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The method of multi objective synthesis of stochastic robust control by multimass electromechanical systems under non-gausian random external disturbances

Aim*. Development of the method of multi objective synthesis of stochastic robust control by multimass electromechanical systems to satisfy various requirements for the operation of such systems in various modes under non-gausian random external disturbances. **Methodology**. The problem of multi objective synthesis of stochastic robust control by multimass electromechanical systems to satisfy various requirements for the operation of such systems in various modes under non-gausian random external disturbances solved based on the choosing of weight matrices in the robust control goal vector.The calculation of the target vector is performed based on the solution of the zero-sum vector antagonistic game. The components of the game payoff vector are variable quality indicators that are applied to the system operation in various modes. The calculation of the components of payoff vector game are performed based on the simulation of the initial system closed by the synthesized stochastic controllers in various operating modes and under various external influences and variations in the parameters of the uncertainty of the initial plant. **Results**. The results of multi objective synthesis of stochastic robust two-mass electromechanical servo systems modes under non-gausian random external disturbances in which differences requirements for the operation of such systems in various modes were satisfied are given. Based on the results of modeling and experimental studies it is established, that with the help of synthesized robust nonlinear controllers, it is possible to improve of quality indicators of two-mass electromechanical servo system in comparison with the system with standard regulators. **Originality.** For the first time the method of multi objective synthesis of stochastic robust control by multimass electromechanical systems to satisfy various requirements for the operation of multimass systems in various modes is developed. **Practical value**. From the point of view of the practical implementation the possibility of solving the problem of multi objective synthesis of stochastic robust control systems to satisfy various requirements for the operation of multimass electromechanical systems in various modes is shown. References 38, figures 3.*
Key words: **multimass electromechanical systems, stochastic robust control, multi objective synthesis, zero-sum vector game solution, computer simulation, experimental research.**

*Ціль. Розробка методу багатокритеріального синтезу стохастичного робастного керування багатомасовими електромеханічними системами для задоволення різноманітних вимог до роботи багатомасових систем в різних режимах при негаусових випадкових зовнішніх збуреннях. **Методологія.** Задача багатокритеріального синтезу стохастичного робастного керування багатомасовими електромеханічними системами для задоволення різноманітних вимог до роботи систем в різних режимах при негаусових випадкових зовнішніх збуреннях вирішена на основі вибору вагових матриць у векторі цілі робастного керування. Обчислення вектору цілі виконується на основі рішення векторної антагоністичної гри з нульовою сумою. Компонентами вектора ціни ігри є показники якості, які пред'являються до роботи системи в різних режимах. Обчислення компонентів вектора ціни гри виконано на основі моделювання вихідної системи, замкненої синтезованими стохастичними регуляторами в різних режимах роботи та при різних зовнішніх впливах та варіаціях параметрів невизначеності вихідної системи. **Результати.** Наведено результати багатокритеріального синтезу стохастичних робастних двомасових електромеханічних сервосистем при негаусових випадкових зовнішніх збуреннях, в яких задовольняються вимоги до роботи таких систем у різних режимах. На основі результатів моделювання та експериментальних досліджень встановлено, що за допомогою синтезованих стохастичних робастних регуляторів можна підвищити якісні показники двомасової електромеханічної сервосистеми в порівнянні з системою зі стандартними регуляторами. **Оригінальність.** Вперше розроблено метод багатокритеріального синтезу стохастичного робастного керування багатомасовими електромеханічними системами для задоволення різноманітних вимог до роботи таких систем у різних режимах. **Практична цінність**. З точки зору практичної реалізації, показано можливість вирішення задачі багатокритеріального синтезу стохастичних робастних систем керування для задоволення різноманітних вимог до роботи багатомасових електромеханічних систем в різних режимах. Бібл. 38, рис. 3.*
Ключові слова: **багатомасові електромеханічні системи, стохастичне робастне керування, багатокритеріальний синтез, рішення векторної антагоністичної гри з нульовою сумою, комп'ютерне моделювання, експериментальні дослідження.**

Introduction. The central problem of the modern theory of robust control is the creation of systems that can function effectively under conditions of uncertainty in the values of parameters, and possibly also in the structure of models of the plant, disturbances and measurement noise.

To date, impressive results have been obtained in the synthesis of robust controllers, which make it possible to obtain guaranteed results in control and identification problems and are designed for the most unfavorable case. However, the payoff for obtaining a guaranteed result in the most unfavorable case is the excessive «caution» of such regulators.

Recently, the theory of stochastic robust control has been intensively developed [1–4]. Stochastic robust control systems have a number of advantages. First, they are robustly stable, i.e. maintain stability when changing the parameters of the control object within certain limits.

Secondly, they have a significantly lower sensitivity to changes in the parameters of the control object compared to optimal systems, despite the fact that the dynamic characteristics of stochastic robust systems may differ slightly from the corresponding characteristics of optimal systems. Therefore, the issues of designing control systems operating under random setting and disturbing influences are relevant.

The problem of designing an anisotropic controller minimizing the anisotropic norm of a closed system was posed and solved in [1–4]. This refers to the synthesis of controllers for systems whose model parameters are fully known. However, there is no complete information about the object model, which entails the need to develop methods that are robust not only to external disturbances, but also robust to the model parameters.

When creating methods for the synthesis of robust control, a new area of application of the theory of

dynamic games has appeared, when one of the players is the control, with the help of which the norm of the goal vector is minimized, and the other player is an external influence, which is considered independent during the synthesis of the system is determined from the condition maximum «evilness» (worst-case disturbance) – the maximum deviation of the norm of the same goal vector.

When synthesizing an anisotropic controller that minimizes the anisotropic norm of a closed system, it is assumed that the goal vector of robust control is given. In as a result of the synthesis, a control is found, with the help of which the anisotropic norm of the target vector, i.e. a scalar value, is minimized. However, for multimass electromechanical control systems, in addition to the requirements for the accuracy of processing or compensation of random input signals, certain requirements are often imposed on the quality requirements of its work in various modes of operation.

