Fault tolerant control of a permanent magnet synchronous machine using multiple constraints Takagi-Sugeno approach

**Introduction.** Fault diagnosis, and fault tolerant control issues are becoming very important to ensure a good supervision of systems and guarantee the safety of human operators and equipments even if system complexity increases. **Problem.** In fact, the presence of faults in actuators, sensors and processes can lead to system performance degradation, system breakdown, economic loss, and even disastrous situations. Furthermore, Actuator saturation or control input saturation is probably the most usual nonlinearity encountered in control engineering because of the physical impossibility of applying unlimited control signals and/or safety constraints. **Purpose.** This article is dedicated to the problem of fault tolerant control for constrained nonlinear systems described by a Takagi-Sugeno model. One of the interests of this type of models is the possibility of extend some tools and methods from linear system case to the nonlinear one. The **novelty** of the work consists in developing a fault tolerant control algorithm for a nonlinear Permanent Magnet Synchronous Machine model using an observer based state-feedback control technique in order to enhance fault and state estimation despite actuator saturation and system disturbances. **Methods.** Indeed a sensor fault detection observer based residual generator is synthesized with a guaranteed L2 performance to attenuate the external disturbances effect from one side and to maximize the residual sensitivity to faults from the other side. Based on Lyapunov function, design conditions are formulated in terms of Linear Matrix Inequalities to ensure stability of the global system. **Practical value.** A detailed study concerning nonlinear permanent magnet synchronous machine model, which is consolidated by simulation results, is conducted to show the used algorithm’s effectiveness guarantying fault estimation and reconfiguration of the control law to maintain stable performance even in the presence of actuator faults, external perturbation and the phenomenon of actuator saturation. References 19, tables 1, figures 5.

**Key words:** Takagi-Sugeno models, actuator saturation, state estimation, actuator faults diagnosis, fault tolerant control, permanent magnet synchronous machine model, linear matrix inequalities.

**Introduction.** Fault detection and isolation (FDI) has been the subject of many research for linear and nonlinear systems. However, in practical cases used only the FDI block for the process is not enough to preserve desired performance, security and system stability. Hence, fault tolerance must be treated and controllers are synthesized to ensure system stability even in failed situations and degraded operations. We can classify the fault tolerant control (FTC) into passive FTC and active FTC [1, 2]. The first approach can be considered as a robust control, and it requires a priori knowledge of faults that can affect the system, the controller is then designed to compensate them, all possible faults are considered as uncertainties, and an adaptive observer is employed to estimate the fault and state signals. The second is called active FTC, we use in this approach a very robust FDI block to know with exactitude the information’s about faults which constitute its major disadvantage. Indeed, a false alarm or an undetected fault can lead to degradation of performance or even instability [3].

During the design of the FTC controller, we must take into account the saturation of the actuator to avoid the undesirable effects that can destabilize the closed-loop system, and degrade the desired performances [4, 5]. Therefore, much attention has been given to stability analysis and controller design for systems with actuator saturation [6]. More-over, several works propose a norm-bounded controller based on multi-model Takagi-Sugeno (T-S) fuzzy approach, due to the exceptional characteristics of T-S fuzzy models for control purposes to avoid control inputs limitations.

Many researches are developed around the T-S multimodel representation [7, 8]. Nonlinear systems described by T-S models have been considered actively and especially in the fields of control, state estimation and
diagnosis of nonlinear systems. This is related to the fact that T-S fuzzy model can approximate exactly any nonlinear system without loss of information. A T-S model can be obtained using the non-linearity sector approach by aggregating the local models using appropriate interpolation functions [9]. These models have a great ability to represent the complex dynamic system.

The stability analysis of a T-S system has been studied in most cases by using a quadratic Lyapunov function, and solutions are almost expressed as linear matrix inequalities (LMI) [10, 11].

Goal. This paper aims to develop a robust control approach subject to multiple system constraints, i.e. actuator faults, input saturation, external perturbation. The system is presented as a T-S fuzzy multi-model, and then an observer-based FTC design method is introduced to preserve the stability of the system with disturbance rejection. The observer and controller gains are obtained through an L2 minimisation by solving LMI conditions.

The main contribution is to develop model-based FTC-scheme for nonlinear dynamic systems described by T-S models and subject to input constraints. Using the Lyapunov theory for T-S systems, the obtained results are less conservative and formulated in terms of LMI conditions. Consequently, the proposed procedure has also two advantages over the previous cited works. Firstly, it is able to estimate time variable fault types. Secondly, for the analysis of the fuzzy systems, to reduce the computational cost of double summation slack matrices, the presented in [12-16] the proposed FTC controller design method can be considered for a large class of nonlinear constrained systems.

