Impact of fractional filter in PI control loop applied to induction motor speed drive

Introduction. One of the main problems of electrical machine control systems is to obtain a satisfactory performance in the rejection of load disturbances, as well as in the set-point tracking tasks. Generally, the development of control algorithms does not take into account the presence of noise. Appropriate filtering is, therefore, essential to reduce the impact of noise on the output of the controller, in addition to the machine output. Recently, there has been a great tendency toward using fractional calculus to solve engineering problems. The filtering is one of the fields in which fractional calculus has received great attention. A review of the index performance evolution (the Integral Square Error, Integral Absolute Error and Integral Time Absolute Error) has allowed a configuration design of the filter. Results. Intensive simulations were performed with a control setup using integer and fractional order filters, which permitted to conclude that the fractional filters give better performance indices compared to the integer one and thus improve the dynamic characteristics of the system. References 27, tables 4, figures 12.

Keywords: fractional filter, first order filter, index performance, induction machine, PI controller.

Introduction. In recent decades several scientific research efforts have focused on the use of fractional order systems in identification, modeling, and control engineering. Applications cover a wide number of physical science fields, including mechanics, electricity, chemistry, biology, economics, modeling, and notably control theory, mechatronics and robotics [1, 2]. Fractional order control is nowadays one of the emerging research topics gathering a growing number of works [3, 4]. The main reason is that fractional order systems allow more powerful control performances and robustness compared to classical integer order ones [5, 6].

Actually, one of the main issues in machine control systems is often to achieve a satisfactory performance in the load disturbance rejection and in the set-point following tasks, simultaneously. The PID algorithm is the core function in low-level controllers. The majority of design strategies ignore measurement noises [7, 8]. Filtering the control loop signals is one possible solution to this problem. In [9], authors investigate how filtering the observed signal affects unwanted control actions caused by measurement noises, the load disturbance response and process uncertainty. The analyses are reduced to a set of design rules.

In another work, Hägglund has proposed a signal filtering in PID control loop [10]. Set-point, process output, and measurable load disturbances are the three basic analog input signals for the controller. Before entering the PID controller, these signals should be filtered. The process output filter is used to remove unwanted components from the signal such as measurement noise and to compensate for undesirable process dynamics.

With the successful implementation of non-integer order fractance devices, interest in using fractional order filters has grown. Seminal works on fractional order filters presented in [11, 12], were concerned with applying filter design theory to the fractional-order domain. Since then, several studies on the design of fractional order filters have been conducted, including [13, 14]. In [15] discussed the design and optimization of fractional filters, as well as their use in adaptive control of industrial processes [16].

Recently, the concept to «fractionalization» was proposed in [17]. It consists in replacing the classical integrator in a control loop by a combination of two fractional order integrators, which adds in fact fractional order filters in the feedback control loop, hence, improving the robustness against noises. Consequently, fractional order filters better approximate the ideal response than the classical ones; this fact makes their generalization for industrial control systems very advantageous.
This present work aims for the improvement of the induction machine speed control robustness applying fractional order filters in a simple PI feedback control scheme. Different structures of introducing the filters are investigated, whether before or after the controller or even in both positions at the same time (see Fig.1).

Figure 1. The basic feedback loop with process, controller and the filters

Fundamentals of fractional calculus. Definitions.
The concept of fractional calculus has been there since the inception of regular (integer-order) calculus, with the first reference most likely attributed to Leibniz and l'Hopital in 1695 [18], where the half-order derivative was discussed. The generalization of integration and differentiation is fractional calculus. The most commonly used definitions for fractional-order integral and derivative are Grunwald-Letnikov (G-L) and Riemann-Liouville (R-L) definition.

Definition of Riemann-Liouville (R-L). The fractional-order integral, in the sense of Riemann–Liouville, is defined as

\[
\int_0^t \frac{dt}{\Gamma(\alpha)} \left( t - \tau \right)^{\alpha-1} f(\tau) \, d\tau,
\]

and the fractional order derivative is expressed as

\[
D^\alpha_f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\alpha} f(\tau) \, d\tau,
\]

with the Gamma function given by

\[
\Gamma(x) = \int_0^\infty e^{-y} y^{x-1} \, dy,
\]

where \( \Gamma(x) \) is the Euler’s Gamma function; \( t_0 \) and \( t \) are the operation bounds; \( \alpha \) is the number identifying the fractional order; \( \tau \) is the time constant.

