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L. Moussaoui, S. Aouaouda, R. Rouaibia

Fault tolerant control of a permanent magnet synchronous machine using multiple constraints Takagi-Sugeno approach

Introduction. *Fault diagnosis, and fault tolerant control issues are becoming very important to ensure a good supervision of systems and guarantee the safety of human operators and equipments even if system complexity increases.* ***Problem.*** *In fact, the presence of faults in actuators, sensors and processes can lead to system performance degradation, system breakdown, economic loss, and even disastrous situations. Furthermore, Actuator saturation or control input saturation is probably the most usual nonlinearity encountered in control engineering because of the physical impossibility of applying unlimited control signals and/or safety constraints.* ***Purpose.*** *This article is dedicated to the problem of fault tolerant control for constrained nonlinear systems described by a Takagi-Sugeno model. One of the interests of this type of models is the possibility of extend some tools and methods from linear system case to the nonlinear one.* ***The novelty*** *of the work consists in developing a fault tolerant control algorithm for a nonlinear Permanent Magnet Synchronous Machine model using an observer based state-feedback control technique in order to enhance fault and state estimation despite actuator saturation and system disturbances.* ***Methods.*** *Indeed a sensor fault detection observer based residual generator is synthesized with a guaranteed L_2 performance to attenuate the external disturbances effect from one side and to maximize the residual sensitivity to faults from the other side. Based on Lyapunov function, design conditions are formulated in terms of Linear Matrix Inequalities to ensure stability of the global system.* ***Practical value.*** *A detailed study concerning nonlinear permanent magnet synchronous machine model, which is consolidated by simulation results, is conducted to show the used algorithm's effectiveness guarantying fault estimation and reconfiguration of the control law to maintain stable performance even in the presence of actuator faults, external perturbation and the phenomenon of actuator saturation.* References 19, tables 1, figures 5.

Key words: **Takagi-Sugeno models, actuator saturation, state estimation, actuator faults diagnosis, fault tolerant control, permanent magnet synchronous machine model, linear matrix inequalities.**

Вступ. *Діагностика несправностей і питання стійкості до відмови стають дуже важливими для забезпечення хорошого контролю систем і гарантії безпеки людей-операторів і обладнання, навіть якщо складність системи зростає.* ***Проблема***. *Насправді наявність несправностей у виконавчих механізмах, датчиках і процесах може призвести до зниження продуктивності системи, поломки системи, економічних втрат і навіть катастрофічних ситуацій. Крім того, насичення виконавчого механізму або насичення керуючого входу, ймовірно, є найбільш поширеною нелінійністю, що зустрічається в техніці керування через фізичну неможливість застосування необмежених керуючих сигналів та обмежень безпеки.* ***Мета***. *Ця стаття присвячена проблемі стійкості до відмови нелінійних систем з обмеженнями, що описуються моделлю Такагі-Сугено. Однією з переваг цього типу моделей є можливість поширення деяких інструментів та методів з випадку лінійної системи на нелінійну.* ***Новизна*** *роботи полягає у розробці алгоритму управління стійкості до відмови для моделі нелінійної синхронної машини з постійними магнітами з використанням методу управління зі зворотним зв'язком станом на основі спостерігача, щоб поліпшити оцінку помилок і станів, незважаючи на насичення приводу і збурення системи.* ***Методи***. *Дійсно, генератор нев'язки на основі спостерігача виявлення несправності датчика синтезується з гарантованою продуктивністю L_2, щоб послабити вплив зовнішніх перешкод з одного боку та максимізувати залишкову чутливість до несправностей з іншого боку. На основі функції Ляпунова умови проєктування формулюються в термінах лінійних матричних нерівностей для забезпечення стійкості глобальної системи.* ***Практична цінність***. *Детальне дослідження нелінійної моделі синхронної машини з постійними магнітами, об'єднане результатами моделювання, проводиться для демонстрації ефективності використовуваного алгоритму, що гарантує оцінку відмов та реконфігурацію закону управління для підтримки стабільної роботи навіть за наявності відмов приводу, зовнішніх збурень та явища насичення приводу.* Бібл. 19, табл. 1, рис. 5.