Problem statement. Let consider the following constrained and disturbed T-S model:

\[ \begin{align*}
\dot{x} &= \sum_{i=1}^{r} \mu_i(\xi(t)) (A_i x + B_i sat(u_f(t))) + d(t) \\
y &= \sum_{i=1}^{r} \mu_i(\xi(t)) C_i x
\end{align*} \]

where \( x \in \mathbb{R}^n \) is the state vector; \( u \in \mathbb{R}^m \) is the control input; \( y \in \mathbb{R}^p \) is the system output; \( A_i, B_i, C_i \) are the constant matrices with appropriate dimensions; \( d(t) \) is the external disturbance signal; \( f_i(t) \) is the actuator failure; \( \xi(t) = [\xi_1(t), ..., \xi_r(t)] \) is the decision variables; \( \mu_i(\xi(t)) \) is the normalized activation function satisfying the sum convex property [17]:

\[ \begin{align*}
0 \leq \mu_i(\xi(t)) \leq 1 & \quad \forall i \in \{1, 2, ..., r\} \\
\sum_{i=1}^{r} \mu_i(\xi(t)) &= 1 \\
\mu_i(\xi(t)) &= \frac{w_i(\xi(t))}{\sum_{i=1}^{r} w_i(\xi(t))} \\
w_i(\xi(t)) &= \prod_{j \neq i} M_{ij}(\xi_j(t))
\end{align*} \]

where \( w_i(\xi(t)) \) are the weights; \( M_{ij}(\xi_j(t)) \) are the fuzzy set. The function \( sat(u_f(t)) \) represents the actuator failure saturation function.

The following lemmas and notations will be used in the rest of this paper:

Lemma 1 [2]. Let consider two matrices \( X \) and \( Y \) and a scalar \( \sigma \) such that the following inequality is verified:

\[ X^T Y + Y^T X < \sigma X^T X + \sigma^{-1} Y^T Y, \sigma > 0. \]

Lemma 2 [18, 19]: Let \( E \) be an \( m \times m \) diagonal matrix whose elements are 1 or 0. Suppose that \( |v_i| < \bar{u}_i \) for all \( i \in I_m \) where \( v_i \) and \( u_i \) are the \( i^{th} \) element of \( v \in \mathbb{R}^n \) and \( u \in \mathbb{R}^m \) respectively. If \( x \in \bigcup_{j=1}^{r} \{H_j \} \) for \( x \in \mathbb{R}^n \), then

\[
\begin{align*}
\text{sat}(u, \bar{u}) &= \sum_{i=1}^{r} \alpha_i (E_i u + \bar{E}_i v) \\
\sum_{i=1}^{r} \alpha_i &= 1, \quad 0 \leq \alpha_i \leq 1 \\
v &= -\sum_{i=1}^{r} \alpha_i H_i x \\
\mathcal{L}(H_j) &= \{x \in \mathbb{R}^n : |h_j^T x| \leq \bar{u}_j \},
\end{align*}
\]

where \( E_i \) and \( \bar{E}_i \) denote all elements of \( E \) and \( \bar{E} \); \( h_j \) is the \( j^{th} \) row of the matrix \( H_j \); \( \alpha_i \) is the weighting functions related to the polytopic representation of the saturation function.

Lemma 3 [18]: An ellipsoid \( \epsilon(P, \rho) \) is inside \( \bigcup_{j=1}^{r} \{H_j \} \) if and only if:

\[ \forall i \in I_m : (h_j^T \epsilon(P, \rho) h_j) \leq \bar{u}_i^2, \]

where \( h_j \) is the \( j^{th} \) row of the matrix \( H_j \).

Assumption 1 [3]: the faults are assumed to have a first time derivative bounded as:

\[ \|f(t)\| \leq f_{\text{max}}, \quad 0 \leq f_{\text{max}} \leq \infty \]

Throughout the paper \( H(Z) \) denotes the Hermitian of a matrix \( Z \), i.e. \( H(Z) = Z + Z^T \); and \( I \) denotes the identity matrix.

Fault tolerant control of T-S multi-model. The design of the controller is based on FTC bloc with multi-constraints (actuator fault, actuator saturation and external disturbance) as shown in Fig. 1, ensuring the convergence of the estimated states and the detection of the faults by the observer.

![Fig. 1. Fault tolerant control scheme](image)

The following observer is adopted to estimate both faults and system states as:

\[
\begin{align*}
\dot{x} &= \sum_{i=1}^{r} \mu_i\left( A x + B_i \text{sat}(u_f(t)) + L e_i \right) \\
y &= C x \\
\dot{f}_e &= T \sum_{i=1}^{r} \mu_i f_i (\hat{e}_r + \delta e) \\
e_y &= y - \hat{y}
\end{align*}
\]
where \( \hat{f} \in \mathbb{R}^n \) is the estimated state; \( \hat{f}_a(t) \) is the estimated fault; \( L_a, T, F, \) and \( \delta \) are the observer gains to be determined.

