Analysis of a DC converter working on a plasma arc

**Introduction.** The article is devoted to the analysis of a stabilized direct current converter operating on a plasma arc. Electroplasma technologies of the new generation cause the need to design workable systems that provide control of technological processes and their dynamic optimization in real time. The improvement of any electroplasma technology begins with the improvement of the operating parameters of the main element of plasma installations - the power source. **Goal** is to build and study a continuous model of a pulsed source of secondary power supply, which works on an electric welding and plasma arc. **Methodology.** In the work, a mathematical description of the converter was performed. The continuous model of the system is substantiated, taking into account its features, namely, the load (gas-discharge gap) is a source of voltage and dynamic resistance. The parameters of the constant part during circuit synthesis are determined: the components of the gain of the constant part, the relative signal coefficient of the current sensor and the PWM gain. Studies of the open system «power source - arc» have been carried out. **Results.** MATLAB objects were created - continuous mathematical models of the object in the form of transfer functions. The obtained transient characteristics for different options: «arc current - control signal» and «inductor current - control signal» showed that open systems are unstable. It was found that in the case of instability, the filling frequency of self-oscillations occurring in the linear mode is close to the frequency of natural oscillations of the circuit. The dependence of the module and the argument of the input resistance of the power part of the pulsed power supply with parallel capacitance to the electric arc and without it, which have matching frequency characteristics, is established. The circuit considered with the initial data adopted in this article has a frequency transfer coefficient of the same type as the first-order non-minimum-phase (phase-shifting) link. Frequency response graphs for the output impedance of the power unit show that this power unit is a broadband frequency-selective system with a bandwidth of \( B_{\text{0.707}} = 100 \text{ kHz} \). **Originality.** Expressions for the frequency transfer function, input and output resistance of the pulse voltage converter operating on an arc load were obtained by the method of averaging and linearization. The frequency amplitude and phase characteristics for the pulse voltage converter with an LC filter and the output according to the arc current and the choke current were studied. The transfer functions of the continuous model in terms of arc current and choke current at the specified parameters are the same, which must be taken into account when designing regulators. **Practical significance.** The frequency characteristics of the input and output resistances and transfer functions can be used when forming a technical task for designing a power source to assess the stability of the «pulse converter - arc» system and rational calculation of input filters. **References:** 22, tables 1, figures 7.

Key words: input and output resistance, filter, impedance, stabilization system, stability, complex load.

В роботі проведено аналіз стабілізованого перетворювача постійного струму, що працює на плазмовому дусі. Обґрунтовано безперервну модель системи з урахуванням її особливостей. Визначені параметри незмінної частини під час аналізу схеми замикання замкнUTOї структури системи електрохімічного джерела енергії (дугового навантаження) із від'ємним диференціальним опором. Проведені дослідження розглянутої системи з розсіянням. Встановлено, що розглядена схема з вихідними даними, прийнятими в даній статті, має частотні характеристики передачі такого ж виду, що і неперервно-фазові хаотичні хвильові схеми перетворювача. Вивчені особливості вихідного і вхідного опорів перетворювача.

Ключові слова: вхідний та вихідний опір, фільтр, імпеданс, система стабілізації, стійкість, комплексне навантаження.

**Reference.** 22, tables 1, figures 7.

**Results.** The frequency characteristics of the input and output resistances and transfer functions can be used when forming a technical task for designing a power source to assess the stability of the «pulse converter - arc» system and rational calculation of input filters. **References:** 22, tables 1, figures 7.

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The complexity of the nonlinear discrete model of the pulse converter (its exact mathematical model), which works on a non-linear load, makes the analysis of the latter in an automatic current stabilization system (closed control system) extremely difficult [5]. At the same time, works [1, 5, 6] show that when the commutation period \( T \) \((T \to 0)\) decreases, nonlinear pulse systems approach linear continuous systems in terms of properties [1, 5]. Considering the high switching frequency of modern converters for electrical technologies, the task of building their limit continuous models turns out to be very promising. Averaged models are the most convenient for practice, as they allow applying well-developed methods of analysis and synthesis of linear continuous systems to nonlinear discrete systems.