Ключові слова: **моделі Такагі-Сугено, насичення приводу, оцінка стану, діагностика несправностей приводу, стійкість до відмов, модель синхронної машини з постійними магнітами, лінійні матричні нерівності.**

Introduction. Faults detection and isolation (FDI) has been the subject of many research for linear and nonlinear systems. However, in practical cases used only the FDI block for the process is not enough to preserve desired performance, security and system stability. Hence, fault tolerance must be treated and controllers are synthesized to ensure system stability even in failed situations and degraded operations. We can classify the fault tolerant control (FTC) into passive FTC and active FTC [1, 2]. The first approach can be considered as a robust control, and it requires a priori knowledge of faults that can affect the system, the controller is then designed to compensate them, all possible faults are considered as uncertainties, and an adaptive observer is employed to estimate the fault and state signals. The second is called active FTC, we use in this approach a very robust FDI block to know with exactitude the information's about faults which constitutes its major disadvantage. Indeed, a false alarm or an undetected fault can lead to degradation of performance or even instability [3].

During the design of the FTC controller, we must take into account the saturation of the actuator to avoid the undesirable effects that can destabilize the closed-loop system, and degrade the desired performances [4, 5]. Therefore, much attention has been given to stability analysis and controller design for systems with actuator saturation [6]. More-over, several works propose a norm-bounded controller based on multi-model Takagi-Sugeno (T-S) fuzzy approach, due to the exceptional characteristics of T-S fuzzy models for control purposes to avoid control inputs limitations.

Many researches are developed around the T-S multimodel representation [7, 8]. Nonlinear systems described by T-S models have been considered actively and especially in the fields of control, state estimation and diagnosis of nonlinear systems. This is related to the fact that T-S fuzzy model can approximate exactly any nonlinear system without loss of information. A T-S model can be obtained using the non-linearity sector approach by aggregating the local models using appropriate interpolation functions [9]. These models have a great ability to represent the complex dynamic system.

The stability analysis of a T-S system has been studied in most cases by using a quadratic Lyapunov function, and solutions are almost expressed as linear matrix inequalities (LMI) [10, 11].

Goal. This paper aims to develop a robust control approach subject to multiple system constraints, i.e. actuator faults, input saturation, external perturbation. The system is presented as a T-S fuzzy multi-model, and then an observer-based FTC design method is introduced to preserve the stability of the system with disturbance rejection. The observer and controller gains are obtained through an L_2 minimisation by solving LMI conditions.

The main contribution is to develop model-based FTC-scheme for nonlinear dynamic systems described by T-S models and subject to input constraints. Using the Lyapunov theory for T-S systems, the obtained results are less conservative and formulated in terms of LMI conditions. Consequently, the proposed procedure has also two advantages over the previous cited works. Firstly, it is able to estimate time variable fault types. Secondly, for the analysis of the fuzzy systems, to reduce the computational cost of double summation slack matrices has been introduced, which leads to a simple design procedure. Furthermore, compared to the approach presented in [12-16] the proposed FTC controller design method can be considered for a large class of nonlinear constrained systems.

Problem statement. Let consider the following constrained and disturbed T-S model:

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} \mu_i(\xi(t))\left(A_i x + B_i sat(u + f_u(t))\right) + d(t) \\ y = \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x \end{cases} \tag{1}$$

where $x \in R^n$ is the state vector; $u \in R^m$ is the control input; $y \in R^p$ is the system output; A_i, B_i, C_i are the constant matrices with appropriate dimensions; $d(t)$ is the external disturbance signal; $f_u(t)$ is the actuator failure; $\xi(t) = [\xi_1(t) \ldots \xi_r(t)]$ is the decision variables; $\mu_i(\xi(t))$ is the normalized activation function satisfying the sum convex propriety [17]:

$$\begin{cases} 0 \le \mu_i(\xi(t)) \le 1 \quad \forall i \in \{1, 2, \cdots, r\} \\ \sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \\ \mu_i(\xi(t)) = \dfrac{w_i(\xi(t))}{\sum_{i=1}^{r} w_i(\xi(t))} \\ w_i(\xi(t)) = \prod_{j=1}^{r} M_{ij}(\xi_j(t)) \end{cases} \tag{2}$$

where $w_i(\xi(t))$ are the weights; $M_{ij}(\xi_j(t))$ are the fuzzy set. The function sat($u + f_u(t)$) represents the actuator failure saturation function.

