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Efficient method for transformer models implementation in distribution load flow matrix

Introduction. *Most distribution networks are unbalanced and therefore require a specific solution for load flow. There are many works on the subject in the literature, but they mainly focus on simple network configurations. Among the methods dedicated to this problem, one can refer to the load flow method based on the bus injection to branch current and branch current to bus voltage matrices.* ***Problem.*** *Although this method is regarded as simple and complete, its drawback is the difficulty in supporting the transformer model as well as its winding connection types. Nevertheless, the method requires the system per unit to derive the load flow solution.* ***Goal.*** *In the present paper, our concern is the implementation of distribution transformers in the modeling and calculation of load flow in unbalanced networks.* ***Methodology.*** *Unlike previous method, distribution transformer model is introduced in the topology matrices without simplifying assumptions. Particularly, topology matrices were modified to take into account all winding types of both primary and secondary sides of transformer that conserve the equivalent scheme of an ideal transformer in series with an impedance. In addition, the adopted transformer models overcome the singularity problem that can be encountered when switching from the primary to the secondary side of transformer and inversely.* ***Practical value.*** *The proposed approach was applied to various distribution networks such as IEEE 4-nodes, IEEE 13-nodes and IEEE 37-nodes. The obtained results validate the method and show its effectiveness.* References 24, tables 4, figures 9.

Key words: **distribution systems, unbalanced load flow, distribution transformer models, topology network matrix.**

Вступ*. Більшість розподільчих мереж незбалансовані і тому потребують спеціального рішення для потоку навантаження. У літературі є багато робіт на цю тему, але переважно вони присвячені простим мережевим конфігураціям. Серед методів, присвячених цій проблемі, можна назвати метод потоку навантаження, заснований на введенні шини в матрицю струму відгалуження і відгалуження струму в матрицю напруги шини.* ***Проблема****. Хоча цей метод вважається простим та повним, його недоліком є складність підтримки моделі трансформатора, а також типів з'єднання його обмоток. Проте метод вимагає системи на одиницю для отримання рішення про потік навантаження.* ***Мета****. У цій статті нас цікавить застосування розподільних трансформаторів для моделювання та розрахунку потоку навантаження у незбалансованих мережах.* ***Методологія.*** *На відміну від попереднього методу, модель розподільного трансформатора вводиться в матриці топології без спрощення припущень. Зокрема, матриці топології були змінені, щоб врахувати всі типи обмоток як первинної, так і вторинної сторін трансформатора, які зберігають еквівалентну схему послідовно ідеально включеного трансформатора з імпедансом. Крім того, прийняті моделі трансформаторів долають проблему сингулярності, з якою можна зіткнутися при перемиканні з первинної на вторинну обмотку трансформатора і навпаки.* ***Практична цінність****. Пропонований підхід був застосований до різних розподільних мереж, таких як IEEE з 4 вузлами, IEEE з 13 вузлами та IEEE з 37 вузлами. Отримані результати підтверджують метод та показують його ефективність.* Бібл. 24, табл. 4, рис. 9.

Ключові слова: **розподільні системи, незбалансований потік навантаження, моделі розподільних трансформаторів, матриця топології мережеві.**

Introduction. Electrical distribution systems are generally unbalanced and therefore require special attention when solving the load flow problem for planning, operation and design studies [1, 2]. The power flow solution method must be robust and efficient to account for the characteristics of distribution systems, i.e., radial or weakly meshed configuration, unbalanced multi-phases, large number of branches and nodes, high R/X ratio. Such load flow method must be able to handle different distribution components with sufficient details, especially the distribution transformer (DT) models whatever its winding connections. Load flow algorithms in distribution networks can be classified into two types: The first class of methods is based on Newton-Raphson algorithms [3, 4]. This well-known approach uses three-phase current injection method in rectangular coordinates [5, 6]. In [7] the author presents a modified version of current injection method. Other linear forms are presented in [8, 9], However, their application is far from being adapted in unbalanced networks and the incorporation of distribution transformer models in nodal admittance matrices has revealed their difficult application and inefficiency to converge due to the singularity problem [10]. Methods of the second type use the forward and backward sweeping (FBS) algorithms [11, 12]. They are based on Kirchhoff's laws. In this class of methods, branch numbering scheme is required for computing currents and node voltages that makes DT modelling, with various winding connection is difficult.