When designing key power sources, continuous linear models of pulse converters are widely used [1, 5]. Their averaged models are substantiated and widely used in various works.

In [9], for example, continuous linearized models of basic converters are given. The method of their construction is based on the assumption that the choke has no active resistance. In addition, the used approach does not allow taking into account the output resistance of the power source, the filter at the input of the converter, etc.

In works [4, 5], the need to take into account the active resistance of the choke circuit during the analysis and synthesis of pulse converter control systems is indicated.

Structural dynamic models of DC pulse converters operating on a complex load with and without taking into account the choke resistance are substantiated, for example, in works [4-7].

In [3], the general issues of power supply and interaction of an electric arc from a current-controlled voltage source and a voltage-controlled current source are considered.

In [10], a power source with a current characteristic, which is made according to the following block diagram, was investigated: a DC source - a transformer - a parallel connection of an AC source - a transformer - a rectifier – an inductance connected in series with an arc load. The active resistance of the choke circuit during the analysis is taken into account, and synthesis of pulse converter control systems is indicated.

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Next, an attempt is made to perform an analysis using a simple continuous model of the converter, which takes into account the active resistance of the keys, the choke and the output resistance of the power source and operating on an arc load. It is assumed that the adjustment of the output parameter is carried out using PWM-2.

The goal of the article is to build and study a continuous model of a pulse source of secondary power supply, which works on an electric welding and plasma arc.

Mathematical description of the converter. Replacing circuit of the power supply system for an arc load, which includes a loaded LC filter, a correction device (CD), a modulator (M) implementing PWM-2, a pulse converter (PC), which forms pulses with the amplitude of the supply voltage \( nU_{in} \) at the input of the filter and duration, which is determined by the switching function \( k_f \) of the modulator, and sensors of voltage \( V_S \) and current \( C_S \) with \( k_f \) and \( R_{CS} \) coefficients, respectively, is shown in Fig. 1, where, for commonality, the resistance \( R \) connected in parallel with the capacitor \( C \) of the output filter is shown.

![Fig. 1. Replacing circuit of the closed structure of the power supply system for arc load with positive differential resistance](image)

In Fig. 1: \( i \) is the current source considered as a disturbance (change) in the load current. The current \( r_f \) includes both the output resistances of the converter and the rectifier, as well as the ohmic losses of the filter choke. Here, the losses resistance may depend on the frequency [11-15]. The load (gas discharge gap) is the series-connected voltage source \( U_b \) and dynamic resistance \( R_{diff} \).

The converter on the circuit is presented in the form of a power four-pole with an inhomogeneity controlled input (Fig. 1) [5, 16], where \( n = w_2 \). The transmission coefficient of the modulator \( PC \), which in our case is defined as \( k_f(\Omega) \), if the error signal does not depend on time, is determined by the relationship:

\[
k_{PWM} = T \frac{U_m}{U_b}
\]

where \( U_m \) is the amplitude (range) of the fractional function (sawtooth voltage), \( T \) is the sweep period.

Usually \( R \gg \omega \) (in particular \( R \) is missing, i.e. \( R \to \infty \)).

Denoting by \( Z_n(s) \) the operator resistance of the parallel connection of \( C \) and \( R_{diff} \):

\[
Z_n(s) = -R_{diff} + [1 - sR_{diff} C] = -R_{diff} / (\omega^2 + 1), \ r > 0,
\]

we obtain the following expression of the transfer function (TF) of the output LC filter in terms of voltages:

\[
K(s) = \frac{Z_n(s)}{Z_{in}(s)} = \frac{Z_n(s)}{\frac{R_f}{C} + sL + Z_n(s)} = k_f \frac{s^2 k_f L C s + L(I - R_{diff} + R_{eq} C + 1)}{k_f + T_f^2 s^2 + 2T_f s + 1} = k_f \frac{s^2 k_f L C s + L(I - R_{diff} + R_{eq} C + 1)}{T_f^2 s^2 + 2T_f s + 1},
\]