The following lemmas and notations will be used in the rest of this paper:

Lemma 1 [2]: Let consider two matrices X and Y and a scalar σ such that the following inequality is verified:

$$X^T Y + Y^T X < \sigma X^T X + \sigma^{-1} Y^T Y, \sigma > 0. \tag{3}$$

Lemma 2 [18, 19]: Let E be an $m \times m$ diagonal matrix whose elements are 1 or 0. Suppose that $|v_i| \le \bar{u}_i$ for all $i \in I_m$ where v_i and u_i are the i^{th} element of $v \in R^m$ and $u \in R^m$ respectively. If $x \in \bigcap_{j=1}^{r} L(H_j)$ for $x \in R^n$, then

$$\begin{cases} sat(u, \bar{u}) = \sum_{s=1}^{2^m} \alpha_s (E_s u + \bar{E}_s v) \\ \sum_{s=1}^{2^m} \alpha_s = 1, \quad 0 \le \alpha_s \le 1 \\ v = -\sum_{j=1}^{r} \mu_j H_j x \end{cases} \tag{4}$$

$$L(H_j) = \{x \in R^n, |h_i^j x| \le \bar{u}_i\}, \tag{5}$$

where E_s denotes all elements of E, $\bar{E}_S = 1 - E_S$; H_j is the $m \times n$ matrix and h_i^j is the i^{th} row of the matrix H_j; α_s is the weighting functions related to the polytopic representation of the saturation function.

Lemma 3 [18]: An ellipsoid $\varepsilon(P, \rho)$ is inside $\bigcap_{j=1}^{r} L(H_j)$ if and only if:

$$\forall i \in I_m : (h_i^j)^T (P/\rho) h_i^j \le \bar{u}_i^2, \tag{6}$$

where h_i^j is the i^{th} row of the matrix H_j.

Assumption 1 [3]: the faults are assumed to have a first time derivative bounded as:

$$\left\| \dot{f}(t) \right\| \le f_{1\max}, \quad 0 \le f_{1\max} \le \infty$$

Throughout the paper $H(Z)$ denotes the Hermitian of a matrix Z, i.e. $H(Z) = Z + Z^T$; and I denotes the identity matrix.

Fault tolerant control of T-S multi-model. The design of the controller is based on FTC bloc with multi-constraints (actuator fault, actuator saturation and external disturbance) as shown in Fig. 1, ensuring the convergence of the estimated states and the detection of the faults by the observer.

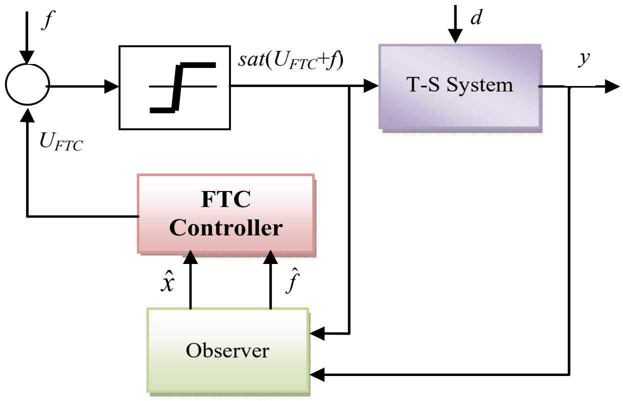

Fig. 1. Fault tolerant control scheme

The following observer is adopted to estimate both faults and system states as:

$$\begin{cases} \dot{\hat{x}} = \sum_{i=1}^{r} \mu_i \left(A_i \hat{x} + B_i sat(u + \hat{f}_u(t)) + L_i e_y \right) \\ \hat{y} = C\hat{x} \\ \dot{\hat{f}}_u = T \sum_{i=1}^{r} \mu_i F_i \left(\dot{e}_y + \delta e_y \right) \\ e_y = y - \hat{y} \end{cases} \tag{7}$$

where $\hat{f} \in R^n$ is the estimated state; $\hat{f}_u(t)$ is the estimated fault; L_i, T, F_i and δ are the observer gains to be determined.