Until now, in the theory of synthesis of anisotropic controllers, the problem of multicriteria synthesis of anisotropic controllers, robust to inaccuracies in setting the parameters of the object model of multimass electromechanical systems has not been solved [5, 6]. Therefore, the development of a method for the multicriteria synthesis of anisotropic controllers those are insensitive to the parameters of the object model of multimass electromechanical systems and allows taking into account all the requirements for the operation of such systems in various operating modes is an urgent problem.

The purpose of the work is to develop the method of multi objective synthesis of stochastic robust control by multimass electromechanical systems to satisfy various requirements for the operation of multimass systems in various modes under non-gausian random external disturbances.

Statement of the problem. The current state of the theory of robust systems is characterized by a wide variety of problem statements and formulations of criteria for the synthesis of robust controllers. It is customary to distinguish between classes of parametric (structured) and non-parametric (unstructured) uncertainties. In practice, there are cases of joint uncertainty (both parametric and non-parametric) [7–10].

Non-parametric uncertainty is understood as such uncertainty in the control object, which affects the structure of the object, changing its order [11–14]. Parametric uncertainty is understood as changes in the coefficients of an object in a certain area [15–19].

The main approach to the synthesis of robust control in the time domain is based on solving the optimal control problem [20–23]. However, unlike the classical approach to the synthesis of optimal control systems, with robust control, in addition to the control vector, the equation of state of the original control object also includes the vector of external influences [24, 25]. Moreover, the vector of external influences characterizes the change in the state of the system due to parametric and structural changes in the model of the control object.

For a discrete robust plant of a multimass system with state vector x_k, control vector u_k, the vector y_k of measurable variables and the vector z_k of the robust control target the difference equation of state is represented in the standard form adopted in robust control theory:

$$\begin{cases} x_{k+1} = Ax_k + B_0\omega_k + B_2u_k, \\ z_k = C_1x_k + D_{12}u_k, \qquad -\infty < k < \infty \quad (1) \\ y_k = C_2x_k + D_{21}\omega_k. \end{cases}$$

The anisotropic regulator is adopted in the form of a discrete dynamical system, which is given by the difference equation:

$$\begin{aligned} \xi_{k+1} &= \hat{A}\xi_k + \hat{B}y_k\cdots, \\ \cdots u_k &= \hat{C}\xi_k. \end{aligned} \qquad (2)$$

The input of this dynamic system (2) is the vector y_k of measuring variables, and its output is the control vector u_k of the control object (1).

An advanced system that includes a control object (1) closed by a controller (2) has a state vector that includes the state vector x_k of the control object and the state vector ξ_k of the controller. The input vectors of this extended system are the vectors ω_k of external signal and parametric perturbations, and the output vector of the system is the target vector z_k of robust control. We introduce the matrix of this extended system in the state space is taken as

$$\left[\begin{array}{cc|cc} A & B_2\hat{C} & B_0 & B_1 \\ \hat{B}C_2 & \hat{A} & \hat{B}D_{21} & 0 \\ \hline C_1 & D_{12}\hat{C} & 0 & 0 \end{array}\right] \equiv \cdots$$

$$\cdots \equiv \left[\begin{array}{c|cc} A_t & B_t & F_t \\ \hline C_t & 0 & 0 \end{array}\right].$$
(3)

When synthesizing an anisotropic controller that minimizes the anisotropic norm of a closed system, the vector z_k of the robust control goal is specified as a vector of some variables that are functions of time and represent a linear transformation of the variables of the state vector x_k and control vector u_k, of the original system (1). In as a result of the synthesis, a control u_k is found, with the help of which the anisotropic norm of the target vector z_k, i.e. a scalar value, is minimized.

Notice, that when synthesizing a robust deterministic controller, the target vector is also specified as a vector function of time, however, during the synthesis, a scalar variable is minimized in the form of the H_∞ norm of this target vector. We also note that under the classical optimal control, a scalar variable is minimized in the form of a H_2 norm of a linear transformation of the variables of the state vector and control of the original system.

However, for multimass electromechanical control systems, in addition to the requirements for the accuracy of processing or compensation of random input signals, certain requirements are often imposed on the quality of transient processes – the time of the first coordination, the time of regulation, re-regulation, etc. Thus, in addition to the requirements for the operation of the system with random input signals, there are certain requirements for working out by the system of deterministic influences – stepwise signals. Moreover, these requirements can differ significantly for transient processes when the system works out «small» and «large» impacts.

In addition, certain requirements are often imposed on multimass electromechanical systems for processing harmonic signals of fixed frequencies, or a given frequency range, which are also deterministic signals.

These requirements can be satisfied with a deterministic approach to the synthesis of the designed system.

Naturally, the designed system must simultaneously satisfy all the requirements for the operation of the system in various modes and with various input signals.

When designing a system, it is necessary to set the goal vector of robust control in such a way that the designed system must simultaneously satisfy all the requirements for the operation of the system in various modes and with various input signals.

Solution method. Consider the method for calculating the target vector of anisotropic robust control that the designed systems satisfy all the requirements for the operation of the system in various modes and with various input signals.

Let us introduce the vector of the desired parameters θ, the components of which are the elements of the C_1 and D_{12} matrices in (1), with the help of which the goal vector z_k of the stochastic robust control is formed in the form of a linear combination of the variables of state vector x_k of the original system and of the component of control vector u_k.

Let us introduce the vector J of quality indicators that apply to the operation of the system in various operating modes. The components of this vector, in particular, can be: the transient times are usually specified when certain input signals are applied: the accuracy of working off the specified minimum speed value the uneven movement of the working body at the minimum speed: minimum value of the stabilization dispersion of a given random change in the reference action is usually required under the action of random disturbing influences caused, for example, by a random change in the road profile.

Let us also introduce the vector B of binary preference relations for pairwise comparison of performance indicators of the system in different modes, with different input signals and with different uncertainty parameters of the original multimass system.