In this paper, \( \alpha \) is taken as a real number that satisfies the restriction \( 0 < \alpha < 1 \) [2, 19]. Besides, it is assumed that \( \alpha = 0 \) and the convention \( D^0_f = D^0_f \) is used.

Definition of Grunwald-Leitnikov (G-L). The fractional-order integral, in the sense of Grunwald-Leitnikov, is defined as [19, 20]:

\[
\int_0^t \frac{dt}{\Gamma(\alpha)} \left( t - \tau \right)^{\alpha-1} f(\tau) \, d\tau,
\]

and the fractional order derivative is expressed as

\[
D^\alpha_f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\infty} (-1)^j \frac{\Gamma(\alpha)}{\beta(\alpha - j + 1)} f(t - jh),
\]

where \( h \) is the sampling period.

For many engineering applications, the control laws are implemented in the frequency domain, using the Laplace transform method. The Laplace transform of the G-L and R-L fractional derivative/integral, under zero initial conditions for order \( \alpha \) is given by [21]:

\[
\mathcal{L}(D)^\alpha_f(t) = s^\alpha F(s).
\]

Approximation methods. One of the most difficult aspects of dealing with fractional order operators and systems is figuring out how to implement them using rational functions. Many researchers have concentrated on this problem and many approximation strategies, such as Oustaloup’s recursive algorithm and Charef’s singularity function approximation method, have been presented [18, 22].

The Oustaloup approximation algorithm is based on the approximate transfer function of a continuous filter for \( s^\alpha \) with rational functions as follows [23] by a rational function:

\[
s^\alpha = \prod_{k=0}^{N} \frac{s + \alpha_0}{s + \alpha_1},
\]

where the poles, zeros, and gain are computed from [18]:

\[
\alpha_0 = \omega_k \alpha_0^{(2 \lambda - 1 - \alpha) / N}; \quad \alpha_1 = \omega_k \alpha_1^{(2 \lambda - 1 - \alpha) / N};
\]

\[
C = \omega_k^{-\alpha},
\]

where \( \omega_k = \sqrt{\omega_0^2 + \omega_1^2} \) is the unity frequencies’ gain and the central frequency of a band of frequencies distributed geometrically; \( \omega_0 \) and \( \omega_1 \) are respectively the upper and lower frequencies, \( \alpha \) is the order of derivative, and \( N \) is the filter order.

Charef and al presented the singularity function approximation method [22], which is very similar to Oustaloup’s method basing on the function approximation of the type \( s^\alpha \) by a quotient of polynomials in \( s \) in a factorized form as follows [24]

\[
s^\alpha = K_d \prod_{k=0}^{N} \frac{(1 + s/z_k)}{(1 + s/p_k)},
\]

computed on the frequency interval \( \omega \in [\omega_0, \omega_1] \), such that

\[
K_d = \omega_k^\alpha,
\]

where \( \omega_k \) is the cutting frequency computed as

\[
\omega_k = \omega_0 \left( \frac{\epsilon^{1/10(1-\alpha)}}{10^{-1}} \right),
\]

and the coefficients are calculated to obtain a maximum deviation of \( \epsilon \) (dB) from the original magnitude response in the frequency domain defining

\[
\alpha = 10^{\epsilon/(10(1-\alpha))}, \quad b = 10^{\epsilon/10\alpha},
\]

The poles and zeros of the approximated rational function are obtained applying

\[
z_0 = \omega_0 \sqrt{\delta}, \quad z_1 = \omega_0 (a/b)^{1}, \quad p_i = a_1 z_0 (a/b)^{1}.
\]

The number of poles and zeros is related to the desired band-width and the error criteria formulated by the expression:
where $\varepsilon$ is the acceptable error chosen by the designer following the desired precision such that $\varepsilon \leq 3$ dB.

**Fractional order filters. Fractional-order low-pass filter.** The generalized transfer function of a fractional order low pass filter was proposed in [11] as:

$$F(s) = \frac{K}{1 + \tau s^\alpha} = \frac{d}{s^\alpha + a},$$

where $d = K/\tau$ and $a = 1/\tau$.