Beside the above mentioned load flow methods, other methods may also be used, such those based on special topological characteristics of distribution networks [13]. The work [14] introduced a new contribution to power flow solution, using the node incidence matrix and a complex vector based model in αβ0 stationary reference frame. The formulation of the admittance matrix in the αβ0 reference and the estimation of the initial network voltage profile complicate the calculation, especially for large networks. The most cited algorithms were referred to in [15-18]. They are based on three matrices namely, bus injection to branch current matrix (BIBC), branch current to bus voltage (BCBV) matrix and distribution load flow matrix (DLF). However, in the latter, DT models and other distribution components cannot be directly incorporated.

The goal. In this paper one proposes a method for unbalanced three-phase power flow solution which can handle DT regardless the type of its windings connection. DT models given by [19], which overcome the singularity problem, have been used. In the proposed load flow method, the BIBC and BCBV network topology matrices have been modified to incorporate DT whose equivalent scheme is branch impedance in series with ideal transformer taking into account the connection type of primary and secondary windings.

The model of components. Distribution line. Typical branch model of distribution lines is shown in Fig. 1, where the line to ground charging capacitance is ignored. The self and mutual elements of the 3×3 phase-impedance matrix are determined by Carson's equations. For neutral distribution line, Kron reduction is used [19].

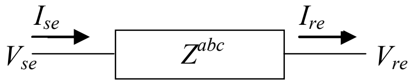

Fig. 1. Typical distribution line branch

From the line model given by Fig. 1, we can write:

$$\boldsymbol{V}_{re} = \boldsymbol{V}_{se} - \Delta \boldsymbol{V}^{sr}; \tag{1}$$

$$\Delta \boldsymbol{V}^{sr} = \boldsymbol{Z}^{abc} \boldsymbol{I}_{se}; \tag{2}$$

$$\boldsymbol{I}_{re} = \boldsymbol{I}_{se}, \tag{3}$$

where $\boldsymbol{I}_{se}$ = $[I_{se}^a \; I_{se}^b \; I_{se}^c]^T$ and $\boldsymbol{I}_{re}$ = $[I_{re}^a \; I_{re}^b \; I_{re}^c]^T$ are respectively the line current vectors at sending and receiving ends; $\boldsymbol{V}_{se}$ = $[V_{se}^a \; V_{se}^b \; V_{se}^c]^T$ and $\boldsymbol{V}_{re}$ = $[V_{re}^a \; V_{re}^b \; V_{re}^c]^T$ are respectively the line to ground voltage vectors at sending and receiving ends; $\Delta \boldsymbol{V}^{sr}$ = $[\Delta V^{aa} \; \Delta V^{bb} \; \Delta V^{cc}]^T$ is the line voltages drop vector.

If there are no full phase components in the distribution system, the elements corresponding to the missed phases in the impedance matrix are set to zero.

Load model. In unbalanced three-phase distribution systems, the loads are specified by the power complex form. All the loads are assumed to be Wye or Delta connected and can be modeled as, constant power, constant current, constant impedance or any combination of the above cited models. Then, for a specified power $S^{\varphi(sp)}$ and voltage V^{φ}, the equivalent load current injected into phase φ can be calculated by (4):

$$I_L^{\varphi} = \left(\frac{S^{\varphi(sp)}}{V^{\varphi}} \right)^*, \tag{4}$$

where φ = $\{\varphi_1 \; \varphi_2 \; \varphi_3\}$ refers to phases $\{a \; b \; c\}$ for Wye load or $\{ab \; bc \; ca\}$ for Delta load. The load currents injected into the i^{th} bus are given by (5):

$$\boldsymbol{I}_L^{\varphi_1 \varphi_2 \varphi_3} = \boldsymbol{D} \cdot \begin{bmatrix} I_L^{\varphi_1} \\ I_L^{\varphi_2} \\ I_L^{\varphi_3} \end{bmatrix}. \tag{5}$$

Depending on the load connection, Wye or Delta, the matrix $\boldsymbol{D}$ is given by (6):

$$\boldsymbol{D} = \begin{cases} \boldsymbol{I} \text{ (identite matrix) for Wye load;} \\ \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad \text{for Delta load.} \end{cases} \tag{6}$$

For single phase and two phase-loads, the currents in the missed phases are set to zero.

Distribution transformer. Three-phase transformer is modeled by connecting three single-phase transformers, in which the transformer magnetizing currents are neglected. To convert the line-to-neutral voltages to phases voltages and the line currents to phases currents, the Wye or Delta windings connection shown in Fig. 2 [20, 21].