In addition, we introduce a vector G of limitations, the components of which can be the limiting values of voltages, currents, rates of change of currents, moments, elastic moments, rates of change of moments (jerks), speeds of various elements of a multimass electromechanical system, their positions, etc.

We also introduce the vector δ of uncertainties in the parameters of the initial system and external influences. The components of this wind, in particular, can be changes in the moments of inertia of the working body.

The dynamic characteristics of the synthesized stochastic robust system are determined by the molel of the initial system, external master and disturbing influences, and, of course, the parameters of the synthesized stochastic robust controllers. The control system designer can only change the stochastic robust control target vector.

For a given value of the vector θ of the desired parameters, with the help of which the goal vector of the stochastic robust control is formed, a stochastic robust controller can be calculated. For this synthesized stochastic controller, the values of the components of the vector J of quality indicators, the vector G of of restrictions and the vector of B of binary preference relations can be calculated in the course of simulating the operation of a closed system in various operating modes,

for various input signals and for various values of the vector parameters of uncertainties of the initial system.

Then, using the given value of the vector θ of these desired parameters, the vector J of the values of quality indicators that are imposed on the system operation, and the vector G of the restrictions when the system is operating in various operating modes and for various setting and disturbing and for various values of the vector δ of the uncertainty of the initial system parameters and external influences the problem of multi objective synthesis of non-linear robust control can be formulated as the zero-sum vector antagonistic game [25, 26].

$$J(\theta,\delta) = [J_1(\theta,\delta), J_2(\theta,\delta),\ldots \\ \ldots J_m(\theta,\delta)]^T. \qquad (4)$$

In this game, the first player is the vector of the desired parameters θ, the components of which are the elements of the C_1 and D_{12} matrices (1), with the help of which the goal vector z_k of the stochastic robust control is computed, and it strategy is to minimize the game pay vector J. The second player is the vector δ of uncertainties in the parameters of the initial system and external influences, and it strategy is to maximize the same game pay vector J. This approach is the standard approach in the robust control synthesis for the «worst» case.

Naturally, in this case, it is necessary to take into account restrictions on the strategy vectors of the first and second players in the form of a vector inequality

$$G(\theta,\delta) \le G_{max} \qquad (5)$$

the components of which are the limiting values of voltages, currents, rates of change of currents, moments, elastic moments, rates of change of moments, speeds of various elements of a multimass electromechanical system.

To correctly calculation of solution of this vector antagonistic game from the set of Pareto-optimal solutions, binary preference relations of local performance criteria B are used.

In conclusion, we note that the computation of the payoff game vector J, the constraint vector G, and the vector B of binary preference relations is algorithmic in nature and requires large computational resources. First, to calculate the stochastic robust control. Then, in order to calculate the values of payoff game vector J, the constraint vector G, and the vector B of binary preference relations it is necessary to simulate the initial non-linear system closed by synthesized stochastic robust controllers for given system operation modes and for given driving and perturbing influences at given values of the stochastic vector certainty of the parameters of the original system.

The algorithm for calculating of the solution of a multicriteria game. Let us consider the algorithm for calculating the solution of a multicriteria game (4) with constraints (5) from the set of Pareto-optimal solutions, taking into account binary preference relations [26–30], based on the particle multiswarm optimization (PSO) algorithm [31–34]. To date, a large number of particle swarm optimization algorithms have been developed – PSO algorithms based on the idea of collective particle swarm intelligence, such as gbest PSO and lbest PSO algorithms. The use of stochastic multi-agent optimization methods to solve vector antagonistic game today causes some difficulties and this area continues to develop

intensively. To solve the initial vector antagonistic game with constraints, we construct an algorithm for stochastic multiagent optimization based on a set of swarms of particles, the number of which is equal to the number of components of the pay vector game.

In the standard optimization algorithm by a swarm of particles, the change in particle velocities is usually carried out according to linear laws [35–38]. To increase the speed of finding a global solution, a nonlinear optimization algorithm by a multiswarm of particles, proposed in [36], is used, in which the motion of i particle – of j swarm is described by the following expressions

$$v_{ij}(t+1) = w_{1j}v_{ij}(t) + c_{1j}r_{1j}(t)*\ldots$$
$$\ldots * H\left(p_{1ij}(t) - \varepsilon_{1ij}(t)\right)\left[y_{ij}(t) - \ldots\right. \quad (6)$$
$$\ldots - x_{ij}(t)\right] + c_{2j}r_{2j}(t)H\left(p_{2ij}(t) - \ldots\right.$$
$$\ldots - \varepsilon_{2ij}(t)\right)\left[y_j^*(t) - x_{ij}(t)\right]$$

$$u_{ij}(t+1) = w_{2j}u_{ij}(t) + c_{3j}r_{3j}(t)H * \ldots$$
$$\ldots * \left(p_{3ij}(t) - \varepsilon_{3ij}(t)\right)\left[z_{ij}(t) - \delta_{ij}(t)\right] + \ldots \quad (7)$$
$$\ldots + c_{4j}r_{4j}(t)H\left(p_{4ij}(t) - \varepsilon_{4ij}(t)\right) * \ldots$$
$$\ldots * \left[z_j^*(t) - \delta_{ij}(t)\right]$$

$$\theta_{ij}(t+1) = \theta_{ij}(t) + v_{ij}(t+1), \quad (8)$$

$$\delta_{ij}(t+1) = \delta_{ij}(t) + u_{ij}(t+1), \quad (9)$$

where $\theta_{ij}(t)$, $\delta_{ij}(t)$ position and $v_{ij}(t)$, $u_{ij}(t)$, velocities of i particle of j swarm.

Note that due to the fact that the solution of the multicriteria game (4) are two vectors of strategies of the first θ and second δ players, each i particle of j swarm has two position components $\theta_{ij}(t)$, $\delta_{ij}(t)$, and two velocity components $v_{ij}(t)$, $u_{ij}(t)$, to find two vectors strategies of the first and second players.