To determine the characteristics of this filter, a frequencies analysis is deemed necessary. For this purpose, the operator $s$ is set as $s = j\omega$ to obtain

$$\mathcal{F}(j\omega) = \frac{d}{\omega^\alpha \cos(\alpha \pi/2) + a + j\omega^\alpha \sin(\alpha \pi/2)}.$$  \hspace{1cm} (20)

The magnitude response and phase of the filter is expressed as

$$|\mathcal{F}(j\omega)| = \frac{d}{\sqrt{\omega^{2\alpha} + 2a\omega^\alpha \cos(\alpha \pi/2) + a^2}},$$

$$\text{Arg}(\mathcal{F}(j\omega)) = -\arctan \frac{\alpha \omega^\alpha \sin(\alpha \pi/2)}{\omega^\alpha \cos(\alpha \pi/2) + a}.$$  \hspace{1cm} (22)

Figure 2 shows the Bode representation of different approximations of the fractional filter and the original one with

$$F(s) = \frac{K}{1 + \tau s^\alpha} \quad (K = 1, \tau = 0.006, \alpha = 0.6).$$

From the frequency representation (Fig. 2), it can be seen that singularity function approximation method provided a better fit, hence this latter is the one to be opted for in this paper.

**Induction machine model and control strategy.**

The dynamic model of the induction machine described in the arbitrary Park referential considering state variables stator current and the rotor flux $\{i_{ds}, i_{qs}, \Phi_{dr}, \Phi_{qr}\}$ is given as follows [25]:

- **voltage equations:**
  \begin{align*}
  V_{ds} &= R_s i_{ds} + \sigma L_s \frac{di_{ds}}{dt} + e_{ds}, \\
  V_{qs} &= R_s i_{qs} + \sigma L_s \frac{di_{qs}}{dt} + e_{qs}, \\
  V_{dr} &= 0 = R_r i_{dr} + \frac{d\Phi_{dr}}{dt} + e_{dr}, \\
  V_{qr} &= 0 = R_r i_{qr} + \frac{d\Phi_{qr}}{dt} + e_{qr}. 
  \end{align*}

- **mechanical equations:**
  \begin{align*}
  J \frac{d\Omega}{dt} + f\Omega &= T_{cm} - C_r, \hspace{1cm} (24)
  T_{cm} &= \frac{M}{L_r}(\Phi_{dr} i_{qs} - \Phi_{qr} i_{ds}), \hspace{1cm} (25)
  \end{align*}

- **auto drive equation:**
  \begin{align*}
  \omega_s &= \omega_r + p\Omega. \hspace{1cm} (26)
  \end{align*}

The $e_d$ and $e_q$ terms are the consequence of electromechanical and electromagnetic coupling between the windings, which is analogous to the electromotive forces produced by a direct current machine, expressed as follows:

\begin{align*}
  e_{ds} &= -\frac{M R_s}{L_r^2} \Phi_{dr} + \sigma L_s \omega_s i_{qs} - \frac{M p \Omega}{L_r} \Phi_{qr}, \\
  e_{qs} &= \frac{M}{L_r} p \Omega \Phi_{dr} + \sigma L_s \omega_s i_{ds} - \frac{M R_s^2}{L_r^2} \Phi_{qr}, \\
  e_{dr} &= -(\omega_s - p \Omega) \Phi_{qr} + \frac{M R_r}{L_r} i_{ds}, \\
  e_{qr} &= (\omega_s - p \Omega) \Phi_{dr} - \frac{M R_r}{L_r} i_{qs}. \hspace{1cm} (27)
  \end{align*}

Thus the induction machine model is illustrated by the causal informational graph (Fig. 3) [26]. All symbols from figures are described in Appendix.

**Strategy of control.** The flux and the current creating the torque must be decoupled in order to control the induction machine. To do this we use a control known as vector control or field oriented drive, which directs the flux along the Park referential’s axis.