where \( r = R_{eq} C > 0 \) is the time constant of the circuit of the output capacitor, \( R_{eq} = \frac{R_f}{C} \) \((-R_{diff}) \), \( k_f = -R_{eq}(\omega^2 + \alpha^2) \) \((-R_{diff}) \) \(R_{eq} > 0 \) is the transmission coefficient of the DC filter, \( Z_n(s) \) is the input operator resistance of the regulator at \( R = \infty \) in the continuous current mode, \( T_f \) and \( \omega \) are the time constant and damping coefficient of the filter, defined as.
where \( Q = \frac{1}{\rho} \sqrt{L/C} = \rho/r_{2x} \) is the \( Q \) factor of the LC filter without taking into account the attenuation introduced by the resistance \( R \) and regeneration; \( d_s = -S_{diff} \sqrt{L/C} \) is the regeneration introduced into the circuit by a load (electric arc); \( d_r = r_{2x} \sqrt{CL/L} \) is the damping introduced into the circuit by resistance; \( \rho = \sqrt{L/C} \) is the characteristics of the circuit forming the filter; \( d = d_s + d_r \) is the complete fading (regeneration) of the circuit; \( S_{diff} = 1 / R_{diff} \) is the differential steepness of the arc volt-ampere characteristic.

For the circuit in Fig. 1 equation that determines the current in the filter choke \( i_{L_f} \) will be as follows:

\[
Z(s) i_L = nD u_{in} + nU_{in} d + Z_n(s) \tau - (Z_n(s) d R_{diff}) u_0, 
\]

where

\[
Z(s) = \frac{Z_n(s) K}{K_0}, 
\]

the symbol \(- \rightarrow \) shows an infinitesimally small change of the variable with respect to the value in the periodic mode, \( d = 2 \pi / T \) is the filling factor, \( D \) is the value of the filling factor in the steady state (periodic) mode [17-20].

At \( C = 0 \), the last equation will take the form

\[
i_{L_f} = k_f \tau - q_{0} L R_{diff} + (nD L \tau R_{diff}) u_{in} + (nU_{in} d R_{diff}) \tau \sigma / L (R_{2x} - R_{diff}) \sigma + 1.
\]

Having chosen as the output variable the arc current \( i_{arc} = i_{arc} \) which does not coincide with the state variable, we obtain the transfer functions for the control influence of the current – control signals:

\[
\frac{\tilde{T}_{arc}}{s} = nU_{in} \tau \tau / (Z_n(s) + Z_s(s) + S_{diff} = nU_{in} K(s) S_{diff},
\]

and by disturbing influences – the sensitivity of the arc current to a change in the input voltage \( u_{in} \) and the disturbing current \( i \) and voltage \( u_{0} \):

\[
\frac{\tilde{T}_{arc}}{s} = nD K(s) S_{diff}, 
\]

\[
\frac{\tilde{T}_{arc}}{s} = nD K(s) S_{diff}, 
\]

\[
\frac{\tilde{T}_{arc}}{s} = -(s^2 L + s r_{2x} C + 1) K(s) S_{diff}, 
\]

where \( Z_n(s) = L + r_{2x} \) is the operator resistance of the choke circuit.

Determination of the parameters of the constant part of the circuit. Let’s find the components of the gain factor of the constant part. Having chosen the steepness of the current sensor \( R_{CS} = 0.75 \text{ mS} \), we determine the CS relative signal coefficient \( k_{s} = 82 \) and the PWM gain

\[
k_{PWM} = F(T / U_{in}) (U_{in}) = F / U_{in} = 1 / 2.5 = 0.4 \text{ B}^{-1},
\]

where the ripple factor \( F \) is taken equal to 1; \( U_{in} = 2.5 \text{ V} \) is the amplitude (span) of sawtooth voltage.