In (6) – (7), $y_{ij}(t)$, $z_{ij}(t)$ and y_j^*, z_j^* are the best local and global positions of i particle found, respectively, by only one i particle and all i particles by j swarm. Moreover, the best components of the local $y_{ij}(t)$ and global $y_j^*(t)$ position i particle of the j swarm are understood in the sense of the strategy of the first player $\theta_{ij}(t)$ to minimize the components of the vector payoff (4). However, in this case, the best components of the local $z_{ij}(t)$ and global z_j^* position of the same i particle of the same j swarm are understood in the sense of the strategy of the second player $\delta_{ij}(t)$ to maximize the same components of the vector payoff (4). This approach corresponds to the motion of particles along the gradient and antigradient when using deterministic algorithms for solving games [24].

Four independent random numbers $r_{1j}(t)$, $r_{2j}(t)$, $r_{3j}(t)$, $r_{4j}(t)$ are in the range [0, 1.0] and determine the stochastic components of the particle velocity components.

Positive constants c_{1j}, c_{2j} and c_{3j}, c_{4j} determine the cognitive and social weights of the particle velocity components.

Note that the peculiarity of the solution of this game is that the components of the strategy vector $\theta_{ij}(t)$ of the first player are the components of which are the elements of the C_1 and D_{12} matrices in (1), with the help of which the goal vector z_k of the stochastic robust control is formed in the form of a linear combination of the

variables of state vector x_k of the original system and of the component of control vector u_k.

Therefore, the values of these constants c_{1j}, c_{2j} and c_{3j}, c_{4j} are chosen taking into account the range of possible changes in the strategies $\theta_{ij}(t)$, $\delta_{ij}(t)$, of the players.

A feature of the solution of this multicriteria game (4) with restrictions (5) is the presence of «ravines» and «ridges» in the vector payoff (4). This is due, firstly, to the fact that the values of the components of the strategies $\theta_{ij}(t)$, $\delta_{ij}(t)$, of the players and differ by more than an order of magnitude. Secondly, a change in some components of the players' strategies leads to insignificant changes in the vector payoff of the game (4).

Therefore, to improve the speed of finding a global solution with small increments of the vector payoff (4) to determine the strategies $\theta_{ij}(t)$, $\delta_{ij}(t)$, of players, in (6) – (7) a non-linear search algorithm (Cuckoo Search) [34] is used for stochastic optimization by a multiswarm of particles. The Heaviside function H [35] is used as a non-linear function for switching the particle motion to the local $y_{ij}(t)$, $z_{ij}(t)$ and global $y_j^*(t)$, z_j^* optima, respectively.

The switching parameters p_{1ij}, p_{3ij} cognitive and social p_{2ij}, p_{4ij} components of the particle velocity components to the local $y_{ij}(t)$, $z_{ij}(t)$ and to the global $y_j^*(t)$, z_j^* optima were taken as increments of changes in the components of the vector payoff (4) for the strategies of the players $\theta_{ij}(t)$, $\delta_{ij}(t)$, when moving, respectively, to the local and global optima.

Random numbers $\varepsilon_{1ij}(t)$, $\varepsilon_{2ij}(t)$, $\varepsilon_{3ij}(t)$ and $\varepsilon_{4ij}(t)$ determine the parameters for switching the motion of particles, respectively, to the local and global optimums. If $p_{1ij} < \varepsilon_{1ij}(t)$ and $p_{2ij} < \varepsilon_{2ij}(t)$, then the component $v_{ij}(t)$ of the velocity of movement of i particle of j swarm does not change at the t step, and this particle moves in the same direction as at the previous optimization step. Similarly, if $p_{3ij} < \varepsilon_{3ij}(t)$ and $p_{4ij} < \varepsilon_{4ij}(t)$, then the component $u_{ij}(t)$ of the velocity of movement of i particle of j swarm does not change at the step t either.

To improve the quality of the process of finding a solution, inertia coefficients w_{1j}, w_{2j} are used, in the range $(0.5 – 0.9)$.

Note that the solutions of the original vector game (4) are the Pareto set of optimal solutions. The narrowing of this set occurs at the stage of search by taking into account constraints (5) on the state variables and control of the original system. As limitations G (5) in this game, first of all, the the limiting values of voltages, currents, rates of change of currents, moments, elastic moments, rates of change of moments (jerks), speeds of various elements of a multimass electromechanical system, their positions, etc are taken into account.

In the considered problem of synthesis of the servo system, in particular, the limitations on the armature current of the drive motor and the speed of rotation of this motor is hindered to decrease the time of the first coordination in the mode of working out large angular mismatches by this servo system.

The maximum particle velocities $v_{ij}(t)$, $u_{ij}(t)$ were also limited, based on the desired accuracy of obtaining solutions to the corresponding components of the strategy vectors $\theta_{ij}(t)$ and $\delta_{ij}(t)$, as well as to improve the convergence of the solution to the game (4).

To find the global optimum solution of the multicriteria game (4) in the course of searching for local optimal solutions to this game, individual particles and swarms exchange information with each other. At the same time, to calculate the speed of a particle of one swarm, information about the global optimum obtained by particles of another swarm is used, which makes it possible to identify all possible Pareto-optimal solutions. To do this, at each step t of the movement of i particle of j swarm, the functions of binary preference relations of local solutions obtained by all swarms are used.

The solution $\theta_j^*(t)$ obtained in the course of solving the game $J(\theta_j^\bullet(t), \delta(t))$ with the help of j swarm is more preferable than the solution $\theta_k^*(t)$ of the game $J(\theta_k^\bullet(t), \delta(t))$ obtained with the help of k swarm, if the condition

$$B\left(J(\theta_j^\bullet(t), \delta(t))\right) < B\left(J(\theta_k^\bullet \\ \ldots(t), \delta(t))\right). \tag{10}$$

In this case, as the global optimal solution $\theta_k^*(t)$ obtained by the k swarm, the global solution $\theta_j^*(t)$ obtained by the j swarm is used, which is more preferable than the global solution $\theta_k^*(t)$ obtained by the k swarm based on the binary preference relation (10).