The rated flux is aligned along the $d$-axis for rotor field orientation, thus $\Phi_{dr} = \Phi_{qr} = 0$. Using the voltage equations of the rotor, we will come to

![Fig. 3. Causal informational graph representation of the induction motor](image-url)
The $q$-axis current is used to adjust the torque as follows:

$$T_{cm} = p \frac{M}{L_r} (\Phi_{dr} i_{qs}). \quad (29)$$

Two magnitudes, the flux and its position, are to be controlled using (28), (29). The current $i_{ds}$ govern the flux, so $V_{ds}$, while the torque is controlled by the current $i_{qs}$, so $V_{qs}$. For an indirect control, we impose: $\Phi_{dr} = \Phi_{ref} = \text{const.}$

The scheme control is depicted in Fig. 4 with speed and currents controllers [26] using the principles of inversion of the causal informational graph. Thus the induction machine model with oriented control field proposed and the filters are illustrated by Simulink scheme block in Fig. 5. The parameters of the induction machine are shown in Table 1.

![Fig. 4. Control scheme of the induction machine](image)

**Fig. 5. Simulink model of the induction machine with field control and filters**

### Design of controllers

One important feature of the proposed method is the use of standard and simple PI controllers, which are very easy to tune.

For the speed control, we have considered the MATLAB tuner for PI controller; whereas, for the current control we have considered the pole assignment approach for the PI parameters’ adjustment. The resulting controllers gains are given in Table 2.

<table>
<thead>
<tr>
<th>Parameters of the controllers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>K_p</strong></td>
</tr>
<tr>
<td>Current controller</td>
</tr>
<tr>
<td>Speed controller</td>
</tr>
</tbody>
</table>

### Design of filters

There are several performance indexes that can be used for this purpose. The most frequently used are the integral absolute error (IAE) index, the integral square error (ISE) index or the integral time absolute error (ITAE) index [27] respectively defined as follow.

The integral absolute error (IAE):

$$J_{IAE} = \int |e(t)| \, dt = \int |r(t) - y(t)| \, dt. \quad (30)$$

The integral square error (ISE):

$$J_{ISE} = \int e^2(t) \, dt = \int (r(t) - y(t))^2 \, dt. \quad (31)$$

The integral time absolute error (ITAE):

$$J_{ITAE} = \int t |e(t)| \, dt = \int t |r(t) - y(t)| \, dt. \quad (32)$$
where \( r(t) \) and \( y(t) \) are respectively the desired value and the output value of the closed control loop.

For the PI control, the filter time constant is a fraction of the system time constant. Thus, the considered integer filter is

\[
F(s) = \frac{K}{1 + \tau s},
\]

with \( K = 1.01 \) for \( \tau = 6 \) ms.

To configure the filters to be introduced in the control loop, the model is simulated with different positions of integer filters, then the three performance indexes are evaluated. It is, therefore, noticed that for a standard PI controller, the minimum value of \( J_{IAE} \) performance index is obtained when the additional filter is placed after the controller, as shown in Fig. 1.

**Simulation results** are carried out using the MATLAB/Simulink environment. To evaluate the added filters, the test of the process in a noisy environment is proposed with PI controllers tuned as in Table 2. For this, the process is simulated in the feedback control closed loop injecting random noise with mean value magnitude of 5 % added to the system output. Considering a fractional 1st order-like low pass filter of transfer function:

\[
F(s) = \frac{K}{1 + \tau s^\alpha},
\]

with \( K = 1.01 \) and \( \tau = 6 \) ms and the order \( \alpha \) varies from 0.05 to 1 with a step of 0.05 as shown in Fig. 6.

From the simulation results represented in Fig. 6, minimum values of \( J_{IAE} \), \( J_{SE} \) and \( J_{ITAE} \) performance indexes are obtained for the order \( \alpha \) values 0.85, 0.8 and 1 respectively.

Simulation results for the machine speed control without additional filters are represented in Fig. 7, 8 for the safe and noisy cases, respectively, while Fig. 9 illustrates the system response with an integer order filter.

Table 3 shows that the optimal index performance values for \( J_{IAE} \) or \( J_{SE} \) are obtained with a fractional filter order \( \alpha = 0.85 \) (filter 1), \( \alpha = 0.8 \) (filter 2), respectively, and the third criterion \( J_{ITAE} \) is minimized in case of integer value \( \alpha = 1 \).

According to these values, the response of the machine is depicted in Fig. 10, 11. The dynamic characteristics are shown in Table 4.