The amplification factor of the constant part at \( k_r = 1 \) and \( F = 1 \)

\[
k_{c} = k_{PWM} n U_{in} (R_{CS} - R_{diff}) / T = -250 - 0.4 \times 10^{-5} / 0.49 = -12.55.
\]

The switching frequency is taken as 26 kHz, the choke inductance of the output LC filter is \( L = 300 \mu \text{H} \), and the capacity of the output capacitor, which depends on the requirements, is \( C = 3 \mu \text{F}, r_{2x} = 0.01 \Omega \), \( R_{diff} = -0.49 \Omega \) [21].

Study of the open system «power source – arc».

Let’s create MATLAB objects – continuous mathematical models of our object in the form of transfer functions (transfer function form). The answer will be the following result in the command window (Table 1).

<table>
<thead>
<tr>
<th>Transfer functions of the object</th>
<th>No.</th>
<th>Transfer function*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>416.4</td>
<td>9 \times 10^{-10} \cdot s^{-2} - 0.000612 \cdot s + 1</td>
</tr>
<tr>
<td>2</td>
<td>308.2</td>
<td>9 \times 10^{-10} \cdot s^{-2} - 0.000612 \cdot s + 1</td>
</tr>
<tr>
<td>3</td>
<td>-0.000366 \cdot s + 308.2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>9 \times 10^{-10} \cdot s^{-2} - 0.000612 \cdot s + 1</td>
<td></td>
</tr>
</tbody>
</table>

*options 1, 2 – TF «arc current – control signals with \( k_{CS} = 33.18 \) and \( k_{CS} = 16.59 \); options 3, 4 – TF «choked current – control signals with \( k_{CS} = 33.18 \) and \( k_{CS} = 16.59 \).

For the TF of object 3-4, the indicators are equal: the order and the instability index are equal to two \( n = s = 2 \), the degree \( r_{1} = n - m = 2 - 1 = 1 \), the indexes of aperiodic and oscillatory neutrality are equal to zero \( \sigma = s = 0 \), the index of non-minimum phase \( s_{ref} = 1 \), the amplification factor \( k = k_{CS} \). The assignment of these indicators and the amplification factor \( k \) reflects the essential features of the TF \( W(s) \) and they can be used to characterize certain properties of the system determined by this TF [22].

Transient characteristics (TC) for options 1, 3 are shown in Fig. 2. Open systems are unstable \( s_{r} = 2 \). Nyquist diagrams for the considered options are shown in Fig. 3. As shown in Fig. 3, the frequency response never covers the point \(-1, j0 \), so closed systems are also unstable.

The input resistance of the power part (PP) according to the diagram in Fig. 1 in operator form

\[
Z_{0}(s) = - (R_{diff} - r_{2}) \cdot s^2 k_f / L C + s (L (R_{diff} - r_{2}) - R_{eq}) + 1,
\]

where \( r_{2} = R_{diff} C > 0 \) is the time constant of the circuit of the output capacitor, \( r_{eq} = r_{2} / (1 - R_{eq}), k_f = R_{eq} / (R_{eq} - r_{2}) \) is the transmission coefficient of the DC filter.

It is easy to show that its output resistance is determined by the expression

\[
Z_{0}(s) = k_{f} k_{PWM} n U_{in} (R_{CS} - R_{diff}) / T = -82.5 \times 0.4 \times 10^{-5} / 0.49 = -12.55.
\]
Fig. 2. TC graphs of unstable systems: option 1 (1), option 3 (3)

Since the losses resistance is quite small, then
\[
Z_0(s) = \frac{sLC}{s^2 - s(R_{\text{diff}}C + 1)(LC)}. \tag{1}
\]

Noting that, in the general case, \(1/\sqrt{LC} = \omega_0\) is the frequency of normal oscillations of the circuit without losses and \(\alpha = -0.5S_{\text{diff}}/C = -0.5\omega_0Q\) is the damping coefficient of the circuit, we rewrite the expression as follows (here, the main conditions were chosen so that instabilities could occur)

\[
Z_0(s) = \frac{sLC}{s^2 - 2\alpha s + \alpha^2} \quad (\alpha > 0).
\]

In this case \(\omega_0 = 1/\sqrt{LC} = (\sqrt{300 \cdot 10^{-6} \cdot 3 \cdot 10^{-6}})^{-1} = 33.33 \cdot 10^3 \text{ s}^{-1}\) or \(f_0 = 5.307 \text{ kHz}\).