An example of such a binary preference can be a situation where a decrease in the time of the transient process when working out small mismatches leads to an increase in the dispersion of the tracking error. Due to an increase in the component of the dispersion for measurement noise due to the expansion of the system bandwidth. In this case, it is more preferable to reduce the dispersion of the tracking error. Since it is this quality indicator that determines the potential accuracy of the tracking system and the possibility of capturing and auto-tracking the target.

In fact, this approach implements the main idea of the method of successively narrowing the area of compromise solutions – Pareto-optimal solutions from the initial set of possible solutions based on information about the relative importance of local solutions that cannot be chosen in accordance with the -formation about binary preference relations (10).

Solutions are removed until a globally optimal solution is found. As a result of applying this approach, the potentially optimal solution will not be removed at each stage of the narrowing. Note that the art of applying the multiswarm particle optimization (PSO) algorithm lies in the reasonable choice of its tuning constants.

In conclusion, we note that the original multicriteria game (9) – (10), taking into account the algorithm for solving it (11) – (13), is a multicriteria stochastic dynamic game, since it clearly has time and random search [26].

Calculation of the Anisotropic norm. Stochastic approach to H_∞-optimization of automatic control systems based on the use of the quality criterion of the stochastic norm of the system. Such a norm quantitatively characterizes the sensitivity of the system output to random input disturbances, the probability distribution of which is not known exactly. The concretization of this approach, obtained by combining the concept of the stochastic norm of the system and the average anisotropy of the signal [1 – 4], leads to a special version of the stochastic norm – the anisotropic norm.

Average anisotropy is a characteristic of the space-time coloring of a stationary Gaussian signal, which is closely related, on the one hand, to the information-theoretic approach to the quantitative description of chaos using the Kolmogorov entropy of probability distributions, and on the other hand, to the principle the isotropy principle of a finite-dimensional Euclidean space. The anisotropic norm of a system characterizes its sensitivity to input Gaussian noises, the average anisotropy of which is limited from above by some non-negative parameter.

This norm is used when a priori information about the input disturbance is that the disturbance is a Gaussian random sequence with zero mean and mean anisotropy bounded from above [4]. The latter is a measure of the correlation of the components of a random vector in a sequence (or, as they say, «coloring») or, which is the same, a measure of the deviation of a sequence of a random variable from Gaussian white noise.

The main concepts of the anisotropic theory of stochastic robust control are the anisotropy of a random vector, the average anisotropy of a random vector, and the anisotropic norm of the system. The anisotropy functional is an entropy measure of the deviation of a probability distribution in Euclidean space from Gaussian distributions with zero mean and scalar covariance matrices.

The average anisotropy of a stationary random sequence characterizes the magnitude of the statistical uncertainty, understood as the discrepancy between the inaccurately known actual noise distribution and the family of nominal perturbation judels in the form of a stationary discrete Gaussian white noise.

The anisotropic norm of a discrete linear stationary system quantitatively determines the capabilities of the system to suppress disturbances by the largest ratio of the power norm of the output of the system to the power norm of its input, provided that the average anisotropy of the input signal does not exceed a given non-negative level.

The keypoint is the definition of the anisotropy of a random vector, the mean anisotropy of a Gaussian sequence, and the anisotropy norm of the transfer function of a closed system. Covariance matrix, therefore, to calculate the anisotropic norm, it is necessary to solve the Ricatti equation – as a necessary condition for the minimum Kullback-Leibler informational deviation [3].

Consider the original system (1) closed by a stochastic controller (2) in the form of the state equation of discrete system

$$\begin{cases} x_{k+1} = Ax_k + B\omega_k; \\ z_k = Cx_k + D\omega_k. \end{cases} \tag{11}$$

To calculate the anisotropic norm of this system must solve the Ricati equation with respect to the matrix

$$R \in R^{n \times n}; \\ R = ARA^T + BB^T - \Lambda\Theta\Lambda^T, \tag{12}$$

where

$$\Lambda = \left(ARC^T + BD^T\right)\Theta^{-1},$$

$$\Theta = CRC^T + DD^T.$$

Then the average anisotropy is calculated based on the following expression

$$\overline{A}(G) = -\frac{1}{2}\ln\det\left(\frac{m\Theta}{Trace\left(CPC^T + DD^T\right)}\right), \tag{13}$$

where P is the controllability Gramian that satisfies the Lyapunov equation

$$P = APA^T + BB^T .$$

Calculating of the payoff vector of the game. To calculate the values of the vector of quality indicators, the initial nonlinear system is modeled, closed by the synthesized anisotropic controller in various operating modes and under various input influences. The central idea of the synthesis of robust control systems is related to the synthesis of a system that minimizes the quality criterion according to the control norm, but maximizes the same criterion according to the norm of the vector of external influences. At the same time, due to the introduction of the norm of the vector of external influences with a minus sign into the Hamilton function, the synthesized system minimizes the sensitivity of the system to the variable parameters of the control object, and therefore ensures the robustness of the system.

This approach corresponds to the game approach to the optimization problem, when the first player «control» minimizes the goal function, and the second player «undefined parameters of the plant» maximizes the same goal function. Moreover, since the original system is described by a system of differential equations - a matrix equation of state, and both players use the same goal function, then such a game is called a zero-sum differential game.

We synthesize a complete anisotropic controller of multimass electromechanical systems with parametric uncertainty by reducing it to the solution of the optimization problem without parametric uncertainty by introducing an additional input, so that the obtained problem is a mixed / optimization problem.