The branch equivalent model of a distribution transformer is as shown in Fig. 3.

On Fig. 3: $\boldsymbol{I}_s$ = $[I_s^a \; I_s^b \; I_s^c]^T$ and $\boldsymbol{I}_p$ = $[I_p^a \; I_p^b \; I_p^c]^T$ are respectively the secondary and primary line currents; $\boldsymbol{V}_s = [V_s^a \; V_s^b \; V_s^c]^T$ and $\boldsymbol{V}_p = [V_p^a \; V_p^b \; V_p^c]^T$ are respectively

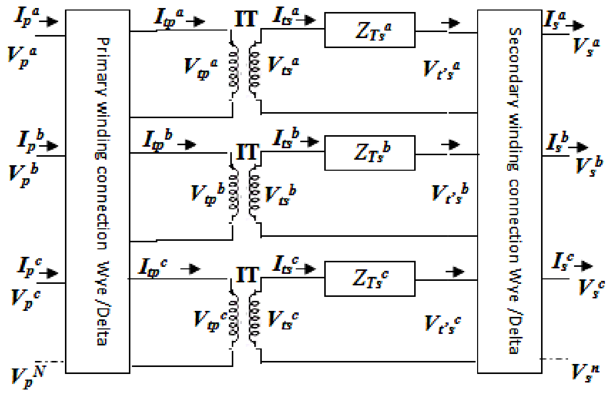

Fig. 2. Three-phase transformer scheme

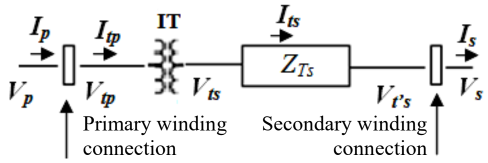

Fig. 3. Transformer simplified branch model

secondary and primary line-to-neutral voltages; $\boldsymbol{I}_{ts}$ = $[I_{ts}^a \; I_{ts}^b \; I_{ts}^c]^T$ and $\boldsymbol{V}_{ts}$ = $[V_{ts}^a \; V_{ts}^b \; V_{ts}^c]^T$ are the transformer secondary phase currents and voltages; $\boldsymbol{V}_{t's} = [V_{t's}^a \; V_{t's}^b \; V_{t's}^c]^T$ are the transformer secondary phase voltages without voltages drop; $\boldsymbol{I}_{tp}$ = $[I_{tp}^a \; I_{tp}^b \; I_{tp}^c]^T$ and $\boldsymbol{V}_{tp} = [V_{tp}^a \; V_{tp}^b \; V_{tp}^c]^T$ are respectively the transformer primary phase currents and voltages; IT is the ideal transformer; $\boldsymbol{Z}_{Ts}$ is the secondary transformer impedance matrix given by:

$$\boldsymbol{Z}_{Ts} = \begin{bmatrix} Z_{Ts}^a & 0 & 0 \\ 0 & Z_{Ts}^b & 0 \\ 0 & 0 & Z_{Ts}^c \end{bmatrix}. \tag{7}$$

Using the DT equivalent branch shown in Fig. 3 and rearranging the accurate transformer equations given by [19], one can write the following equations.

Current equations. For the secondary and primary line currents, one can write the following relationships:

$$\boldsymbol{I}_p = \boldsymbol{K}_I \boldsymbol{I}_{ts}; \tag{8}$$

$$\boldsymbol{I}_{ts} = \boldsymbol{K}_L \boldsymbol{I}_s. \tag{9}$$

Substituting $\boldsymbol{I}_{ts}$ by its expression (9) into (8) leads to:

$$\boldsymbol{I}_p = \boldsymbol{K}_I \boldsymbol{K}_L \boldsymbol{I}_s, \tag{10}$$

where $\boldsymbol{K}_I$ is the current transformation matrix. This matrix also takes into account the transition from $\boldsymbol{I}_{tp}$ to $\boldsymbol{I}_p$. It is as given by Table 1. $\boldsymbol{K}_L$ is the matrix transforming the secondary line currents into phase delta currents. It is equal to:

$$\boldsymbol{K}_L = \begin{cases} \boldsymbol{I} \text{ (identite matrix) for Wye connection;} \\ \frac{1}{3} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 2 & 0 \\ -2 & -1 & 0 \end{bmatrix} \quad \text{for Delta connection.} \end{cases} \tag{11}$$