The proposed FTC control law is given by:

$$U_{FTC} = -\sum_{i=1}^{r} \mu_i K_i \hat{x} - \hat{f}_u(t); \quad (8)$$

$$V_{FTC} = -\sum_{i=1}^{r} \mu_i H_i \hat{x}, \quad (9)$$

where K_i, H_i are the $m \times n$ gains matrices to be determined.

In this work, the saturation function can be written as polytopic representation defined by:

$$sat(u + \hat{f}_u(t)) = E_{\alpha s}\left(U_{FTC} + \hat{f}_u(t)\right) + \bar{E}_{\alpha s} V_{FTC} \quad (10)$$

with:

$$E_{\alpha s} = \sum_{s=1}^{2^m} \alpha_s E_s; \quad (11)$$

$$\bar{E}_{\alpha s} = \sum_{s=1}^{2^m} \alpha_s \bar{E}_s. \quad (12)$$

Using (10), the system (1) will be:

$$\dot{x} = \sum_{i=1}^{r} \mu_i \left(A_i x + B_i \left[E_{\alpha s}\left(U_{FTC} + f_u(t)\right) + \bar{E}_{\alpha s} V_{FTC} \right] \right) + d(t). \quad (13)$$

Using (8), (9) one can obtain:

$$\dot{x} = \sum_{i=1}^{r} \mu_i \begin{pmatrix} \left(A_i - B_i\left(E_{\alpha s}K_i + \bar{E}_{\alpha s}H_i\right)\right)x \\ +B_i\left(E_{\alpha s}K_i + \bar{E}_{\alpha s}H_i\right)e_x + B_i E_{\alpha s} e_f \end{pmatrix} + d(t) \quad (14)$$

with:

$$e_x = x - \hat{x}; \quad (15)$$

$$e_f = f_u - \hat{f}_u. \quad (16)$$

Theorem 1. The system that generate state error e_x, and fault error e_f is stable and asymptotically converge to zero and achieve a disturbance rejection level $\gamma > 0$ if there exist a symmetric positive definite matrix X, matrices $\bar{L}_i$, $\bar{K}_i$, Z_j, T, F_i and scalars δ, σ_{1i} ... σ_{3i}, solutions of the following LMI constraints:

$$\min_{X, \bar{L}_i, \bar{K}_i, Z_j, T, F_i, \delta, \sigma_{1i}, \ldots \sigma_{3i}} \gamma;$$

$$\begin{bmatrix} \bar{u}_i^2 & z_i^j \\ (z_i^j)^T & X \end{bmatrix} \geq 0, \quad \forall i \in I_m; \quad (17)$$

$$\begin{bmatrix} D''_{11} & * & * & * & * & * & * & * & * & * & * \\ D''_{21} & D''_{22} & * & * & * & * & * & * & * & * & * \\ D''_{31} & D''_{32} & D''_{33} & * & * & * & * & * & * & * & * \\ I & I & -\frac{1}{\delta^T}C^T & -\gamma^2 I & * & * & * & * & * & * & * \\ X & 0 & 0 & 0 & I & * & * & * & * & * & * \\ 0 & X & 0 & 0 & 0 & \sigma_{1i}I & * & * & * & * & * \\ 0 & \bar{L}_i & 0 & 0 & 0 & 0 & \sigma_{2i}I & * & * & * & * \\ 0 & X & 0 & 0 & 0 & 0 & 0 & \sigma_{3i}I & * & * & * \\ 0 & 0 & \frac{1}{\delta}F_i C A_i & 0 & 0 & 0 & 0 & 0 & \sigma_{1i}^{-1}I & * & * \\ 0 & 0 & \frac{1}{\delta}F_i C & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{2i}^{-1}I & * \\ 0 & 0 & F_i C & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{3i}^{-1}I \end{bmatrix} < 0; \quad (18)$$

with:

$$D''_{11} = H\left(A_i X - B_i E_{\alpha s}\bar{K}_i - B_i \bar{E}_{\alpha s} Z_j\right);$$
$$D''_{21} = (\bar{K}_i^T E_{\alpha s}^T + Z_j^T \bar{E}_{\alpha s}^T) B_i^T;$$
$$D''_{31} = E_{\alpha s}^T B_i^T;$$
$$D''_{22} = A_i X + X A_i^T - \bar{L}_i - \bar{L}_i^T = H\left(A_i X - \bar{L}_i\right); \quad (19)$$
$$D''_{32} = E_{\alpha s}^T B_i^T;$$
$$D''_{33} = -H\left(\frac{1}{\delta} F_i C B_i E_{\alpha s}\right).$$

The controller gains are obtained by $K_i = \bar{K}_i \cdot X^{-1}$, $H_j = Z_j \cdot X^{-1}$ and the observer gains are derived by $L_i = \bar{L}_i \cdot (CX)^{-1}$.

Proof: let $X = (P_1/\rho)^{-1}$ and $Z_j = H_j \cdot X$ the inequality (6) can be written as:

$$\bar{u}_i^2 X - \left(z_i^j\right)^T z_i^j \geq 0, \quad (20)$$

where z_i^j is the i^{th} row of the matrix Z_j and by Schur complement, inequality (20) can be written as (17).

The goal is to determine the unknowns' parameters of the controller L_i, T, F_i and δ. Let us choose the following Lyapunov function:

$$V = x^T P_1 x + e_x^T P_1 e_x + \frac{1}{\delta} e_f^T T^{-1} e_f > 0, \quad (21)$$

where P_1 is the symmetric positive definite matrix.

The derivative of V with respect to time t is given by:

$$\begin{aligned} \dot{V} = {} & \dot{x}^T P_1 x + x^T P_1 \dot{x} + \dot{e}_x^T P_1 e_x + e_x^T P_1 \dot{e}_x + \\ & + \frac{1}{\delta} \dot{e}_f^T T^{-1} e_f + \frac{1}{\delta} e_f^T T^{-1} \dot{e}_f < 0 \end{aligned} \quad (22)$$

According to (14)–(16), $\dot{V}$ becomes:

$$\begin{aligned} \dot{V} = {} & x^T \Psi_{ij}^T P_1 x + e_f^T E_{\alpha s}^T B_i^T P_1 x + e_x^T (K_i^T E_{\alpha s}^T + \\ & + H_i^T \bar{E}_{\alpha s}^T) B_i^T P_1 x + d^T P_1 x + x^T P_1 \Psi_{ij} x + x^T P_1 B_i E_{\alpha s} e_f + \\ & + x^T P_1 B_i (E_{\alpha s} K_i + \bar{E}_{\alpha s} H_i) e_x + x^T P_1 d + e_x^T \phi_i^T P_1 e_x + \\ & + e_f^T E_{\alpha s}^T B_i^T P_1 e_x + d^T P_1 e_x + e_x^T P_1 \phi_i e_x + e_x^T P_1 B_i E_{\alpha s} e_f + \\ & + e_x^T P_1 d + \frac{1}{\delta} \dot{f}_u^T T^{-1} e_f - \frac{1}{\delta}^T \dot{e}_y^T F_i^T e_f - e_y^T F_i^T e_f + \\ & + \frac{1}{\delta} e_f^T T^{-1} \dot{f}_u - \frac{1}{\delta} e_f^T F_i \dot{e}_y - e_f^T F_i e_y \end{aligned} \quad (23)$$

with:

$$\Psi_{ij} = A_i - B_i\left(E_{\alpha s} K_i + \bar{E}_{\alpha s} H_i\right); \quad (24)$$

$$\phi_i = A_i - L_i C. \quad (25)$$

The dynamic of the state estimation error, output error and the fault estimation error are calculated by:

$$\dot{e}_x = \sum_{i=1}^{r} \mu_i \left(\phi_i e_x + B_i E_{\alpha s} e_f\right) + d(t); \quad (26)$$

$$\dot{e}_y = C\dot{e}_x = C\left(\sum_{i=1}^{r} \mu_i \left(\phi_i e_x + B_i E_{\alpha s} e_f\right) + d(t)\right); \quad (27)$$