We use the results of differential games to solve the problem of synthesis of anisotropic regulators. To find the worst input, forming vector of structural perturbations, that maximizes the anisotropic norm of the system, solve the following first Riccati's equation

$$\widetilde{Y} = A_t^T \widetilde{Y} A_t + L_t^T \Pi L_t + Q , \qquad (14)$$

where

$$L_t = \Pi^{-1} F_t^T \widetilde{Y} A_t , \quad \Pi = \Gamma - F_t^T \widetilde{Y} F_t ,$$

To find the worst input, forming vector of parametric perturbations, that maximizes the anisotropic rate of the system, solve the following second Riccati's equation

$$R = A_\omega^T R A_\omega + q C_\omega^T C_\omega + L^T \Sigma^{-1} L , \qquad (15)$$

where

$$L = \Sigma \left(B_\omega^T R A_\omega + q D_\omega^T C_\omega \right),$$

$$\Sigma = \left(I_{m1} - B_\omega^T R B_\omega \right)^{-1} .$$

For the synthesis of anisotropic observers of the state of multimass electromechanical systems with parametric uncertainty, which are necessary for the realization of anisotropic regulators of complete order, we solve the third Riccati equation

$$S = \widetilde{A}_{11} S \widetilde{A}_{11}^T + \widetilde{B}_1 \widetilde{B}_1^T - \Lambda \Theta \Lambda^T , \qquad (16)$$

where

$$\Theta = \widetilde{C}_{21} S \widetilde{C}_{21}^T + \widetilde{D} \widetilde{D}^T ,$$

$$\Lambda = \left(\widetilde{A}_{11} S \widetilde{C}_{21}^T + \widetilde{B}_1 \widetilde{D}^T \right) \Theta^{-1} .$$

To synthesize the optimal anisotropic state regulator that minimizes the anisotropic norm of the system, we solve the fourth Riccati equation

$$T = A_u^T T A_u + C_u^T C_u - N^T \Psi N , \qquad (17)$$

where

$$\Psi = B_u^T T B_u + D_{12}^T D_{12} ,$$

$$N = -\Psi^{-1} \left(B_u^T T A_u + D_{12}^T C_u \right).$$

The obtained algorithm for the synthesis of anisotropic regulators is reduced to an iterative solution of the problem of stochastic robust optimization. The solution of the problem begins for fixed values of tolerance parameters, the values of which must be large enough for the equations to have solutions. Then we solve a system of four related Riccati equations (14) – (17), Lyapunov equation and special expression (13). This system of equations is numerically solved using the homotopy method, which includes vectorization of matrices and iterations according to Newton's method.

The optimal anisotropic controller of the complete order is the optimal estimator of the optimal control law in the problem with complete information about the system state vector for the case of the worst input. The principle of separation in the mixed problem of robust control does not mean the independence of the Riccati equations. Unlike classical H_2 and H_∞ optimization, the problem of estimator synthesis and the problem of synthesis of the optimal static regulator in the form of feedback cannot be solved independently of each other.

This generalized distribution principle allows us to interpret the results obtained in terms of the theory of differential games. Control of the system in the presence of external perturbations is considered as a differential game between two players – nature and the regulator. The optimal strategy of the first player – the regulator, is to obtain optimal control, and the optimal strategy of the second player – nature, is to obtain the «worst» perturbation in the form of parametric uncertainty of the control object. At the same time, each of the players knows about the optimal strategy of his opponent.

Simulation results. Let us consider multi objective synthesis of stochastic robust control by two-mass electromechanical servo systems to satisfy various requirements for the operation of such systems in various modes under non-gausian random external disturbances. One of the most stressful modes of operation of such a servo system is the guidance mode with a random velocity celm under the action of random non-Gaussian perturbations on the plant. On Fig. 1 are shown the implementation of random processes of state variables: rotation speed $\omega_M(t)$ of the motor (a), rotation speed $\omega_P(t)$ of the plant (b) and the disturbances $M_D(t)$ acting on the plant (c).

An analysis of these implementations shows that the system accurately processes the low-frequency components of the master random process; however, when processing the high-frequency components, the system error is about 20 %.

In this example anisotropic controllers demonstrated the best quality of suppression of external disturbances and tracking at the lowest control costs, and closed systems with anisotropic controllers have greater noise immunity.

Taking into account apriori information about the input action, which consists in the limitation from above of the functional of the average anisotropy of the input perturbation by a known parameter, makes it possible to

obtain controllers that are more robust with respect to the specified action than H_2 controllers and less conservative than H_∞ regulators, which makes them very attractive for practical use instead of classical LQR regulators
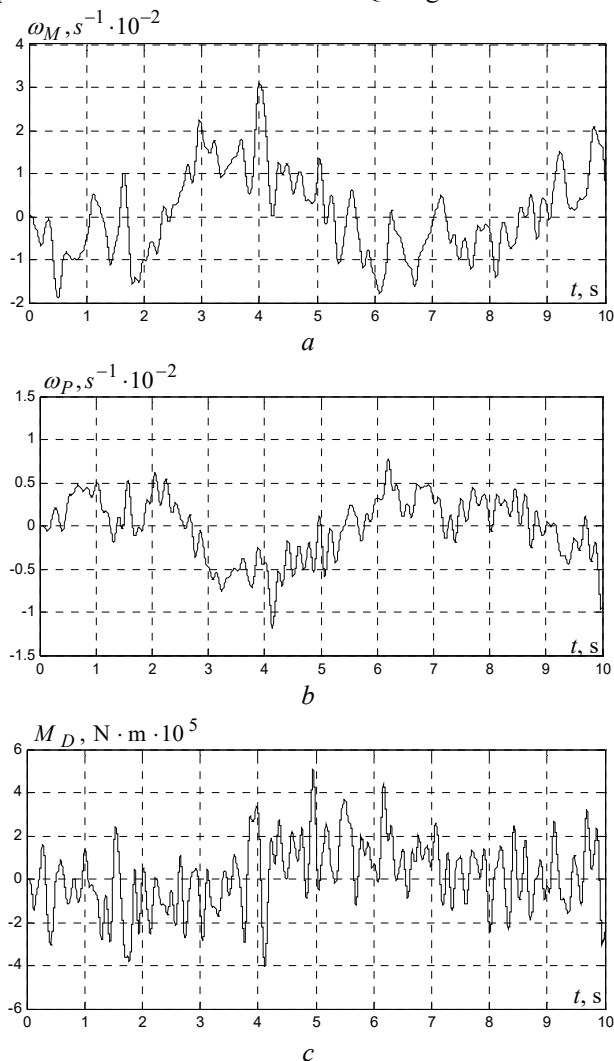

Fig. 1. The implementation of random processes of state variables of two-mass electromechanical servo systems

Note that the solutions of the original vector game (4) are the Pareto set of optimal solutions. The narrowing of this set occurs at the stage of search by taking into account constraints (5) on the state variables and control of the original system. In particular, the decrease in the time of the first coordination in the mode of working out large angular mismatches by the servo system is hindered by limitations on the armature current of the drive motor and the speed of rotation of this motor.