Therefore, in the case of instability, the filling frequency of self-oscillations occurring in the linear mode is close to the frequency of normal oscillations of the circuit \(f_0\).

If \(C = 0\), then
\[
Z_0(s) = sL(-\tau_{\text{load}}S + 1)^{-1}, \tag{2}
\]
where \(\tau_{\text{load}} = L/R_{\text{diff}} > 0\) is the load circuit time constant.

Options (Fig. 4) of the dependencies of the module and the argument of the input resistance of the PP of the pulse power supply with and without parallel capacitance to the electric arc have frequency characteristics (FC) that coincide.

The closeness of FC indicates similarity and small differences in transient processes according to the main quality indicators. Thus, the circuit under consideration with the initial data adopted in this article has a frequency transfer coefficient of the same type as a first-order non-minimum phase (phase-shifting) link
\[
Z(j\omega) = k(\omega_0 - 1),
\]
where \(k = R_{\text{diff}}, \tau = L/R_{\text{diff}} > 0\).

Frequency response graphs for the output impedance of the PP, constructed according to expressions (1), (2), are shown in Fig. 5. Note that for \(C \neq 0\), this PP is a broadband frequency-selective system \((B_{0.707} / f_0 >> 1)\) with bandwidth \(B_{0.707} = 100 \text{ kHz}\).

Logarithmic characteristics of the module and phase of the input conductance of the PP of the PC for the case of the absence of capacitance at the output of the pulse current stabilizer – unstable with negative self-alignment of the aperiodic link
\[
Y_{\text{pp}}(s) = 1/Z_0(s) = k(-1 + j\omega)^{-1} \tag{3}
\]
have been constructed in Fig. 6, a. the Nyquist diagram is shown in Fig. 6, b. According to (3), the asymptotic
characteristic has a break at the point $\omega_1 = 1 / \tau$. Bandwidth is 0 – 465 Hz.

Figure 7 shows the frequency response and phase response of the «power source – arc» system in the open state.

From graphs shown in Fig. 7 it can be seen that the closed system for the considered options will be unstable ($L(\omega) > 0$, $\phi_{\text{margin}}(\omega) < 0$, $L \to \infty$, $\phi_{\text{margin}} = -51.9^\circ - 90.2^\circ$), which introduces additional complications in the construction of the converter control circuit [21].

From the nature of $\phi(\omega)$ and $A(\omega)$ (Fig. 7), it follows that the construction of closed current stabilization systems of the considered converter causes difficulties, since it is very difficult to ensure a sufficient margin of stability at a high amplification factor in a closed circuit due to a very rapid increase in the phase shift (the angle $|\phi|$ exceeds $180^\circ$) with an almost unchanged frequency response of the converter.

Thus, the considered continuous models can be used in the process of designing devices based on pulse voltage converters for loads with negative differential resistance: sources of secondary power supply, etc., to conduct research on the main properties of these converters.

Conclusions.
1. Expressions for the frequency transfer function, input and output resistances of a pulse voltage converter operating on an arc load with negative differential resistance were obtained by the method of averaging and linearization.
2. The transfer functions of the converter with output by arc current and choke current are not minimally in phase. This explains the nature of the phase characteristics of the converter and the complexity of the synthesis of current regulators for it.
3. The frequency response and phase response for the pulse voltage converter by arc current and choke current were studied.
4. The transfer functions of the continuous model by arc current and choke current at the given parameters are the same, which must be taken into account when designing regulators.

Conflict of interest.
The authors of the article declare that there is no conflict of interest.

REFERENCES

How to cite this article:
Vershchagin E.M., Kostichenko V.I., Novogretskyi S.M. Analysis of a DC converter working on a plasma arc. Electrical Engineering & Electromechanics, 2023, no. 5, pp. 31-36. doi: https://doi.org/10.20998/2074-272X.2023.5.05.