A comparative output response for these different cases is given in Fig. 12. It indicates an improvement of the dynamic characteristics (overshoot and oscillations) when using the second fractional order filter minimizing the \( J_{SE} \) criterion.
The main contribution of this paper is the application of fractional filtering to standard PI control loop for an induction motor speed drive. To evaluate its impact and benefit, intensive simulations have been realized with a control configuration using an integer and fractional order filters.

As a result of the conducted simulation, it is concluded that the fractional filters give better index performances ($J_{IAE} = 4.7587$ and $J_{ISE} = 245.1812$ for the fractional filter 1 and $J_{IAE} = 4.8903$ and $J_{ISE} = 244.6224$ for the fractional filter 2), as compared to integer one ($J_{IAE} = 5.0743$ and $J_{ISE} = 292.8749$), which improves the system robustness against noises and disturbance.

As regards the dynamic characteristics and though a slight degradation of the rise and the settling times, a significant amelioration of the overshoot is obtained, namely a lesser overshoot of 10.04 %, compared to a result of 23.22 % and 11.83 % with integer filter and without filtering, respectively.

This study may provide an opportunity for further research on considering other sources of disturbance.

**Conflict of interest.** The authors declare that they have no conflicts of interest.

### Appendix

<table>
<thead>
<tr>
<th>$R_s$</th>
<th>Stator resistance</th>
<th>$X_{dr}$</th>
<th>Rotor variable on the d axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_r$</td>
<td>Rotor resistance</td>
<td>$X_{dq}$</td>
<td>Stator variable on the q axis</td>
</tr>
<tr>
<td>$L_s$</td>
<td>Stator inductance</td>
<td>$X_{mes}$</td>
<td>Measured variable</td>
</tr>
<tr>
<td>$L_r$</td>
<td>Rotor inductance</td>
<td>$X_{est}$</td>
<td>Estimated variable</td>
</tr>
<tr>
<td>$M$</td>
<td>Mutual inductance</td>
<td>$X_{ref}$</td>
<td>Reference value</td>
</tr>
<tr>
<td>$p$</td>
<td>Number of pole pairs</td>
<td>$\Phi$</td>
<td>Flux</td>
</tr>
<tr>
<td>$f$</td>
<td>Friction coefficient</td>
<td>$\Phi_{ref}$</td>
<td>Flux reference</td>
</tr>
<tr>
<td>$J$</td>
<td>Moment of inertia</td>
<td>$T_m$</td>
<td>Electromagnetic torque</td>
</tr>
</tbody>
</table>

\[
s = 1 - \frac{M^2}{L_s L_r} \quad \text{Blondel’s dispersion coefficient} \quad C_L \quad \text{Load torque}
\]

<table>
<thead>
<tr>
<th>$V$</th>
<th>Voltage</th>
<th>$a_0$</th>
<th>Stator pulse</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Current</td>
<td>$a_0$</td>
<td>Rotor pulse</td>
</tr>
<tr>
<td>$(d,q)$</td>
<td>Park axis</td>
<td>$\Omega$</td>
<td>Speed</td>
</tr>
<tr>
<td>$X_{dr}$</td>
<td>Stator variable on the d axis</td>
<td>$\Omega_{ref}$</td>
<td>Speed reference</td>
</tr>
<tr>
<td>$X_{dq}$</td>
<td>Stator variable on the q axis</td>
<td>$\tau$</td>
<td>Time constant</td>
</tr>
</tbody>
</table>

### References


Hassaina, Saida Hassainia, Senior Lecturer, Samir Ladaci1, Professor, Sihem Kechida2, Professor, Khaled Khelil1, Professor, 1 Laboratory of Electrotechnics and Renewable Energies, Mohamed Cherif Messaadia University, Souk Ahras, Algeria, e-mail: saida.hassainia@univ-soukahras.dz (Corresponding Author), khaled.khelil@univ-soukahras.dz 2 Laboratory of Signal Processing, National Polytechnic School of Algiers, Algeria, e-mail: samir.ladaci@gmail.com 3 Laboratory of Automatics and Informatics, 8 Mai 1945 University, Guelma, Algeria, e-mail: kechida.sihem@univ-guelma.dz

How to cite this article:

Received 08.03.2022
Accepted 20.06.2022
Published 07.09.2022