite ends of the object to make these infuences as small as possible. Because there is no bounds of what object shapes could be used for the measurement the uncertainties of the given terminal placements should be computed for each case and decided if it is appropriate for the given application. The obtained results are similar to the resistivities of other metals of the same category. Because the measurement yielded a similar value of resistivity when the terminals were placed on the sheet axis (original measurement and sheet marked *a*)) and perpendicular to it (the sheet marked *b*)) we can assume that the electrical steel is electrically isotropic.

Conflict of interest. The authors of the article declare that there is no conflict of interest.

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