Further narrowing of this set at the search stage occurs on the basis of binary preference relations. In particular, in the mode of working out small mismatch angles, with a decrease in the time of the first matching, the dispersion of the tracking error increased due to the expansion of the system bandwidth at the amplification of the noise of the meters. For the tracking system under consideration, the error variance is a more preferable criterion, since it determines the potential accuracy of the tracking system. Therefore, the solution corresponding to the shorter time of the first matching in the mode of working out small angles was discarded.

Experimental research. For experimental research of the dynamic characteristics of synthesized stochastic systems the research stand of a stochastic two-mass electromechanical system was developed. The scheme of this stand of a stochastic two-mass electromechanical system is shown in Fig. 2.
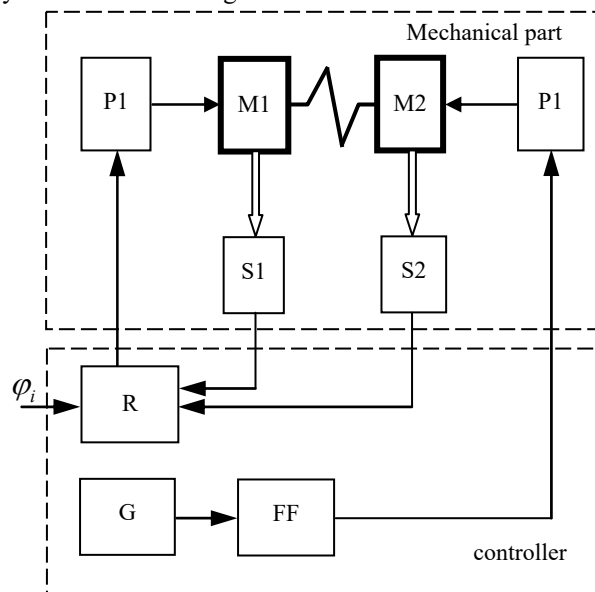

Fig. 2. The scheme of research stand of a stochastic two-mass electromechanical system

The mechanical part of the stand is made on the basis of two identical micromotors of direct current motor – DCM 25-N2.

The converter of electric motor into mechanical is the micromotor M1, and the micromotor M2 forms the amount of load for M1. The shafts of engines M1 and M2 are connected by an elastic transmission. With the help of the second motor M2 the loading moment is created.

The angle of turn of shaft of motors M1 and M2 is measured the optical sensors of angle of turn S1 and S2. The sources of power of P1 and P2 provide the power of current load of motors M1 and M2. Forming of controls influence for the sources of power is executed in the regulator of position (G). To simulate a random effect on the system at the input of the M2 is a random signal from the output of the forming filter (FF) in the form of an oscillating link. A white noise signal from a random signal generator (RSG) is fed to the input of the generating filter. Characteristics of random change of loading moment are defined by parameters of the forming filter and actually the motor M2.

Experimental studies of the dynamic characteristics of a two-mass electromechanical system with typical regulators and with synthesized anisotropic regulators were carried out at the stand.

On Fig. 3 are shown the implementation of random processes of state variables of research stand of a stochastic two-mass electromechanical system. In Fig. 3 are shown the following state variables: rotation speed $\omega_{M1}(t)$ of the motor M1 (a); rotation speed $\omega_{M2}(t)$ of the motor M2 (b), the armature current $I_{M1}(t)$ of the motor M1 (c) and the armature current $I_{M2}(t)$ of the motor M2 (d).

It is shown that the use of stochastic robust control of the stand of a two-mass electromechanical system with

a random change in the load moment allows to reduce the control error of the rotation speed of the second motor by more than 1.5 times and to reduse the error in regulating the angle of rotation of the shaft of the second motor by more than 2 times in comparison with a system with typical regulators.
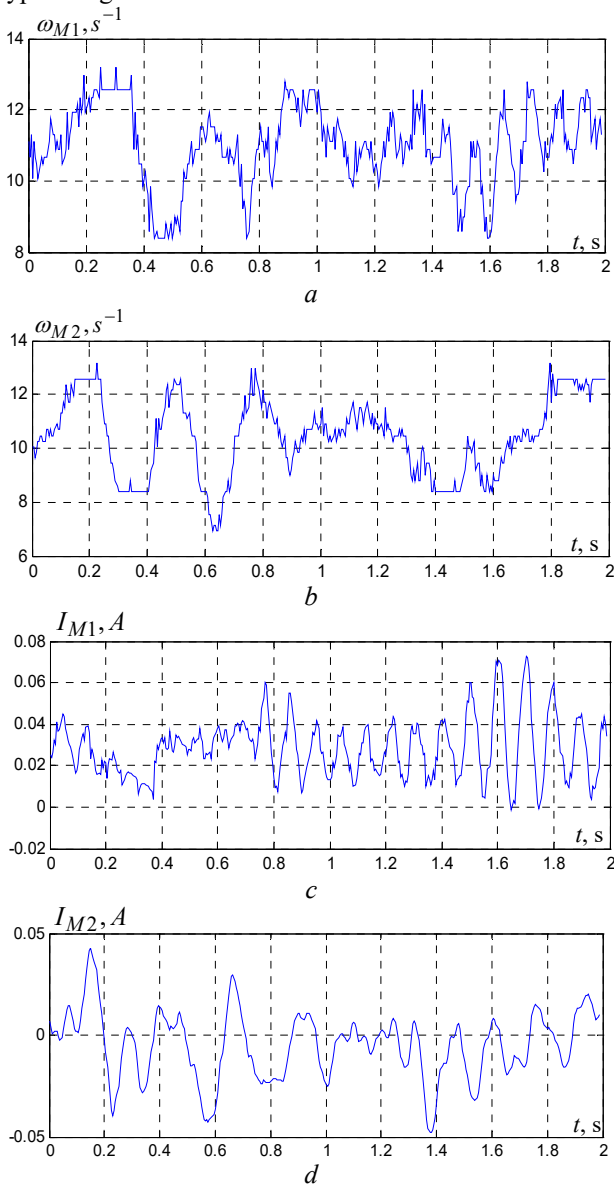