The proposed FTC control law is given by:
\[
U_{FTC} = -\sum_{i=1}^{m} \mu_i K_i \hat{x} + \hat{f}_a(t); \tag{8}
\]
\[
V_{FTC} = -\sum_{i=1}^{m} \mu_i H_i \hat{x}, \tag{9}
\]
where \( K_i, H_i \) are the \( m \times n \) gains matrices to be determined.

In this work, the saturation function can be written as polytopic representation defined by:
\[
sat(u + f_a(t)) = E_{as} \left[ U_{FTC} + f_a(t) \right] + E_{es} V_{FTC} \tag{10}
\]
with:
\[
E_{as} = \sum_{i=1}^{m} \alpha_i E_i; \tag{11}
\]
\[
E_{es} = \sum_{i=1}^{m} \beta_i F_i; \tag{12}
\]

Using (10), the system (1) will be:
\[
\dot{x} = \sum_{i=1}^{m} \mu_i \left( A_i x + B_i (E_{as} (U_{FTC} + f_a(t)) + E_{es} V_{FTC}) \right) + d(t) \tag{13}
\]

Using (8), (9) one can obtain:
\[
\dot{x} = \sum_{i=1}^{m} \mu_i \left( A_i x + B_i (E_{as} H_i) \right) x + B_i (E_{as} H_i) e_x + B_i E_{as} f_a \tag{14}
\]
with:
\[
e_x = x - \hat{x}; \tag{15}
\]
\[
ez = f_a - \hat{f}_a. \tag{16}
\]

**Theorem 1.** The system that generate state error \( e_x \), and fault error \( e_f \) is stable and asymptotically converge to zero and achieve a disturbance rejection level \( \gamma > 0 \) if there exist a symmetric positive definite matrix \( X \), matrices \( L_i, \bar{K}_i, Z_i, T, F_i \) and scalars \( \delta, \sigma_{i1}, \ldots, \sigma_{is} \), solutions of the following LMI constraints:

\[
\min_{\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{is}, \lambda_{j1}, \ldots, \lambda_{js}, \gamma} \lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{is}, \gamma \geq 0, \quad \forall i \in I_a; \tag{17}
\]
\[
\begin{bmatrix}
\Pi_i & z_i & X_i \\
\bar{z}_i & y_i & X_i \\
\lambda_{i1} & \lambda_{i2} & \cdots & \lambda_{is} & \gamma
\end{bmatrix} \leq 0
\]

\[
D_{i1} = H (A_i x - B_i E_{as} \bar{K}_i - B_i \bar{E}_{as} Z_i); \\
D_{i2} = (\bar{E}_{as} E_{as})^T + Z_i \bar{E}_{as} E_{as} \tag{18}
\]

\[
D_{i3} = A_i X + X A_i^T - \bar{T}_i - \bar{T}_i^T = H (A_i X - \bar{L}_i); \\
D_{i4} = E_{as} B_i^T; \\
D_{i5} = -H (\frac{1}{\delta} F C B_{E_{as}}) \tag{19}
\]

The controller gains are obtained by \( K_i = \bar{K}_i \cdot X_i \), \( H_j = Z_j X_i \) and the observer gains are derived by \( L_i = \bar{L}_i \cdot (C \bar{X}_i) \).

**Proof:** Let \( X = (P_i \rho)^{-1} \) and \( Z_j = H_j X \) the inequality (6) can be written as:
\[
\rho^2 X \left( \frac{z_i}{\gamma} \right)^T \frac{z_i}{\gamma} \geq 0, \tag{20}
\]
where \( z_i \) is the \( i \)th row of the matrix \( Z_j \) and by Schur complement, inequality (20) can be written as (17).

The goal is to determine the unknowns’ parameters of the controller \( L_i, T, F_i, \) and \( \delta \). Let us choose the following Lyapunov function:
\[
V = x^T P_x + e^T P_e + \frac{1}{\delta} e^T T^T e_f > 0, \tag{21}
\]
where \( P_i \) is the symmetric positive definite matrix.