Voltage equations. The relations between secondary and primary line-to-neutral voltages can be obtained as:

$$\boldsymbol{V}_{ts} = \boldsymbol{K}_v \boldsymbol{V}_p; \tag{12}$$

$$\boldsymbol{V}_{t's} = \boldsymbol{V}_{ts} - \boldsymbol{Z}_{Ts} \boldsymbol{I}_{ts}; \tag{13}$$

$$\boldsymbol{V}_s = \boldsymbol{K}_w \boldsymbol{V}_{t's}. \tag{14}$$

Table 1. $\boldsymbol{K_I}$, $\boldsymbol{K_L}$, $\boldsymbol{K_v}$, $\boldsymbol{K_w}$ for some common industrial distribution transformers

Connection	Coefficients				
	$\boldsymbol{K_I}$	$\boldsymbol{K_L}$	$\boldsymbol{K_v}$	$\boldsymbol{K_w}$	n_T
YG-yg	$\frac{1}{n_T}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\frac{1}{n_T}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\frac{V_{LN}^{High\ Side}}{V_{LN}^{Low\ Side}}$
D-yg	$\frac{1}{n_T}\begin{bmatrix}1&-1&0\\0&1&-1\\-1&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\frac{1}{n_T}\begin{bmatrix}1&0&-1\\-1&1&0\\0&-1&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\frac{V_{LL}^{High\ Side}}{V_{LN}^{Low\ Side}}$
Y-d	$\frac{1}{n_T}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1&-1&0\\1&2&0\\-2&-1&0\end{bmatrix}$	$\frac{1}{n_T}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}2&1&0\\0&2&1\\1&0&2\end{bmatrix}$	$\frac{V_{LN}^{High\ Side}}{V_{LL}^{Low\ Side}}$
gY-d	$\frac{1}{n_T}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1&-1&0\\1&2&0\\-2&-1&0\end{bmatrix}$	$\frac{1}{n_T}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}2&1&0\\0&2&1\\1&0&2\end{bmatrix}$	$\frac{V_{LN}^{High\ Side}}{V_{LL}^{Low\ Side}}$
D-d	$\frac{1}{n_T}\begin{bmatrix}1&0&-1\\-1&1&0\\0&-1&1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}1&-1&0\\1&2&0\\-2&-1&0\end{bmatrix}$	$\frac{1}{n_T}\begin{bmatrix}1&-1&0\\0&1&-1\\-1&0&1\end{bmatrix}$	$\frac{1}{2}\begin{bmatrix}2&1&0\\0&2&1\\1&0&2\end{bmatrix}$	$\frac{V_{LL}^{High\ Side}}{V_{LL}^{Low\ Side}}$

Here V_{LN} is the rated-to-neutral voltage; V_{LL} is the rated line-to-line voltage.

Combining (12), (13) and (14), one can write:

$$\boldsymbol{V}_s = \boldsymbol{K}_w\boldsymbol{K}_v\boldsymbol{V}_p - \boldsymbol{K}_w\boldsymbol{Z}_{Ts}\boldsymbol{K}_L\boldsymbol{I}_s, \qquad (15)$$

where $\boldsymbol{K}_v$ is the voltage transformation matrix given in Table 1. It takes into account the primary winding connection type i.e. transition from $\boldsymbol{V}_{tp}$ to $\boldsymbol{V}_p$. $\boldsymbol{K}_w$ is the matrix which transforms the phase delta voltages to secondary line-to-neutral voltages. Like $\boldsymbol{K}_L$, $\boldsymbol{K}_w$ matrix is given by:

$$\boldsymbol{K}_w = \begin{cases} \boldsymbol{I} \text{ (identite matrix) for Wye connection;} \\ \frac{1}{3}\begin{bmatrix}2&1&0\\0&2&1\\1&0&2\end{bmatrix} \quad \text{for Delta connection.} \end{cases} \qquad (16)$$

The relations (10) and (15) remain applicable regardeless the transformer configuration even for those having voltage or current zero-sequence component interrupted like Wye-Delta and ground Wye-Delta.