Fig. 3. The implementation of random processes of state variables of research stand of a stochastic two-mass electromechanical system

Discussion. The construction of various robust controllers under the action of external uncontrolled disturbances showed that these controllers, along with a number of undeniable advantages, also have disadvantages. The main feature of H_∞-regulators is the fact that in the process of functioning of a robust system only a priori information about possible external disturbances is used. This leads to the fact that robust control systems are characterized by some conservatism.

This is explained by the fact that robust systems must remain operational (maintain robust stability and a given level of quality) under the maximum permissible perturbations, without «having information» about when this perturbation will occur, i.e. the regulator is «always ready» for the worst case.

This is a common characteristic feature of all minimax controllers (which include H_∞-regulators).

When using anisotropic controllers built at an appropriate level of average anisotropy of the input signal, a significantly smaller control achieves almost the same quality of transient processes in terms of controlled variables compared to a H_∞ controller. The calculation of anisotropic robust controllers and anisotropic robust observers is similar to the calculation of deterministic robust controllers and deterministic robust observers in the four Ricatti approach. The solution of the Ricatti equation used as a necessary condition for the extremum of the corresponding quality criterion in the corresponding variables. When synthesizing an anisotropic robust controller and an anisotropic robust observer, a game approach is used.

The first game related to the synthesis of a robust anisotropic controller. The payoff of the first game is the anisotropic norm of the target vector. In this game, the first player is the anisotropic controller and its strategy is to minimize the anisotropic norm of the control goal vector. To calculate this controller, the fourth Ricatti equation (17) is solved. The second player in this game is the vector of external structural disturbances acting on the original system, and its strategy is to maximize the same anisotropic norm of the target vector over the vector of external disturbances in accordance with the wirst case. To calculate this vector of external perturbations, the first Ricatti equation (14) solved.

The second game related to the synthesis of a robust anisotropic observer. The payoff of the second game is the anisotropic norm of the error vector of restoring the state vector of the original system with the help of an observer. In this game, the anisotropic observer is the first player, and his strategy is to minimize the anisotropic norm of the error vector of restoring the state vector of the original system with the help of the observer from the gain vector of the synthesized aniso-triple observer. To calculate this observer, the third Ricatti equation (16) is solved.

The second player in this game is the vector of external noise measuring the output vector of the original system and acting on the observer, and its strategy is to maximize the same anisotropic norm of the vector of the error vector of restoring the state vector of the original system with the help of the observer from the vector of external parametric disturbances in accordance with the wirst case by chance. To calculate this vector of external paramrtric disturbances, the second Ricatti equation (15) is solved. Thus, the solution of the stochastic robust optimization problem is reduced to the calculation of four algebraic Riccati equations, the Lyapunov equation, and an expression of a special form for calculating the level of anisotropy of the input signal.

Thus, with zero average anisotropy of the input signal corresponding to a signal of the white noise type, the anisotropic controller is the optimal stochastic controller that minimizes the H^2 norm. With an infinite average anisotropy of the input signal corresponding to a fully defined deterministic signal, the anisotropic controller is the optimal deterministic robust controller that minimizes the H^∞ norm. With values of the average anisotropy of the input signal in the $0 < a < \infty$ range, the anisotropic controller occupies an intermediate position between the controllers that minimize H^2 and H^∞ norms.

Conclusions.

1. For the first time the method of multi objective synthesis of stochastic robust control by multimass electromechanical systems to satisfy various requirements for the operation of such systems in various modes under non-gausian random external disturbances is developed.

2. The problem of multi objective synthesis of stochastic robust control by multimass electromechanical systems to satisfy various requirements for the operation of such systems in various modes under non-gausian random external disturbances is formulated by solving a zero-sum vector antagonistic game.

3. The computation of the game payoff vector, the constraint vector, and the vector of binary preference relations is algorithmic in nature and requires large computational resources. To calculate the stochastic robust control it is need to solve the algebraic Riccati equations, the Lyapunov equation, and an equation of a special form for calculating the level of anisotropy of the input signal. Then as a result of modeling a closed system, the vector of the values of the quality indicators that are imposed on the system operation, and the vector of the restrictions, when the system is operating in various operating modes and for various setting and disturbing and for various values of the vector of the initial uncertainty of the system parameters and external influences are calculated.

4. The results of multi objective synthesis of stochastic robust control by servo two-mass electromechanical systems under non-gausian random external disturbances in which differences requirements for the operation of such systems in various modes were satisfied are given. The results of modeling and experimental studies of and realizations of state variables of of servo two-mass electromechanical system under random external influences are presented.

5. Based on the results of modeling and experimental studies of a servo two-mass electromechanical system with a random change in the load moment it is established, that with the help of synthesized robust stochastic controllers, it is possible to reduce the error of the rotation speed of the plant by more than 1.5 times and to reduse the error in regulating the angle of rotation of the shaft of the plany by more than 2 times in comparison with a system with typical regulators.

Conflict of interest. The authors declare that they have no conflicts of interest.

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