$$\dot{e}_f = \dot{f}_u - \dot{\hat{f}}_u. \quad (28)$$

Now, using Lemma 1 and Assumption 1 we can write:

$$\begin{aligned} & \frac{1}{\delta} \dot{f}_u^T T^{-1} e_f + \frac{1}{\delta} e_f^T T^{-1} \dot{f}_u < \frac{1}{\delta} e_f^T G e_f + \\ & + \frac{1}{\delta} f_{1\max}^2 \lambda_{\max}\left(T^{-1} G^{-1} T^{-1}\right) \end{aligned} \quad (29)$$

Based on the dynamic errors defined by (26)–(28) and the inequality (29) the time derivative of the Lyapunov function (23) is rewritten as follows after some algebraic manipulation using Lemma 1:

$$\begin{bmatrix} \delta_{11} & * & * & * \\ \delta_{21} & \delta_{22} & * & * \\ E_{\alpha s}^T B_i^T P_1 & \delta_{32} & \delta_{33} & * \\ P_1 & P_1 & -\frac{1}{\delta^T} C^T F_i^T & -\gamma^2 \end{bmatrix} < 0 \quad (30)$$

with:

$$\delta_{11} = \Psi_{ij}{}^T P_1 + P_1 \Psi_{ij} + I;$$
$$\delta_{21} = \left(K_i^T E_{\alpha s}{}^T + H_i^T \bar{E}_{\alpha s}{}^T\right) B_i^T P_1;$$
$$\delta_{22} = \phi_i^T P_1 + P_1 \phi_i;$$
$$\delta_{32} = E_{\alpha s}{}^T B_i^T P_1 - \frac{1}{\delta} F_i C \phi_i - F_i C; \qquad (31)$$
$$\delta_{23} = P_1 B_i E_{\alpha s} - \frac{1}{\delta^T} \phi_i^T C^T F_i^T - C^T F_i^T;$$
$$\delta_{33} = -\frac{1}{\delta^T} E_{\alpha s}{}^T B_i^T C^T F_i^T - \frac{1}{\delta} F_i C B_i E_{\alpha s} + \frac{1}{\delta} G.$$

Multiplying (30) left and right by diag(X X 1 1), where $P_1^{-1} = X$ the following inequality is derived:

$$\begin{bmatrix} D_{11} & * & * & * \\ \left(\bar{K}_i^T E_{\alpha s}{}^T + Z_j^T \bar{E}_{\alpha s}{}^T\right) B_i^T & D_{22} & * & * \\ E_{\alpha s}{}^T B_i^T & D_{32} & D_{33} & * \\ I & I & -\frac{1}{\delta^T} C^T F_i^T & -\gamma^2 \end{bmatrix} < 0 \qquad (32)$$

with:

$$D_{11} = H\,(A_i X - B_i E_{\alpha s} \bar{K}_i - B_i \bar{E}_{\alpha s} Z_j) + XIX;$$
$$D_{22} = A_i X + X A_i^T - \bar{L}_i - \bar{L}_i^T = H\,(A_i X - \bar{L}_i);$$
$$D_{32} = E_{\alpha s}{}^T B_i^T - \frac{1}{\delta} F_i C A_i X + \frac{1}{\delta} F_i C L_i C X - F_i C X; \qquad (33)$$
$$D_{33} = -H\,(\frac{1}{\delta} F_i C B_i E_{\alpha s}).$$

Applying Lemma 1 the inequality (32) becomes:

$$\begin{bmatrix} D'_{11} & * & * & * \\ D'_{21} & D'_{22} & * & * \\ D'_{31} & D'_{32} & D'_{33} & * \\ I & I & -\frac{1}{\delta^T} C^T F_i^T & -\gamma^2 \end{bmatrix} < 0 \qquad (34)$$

with:

$$D'_{11} = H\,(A_i X - B_i E_{\alpha s} \bar{K}_i - B_i \bar{E}_{\alpha s} Z_j) + XIX;$$
$$D'_{21} = \left(\bar{K}_i^T E_{\alpha s}{}^T + Z_j^T \bar{E}_{\alpha s}{}^T\right) B_i^T;$$
$$D'_{31} = E_{\alpha s}{}^T B_i^T;$$
$$D'_{22} = H\,(A_i X - L_i C X) = A_i X + X A_i^T - \bar{L}_i - \bar{L}_i^T + ;$$
$$+ \sigma_{1i}{}^{-1} XX + \sigma_{2i}{}^{-1} \bar{L}_i \bar{L}_i^T + \sigma_{3i}{}^{-1} XX \qquad (35)$$
$$D'_{32} = E_{\alpha s}{}^T B_i^T;$$
$$D'_{33} = -H\,(\frac{1}{\delta} F_i C B_i E_{\alpha s}) + \sigma_{1i} \frac{1}{\delta} F_i C A_i A_i^T C^T F_i^T \frac{1}{\delta^T} + .$$
$$+ \sigma_{2i} \frac{1}{\delta} F_i C C^T F_i^T \frac{1}{\delta^T} + \sigma_{3i} F_i C C^T F_i^T$$

Now, applying Schur complement on terms D'_{11}, D'_{22} and D'_{33}, the theorem 1 is hold.

Simulation results. In order to show the effectiveness of the proposed FTC approach, a PMSM non linear model is considered. The nonlinear PMSM model is given by:

$$\frac{di_d}{dt} = \dot{x}_1 = f_1(x) + g_1 u_d$$
$$\frac{di_q}{dt} = \dot{x}_2 = f_2(x) + g_2 u_q \qquad (36)$$
$$\frac{dw_r}{dt} = \dot{x}_3 = f_3(x)$$

with:

$$F(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix} = \begin{pmatrix} a_1 x_1 + a_2 x_2 x_3 \\ b_1 x_2 + b_2 x_1 x_3 + b_3 x_3 \\ c_1 x_3 + c_2 x_1 x_2 + c_3 x_2 + c_4 C_r \end{pmatrix};$$
$$G = \begin{pmatrix} g_1 & 0 \\ 0 & g_2 \\ 0 & 0 \end{pmatrix};$$

$a_1 = -R / L_d$; $a_2 = pL_q / L_d$

$b_1 = -R / L_q$; $b_2 = - pL_d / L_q$; $b_3 = -p\varphi_f / L_q$

$c_1 = -f / J$; $c_2 = k_1 / J$; $c_3 = k_2 / J$; $c_4 = -1 / J$

$g_1 = 1 / L_d$; $g_2 = 1 / L_q$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} \frac{di_d}{dt} \\ \frac{di_q}{dt} \\ \frac{dw_r}{dt} \end{pmatrix} = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix} + \begin{pmatrix} g_1 & 0 \\ 0 & g_2 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_d \\ u_q \end{pmatrix}.$$

The different parameters values are given in Table 1.

Table 1

PMSM parameters

Parameter	Description	Value
R	Stator resistance, Ω	1.4
L_d	Direct stator inductance, H	0.0066
L_q	Quadratic stator inductance, H	0.0058
f	Coefficient of friction, N·m/rad·s	0.00038
p	Number of pole pairs	3
J	Moment of inertia, kg·m^2	0.00176

Furthermore, to see the effectiveness of the proposed approach by applying the FTC law with the derived controller and observer gains, Fig. 2–4 show the estimated states and real ones obtained with the FTC law.