Proposed method. Two basic topology matrices are required for solving three-phase power flow problem namely, bus injection currents to branch currents matrix $\boldsymbol{B_I}$ and the matrix $\boldsymbol{B_v}$ that links branch voltage drops to bus voltages mismatch. The currents and voltages relations are as given below:

$$\boldsymbol{I} = \boldsymbol{B}_I\boldsymbol{I}_{bus}; \qquad (17)$$

$$\Delta\boldsymbol{V}_{bus} = \boldsymbol{B}_v\Delta\boldsymbol{V}. \qquad (18)$$

To update bus voltages, the following equation, where $\boldsymbol{V}_{bus}^{nl}$ is the no-load bus voltage vector, is used:

$$\boldsymbol{V}_{bus} = \boldsymbol{V}_{bus}^{nl} - \Delta\boldsymbol{V}_{bus}. \qquad (19)$$

Combining (17) and (18) with Ohm's law given by (20), bus voltages mismatch $\Delta\boldsymbol{V}_{bus}$ given by (21) is derived.

$$\Delta\boldsymbol{V} = \boldsymbol{ZI}; \qquad (20)$$

$$\begin{cases} \Delta\boldsymbol{V}_{bus} = \boldsymbol{mDLF}\cdot\boldsymbol{I}_{bus}; \\ \boldsymbol{mDLF} = \boldsymbol{B}_v\boldsymbol{ZB}_I. \end{cases} \qquad (21)$$

Equations (21) are similar to those given in literature by the following equations [10, 18]:

$$\begin{cases} \Delta\boldsymbol{V}_{bus} = \boldsymbol{DLF}\cdot\boldsymbol{I}_{bus}; \\ \boldsymbol{DLF} = \boldsymbol{BCBV}\cdot\boldsymbol{BIBC}. \end{cases} \qquad (22)$$

In the method given by [16], BCBV and BIBC matrices are built based on DT equivalent scheme given in Fig. 3. To simplify the modeling of the network and after decoupling the phases of the DT, one substituted the mutual impedances by injecting currents in the nodes. This leaves, in the model, only the DTs whose equivalent scheme is an ideal transformer in series with impedance. The trick used to rule out the ideal transformer is the per-unit system, which makes the ratio of the transformer equal to 1. Then, only the series impedance remains in the DT model. Unlike the method described above and governed by (22), in the proposed method whose mathematical model is given by (21), DTs are also substituted by their equivalent scheme of Fig. 3 without any simplifications. Matrices named $\boldsymbol{B_I}$ and $\boldsymbol{B_v}$, the details of which are developed in the following sections, are then constructed. Thus, in (21), the mDLF matrix is the modified of version of the DLF matrix that appears in (22).

$\boldsymbol{B_I}$ and $\boldsymbol{B_v}$ matrices building. The construction of $\boldsymbol{B_I}$ and $\boldsymbol{B_v}$ matrices is illustrated using the one-line diagram of the radial distribution system shown in Fig. 4 and where $\boldsymbol{V_1}$ is the substation voltage.

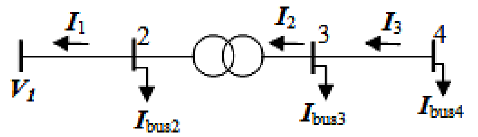

Fig. 4. Radial distribution network

$\boldsymbol{B_I}$ matrix building. The $\boldsymbol{B_I}$ matrix building begin by calculating the bus injected currents $\boldsymbol{I}_{bus2}^{abc}$, $\boldsymbol{I}_{bus3}^{abc}$ and $\boldsymbol{I}_{bus4}^{abc}$ using (4) and (5) and integrating them in bus current vector $\boldsymbol{I}_{bus}$ as it is shown below:

$$\boldsymbol{I}_{bus} = \left[\boldsymbol{I}_{bus2}^{abc} \quad \boldsymbol{I}_{bus3}^{abc} \quad \boldsymbol{I}_{bus4}^{abc}\right]. \qquad (23)$$

Then, the relationships between bus currents and branch currents are determined using (3) and (10). They are as given below:

$$\begin{cases} \boldsymbol{I}_3^{abc} = \boldsymbol{I}_{bus4}^{abc}; \\ \boldsymbol{I}_2^{abc} = \boldsymbol{I}_{bus3}^{abc} + \boldsymbol{I}_{bus4}^{abc}; \\ \boldsymbol{I}_1^{abc} = \boldsymbol{I}_{bus2}^{abc} + \boldsymbol{K}_I \boldsymbol{K}_L \boldsymbol{I}_2^{abc}. \end{cases} \quad (24)$$