The derivative of \( V \) with respect to time \( t \) is given by:
\[
\dot{V} = \dot{x}^T P_x + x^T x \dot{e}_x + e^T \dot{P}_e + \frac{1}{\delta} \dot{e}^T T^T e_f < 0
\]

According to (14)–(16), \( \dot{V} \) becomes:
\[
\dot{V} = x^T \Psi_i P_x + e^T \bar{E}_{as} B_i^T P_x + e^T (K_i^T E_{as})^T + H_i^T \bar{E}_{as} B_i^T P_x + d^T P_x + x^T P_x + \bar{X}_i \Phi_i P_x + \bar{X}_i P_x + \bar{X}_i \Phi_i \dot{P}_e + e^T \bar{P}_e + e^T \Phi_i \dot{P}_e + e^T (K_i^T E_{as})^T + e^T \bar{P}_e + h^T \dot{f}_x \dot{T}^T e_f - \frac{1}{\delta} \dot{e}^T F_i^T e_f - e^T F_i^T e_f + \frac{1}{\delta} \dot{e}^T \dot{f}_x \dot{T}^T e_f - \frac{1}{\delta} \dot{e}^T F_i \dot{e}_f - e^T F_i \dot{e}_f + \frac{1}{\delta} \dot{e}^T \dot{f}_x \dot{T}^T e_f + \frac{1}{\delta} \dot{e}^T F_i \dot{e}_f - e^T F_i \dot{e}_f
\]

with:
\[
\Psi_i = A_i - B_i (E_{as} K_i + E_{as} H_i); \tag{24}
\]
\[
\Phi_i = A_i - L_i C. \tag{25}
\]

The dynamic of the state estimation error, output error and the fault estimation error are calculated by:
\[
\dot{e}_x = \sum_{i=1}^{m} \mu_i \left( \Phi_i e_x + B_i E_{as} e_f \right) + d(t); \tag{26}
\]
\[
\dot{e}_x = C_i e_x = C \left( \sum_{i=1}^{m} \mu_i \left( \Phi_i e_x + B_i E_{as} e_f \right) + d(t) \right); \tag{27}
\]
\[
\dot{e}_f = \dot{f}_x - \dot{f}_a. \tag{28}
\]

Now, using Lemma 1 and Assumption 1 we can write:
\[
\frac{1}{\delta} \dot{e}_x^T \dot{T}^T e_f + \frac{1}{\delta} \dot{e}^T \dot{T}^T e_f < \frac{1}{\delta} \dot{e}^T \bar{P}_e + \frac{1}{\delta} \dot{e}^T \dot{f}_x \dot{T}^T e_f - e^T F_i \dot{e}_f + e^T F_i \dot{e}_f
\]

Based on the dynamic errors defined by (26)–(28) and the inequality (29) the time derivative of the Lyapunov function (23) is rewritten as follows after some algebraic manipulation using Lemma 1:
\[
\begin{bmatrix}
\delta_{i1} & * & * & * \\
\delta_{i2} & \delta_{i3} & * & * \\
E_{as} B_i^T P_i & \delta_{i3} & \delta_{i4} & * \\
\end{bmatrix} \leq 0
\]

with:
\[
\delta_{i1} = \frac{1}{\delta} F C B_{E_{as}} E_{as}. \tag{30}
\]
\[ T_{ij} I_P = \Phi + F \delta \]
\[ \alpha = \frac{1}{\delta} F_C \delta F_C F_C - C_i F_i \]
\[ D_{ij} = H (A_i X - B_i E_{i,}\Phi - B_i F_{i,} Z_i) + XIX \]
\[ D'_{ij} = \left( \Phi + \Phi' \right) B'_{ij} \]
\[ \delta_{ij} = \frac{1}{\delta} F_C A_i C_i F_i \]
\[ \delta_{ij} = \frac{1}{\delta} F_C A_i C_i F_i + \frac{1}{\delta} G \]
\[ \frac{d}{dt} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{pmatrix} + \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \begin{pmatrix} u_a \\ u_b \end{pmatrix} \]

Simulation results. In order to show the effectiveness of the proposed FTC approach, a PMSM non linear model is considered. The nonlinear PMSM model is given by:

\[ \begin{align*}
\frac{d}{dt} x_1 &= f_1(x) + g_1 u_a \\
\frac{d}{dt} x_2 &= f_2(x) + g_2 u_b \\
\frac{d}{dt} x_3 &= f_3(x)
\end{align*} \]
It should be highlighted that the proposed FTC design approach gives promising results while preserving the stability and guarantee the disturbance rejection. The fault has been estimated by the observer (5) as illustrated in Fig. 5.

Conclusions.
This paper is dedicated to the design of fault tolerant control strategy for nonlinear systems described by Takagi-Sugeno models. The systematic procedure is presented to deal with the faulty actuator and input system saturation and applied into permanent magnet synchronous machine. The proposed fault tolerant control design approach is based on a robust observer to estimate both the actuator faults and the system states used to synthesize the controller law. The main advantage of the proposed approach is to synthesize the control law by taking into consideration the saturation limits. Using Lyapunov theory, sufficient conditions are derived by solving an linear matrix inequalities optimization problem. Sufficient conditions are derived by taking into consideration the saturation limits. Using Lyapunov theory, sufficient conditions are derived by solving an linear matrix inequalities optimization problem.

REFERENCES


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