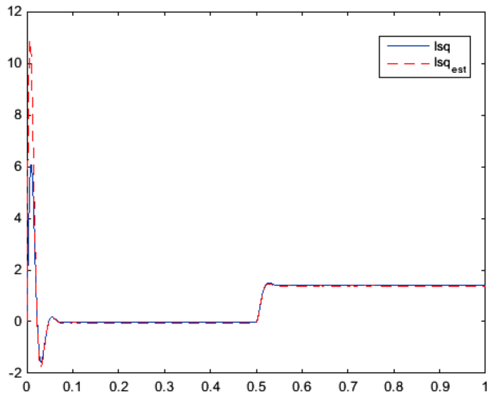

Fig. 2. Stator current I_q and their estimation

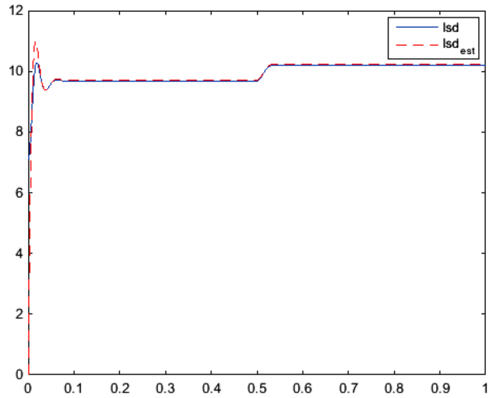

Fig. 3. Stator current I_d and their estimation

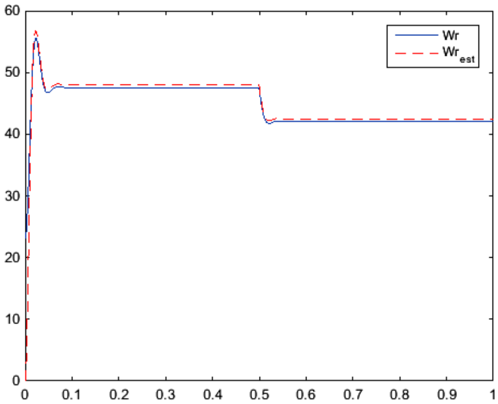

Fig. 4. Motor speed and their estimation

It should be highlighted that the proposed FTC design approach gives promising results while preserving the stability and guarantee the disturbance rejection. The fault has been estimated by the observer (5) as illustrated in Fig. 5.

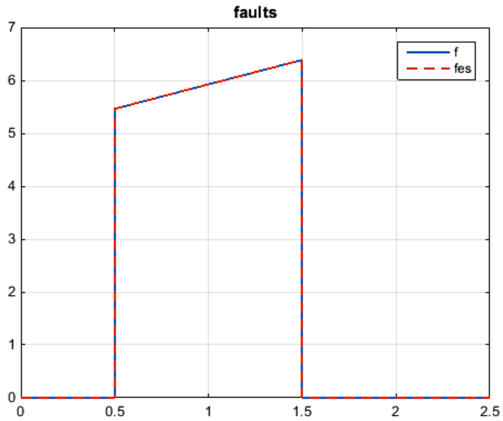

Fig. 5. Fault and their estimation

Conclusions.

This paper is dedicated to the design of fault tolerant control strategy for nonlinear systems described by Takagi-Sugeno models. The systematic procedure is presented to deal with the faulty actuator and input system saturation and applied into permanent magnet synchronous machine. The proposed fault tolerant control design approach is based on a robust observer to estimate both the actuator faults and the system states used to synthesize the controller law. The main advantage of the proposed approach is to synthesize the control law by taking into consideration the saturation limits. Using Lyapunov theory, sufficient conditions are derived by solving an linear matrix inequalities optimization problem by taking into account the saturation level. It can be noticed from the simulation results that the proposed fault tolerant control strategy is clearly approved on a permanent magnet synchronous machine model. It is clear that the values of the speed and currents estimated by the observers follow correctly the objectives even when changing the torque at 0.5 s, also estimated faults follow correctly generated fault between 0.5–1.5 s.

As future works, the proposed approach may be generalized to Takagi-Sugeno systems with unmeasurable decision variable and/or subject to sensor saturations.

Conflict of interest. The authors declare no conflict of interest.

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Lotfi Moussaoui[1]*, Doctor and Senior Lecturer in Automatic,*
Sabrina Aouaouda[1]*, PhD, Professor,*
Reda Rouaibia[1]*, Doctor and Senior Lecturer in Automatic,*
[1] Department of Electrical Engineering,
Faculty of Science and Technology,
Mohamed-Cherif Messaadia University Souk Ahras,
BP 1553, Souk Ahras, Algeria.
e-mail: l.moussaoui@univ-soukahras.dz (Corresponding Author),
sabrina.aouaouda@univ-soukahras.dz,
r.rouaibia@univ-soukahras.dz

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