One can also write $\boldsymbol{I}_1^{abc}$ in the following form:

$$\boldsymbol{I}_1^{abc} = \boldsymbol{I}_{bus2}^{abc} + \boldsymbol{K}_I \boldsymbol{K}_L \boldsymbol{I}_{bus3}^{abc} + \boldsymbol{K}_I \boldsymbol{K}_L \boldsymbol{I}_{bus4}^{abc}. \quad (25)$$

Equations (24) can also be rewritten in the following matrix form:

$$\begin{bmatrix} \boldsymbol{I}_1^{abc} \\ \boldsymbol{I}_2^{abc} \\ \boldsymbol{I}_3^{abc} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{K}_I \boldsymbol{K}_L & \boldsymbol{K}_I \boldsymbol{K}_L \\ 0 & \boldsymbol{I} & \boldsymbol{I} \\ 0 & 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_{bus2}^{abc} \\ \boldsymbol{I}_{bus3}^{abc} \\ \boldsymbol{I}_{bus4}^{abc} \end{bmatrix}. \quad (26)$$

This matrix form brings us to the relationship given by (17) and thereby one can deduce $\boldsymbol{B}_I$ by identification:

$$\boldsymbol{B}_I = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{K}_I \boldsymbol{K}_L & \boldsymbol{K}_I \boldsymbol{K}_L \\ 0 & \boldsymbol{I} & \boldsymbol{I} \\ 0 & 0 & \boldsymbol{I} \end{bmatrix}. \quad (27)$$

It is worth noting that 0 and $\boldsymbol{I}$ in (27) are 3×3 matrices.

$\boldsymbol{B}_v$ matrix building. To build $\boldsymbol{B}_v$ matrix, one calculates first branch voltages drops. For the considered example (Fig. 4), the branch voltage drops are given as:

$$\begin{cases} \Delta \boldsymbol{V}_1^{abc} = \boldsymbol{Z}_1^{abc} \boldsymbol{I}_1^{abc}; \\ \Delta \boldsymbol{V}_2^{abc} = \boldsymbol{Z}_{Ts}^{abc} \boldsymbol{I}_2^{abc}; \\ \Delta \boldsymbol{V}_3^{abc} = \boldsymbol{Z}_3^{abc} \boldsymbol{I}_3^{abc}, \end{cases} \quad (28)$$

where $\boldsymbol{Z}_1^{abc}$, $\boldsymbol{Z}_3^{abc}$ are the 1st and 3rd branch impedance matrices; $\boldsymbol{Z}_{Ts}^{abc}$ is the 2nd branch impedance matrix transformer included.

In a matrix form, the (28) becomes:

$$\begin{bmatrix} \Delta \boldsymbol{V}_1^{abc} \\ \Delta \boldsymbol{V}_2^{abc} \\ \Delta \boldsymbol{V}_3^{abc} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}_1^{abc} & 0 & 0 \\ 0 & \boldsymbol{Z}_{Ts}^{abc} & 0 \\ 0 & 0 & \boldsymbol{Z}_3^{abc} \end{bmatrix} \begin{bmatrix} \boldsymbol{I}_1^{abc} \\ \boldsymbol{I}_2^{abc} \\ \boldsymbol{I}_3^{abc} \end{bmatrix}. \quad (29)$$

The identification of the matrix form (29) to that given by (20) allows us deducing the $\boldsymbol{Z}$ matrix. As shown in (29), it should be pointed out that the full matrix $\boldsymbol{Z}$ is built by gathering, on its diagonal, all branch impedance matrices.

After which, the branch to bus voltages are calculated according to (1) and (15). One can write in this case:

$$\begin{cases} \boldsymbol{V}_2^{abc} = \boldsymbol{V}_1^{abc} - \Delta \boldsymbol{V}_1^{abc}; \\ \boldsymbol{V}_3^{abc} = \boldsymbol{K}_w \boldsymbol{K}_v \boldsymbol{V}_2^{abc} - \Delta \boldsymbol{V}_2^{Tr(abc)}; \\ \boldsymbol{V}_4^{abc} = \boldsymbol{V}_3^{abc} - \Delta \boldsymbol{V}_3^{abc}. \end{cases} \quad (30)$$

Combining the (30) gives:

$$\begin{bmatrix} \boldsymbol{V}_2^{abc} \\ \boldsymbol{V}_3^{abc} \\ \boldsymbol{V}_4^{abc} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{K}_w \boldsymbol{K}_v \\ \boldsymbol{K}_w \boldsymbol{K}_v \end{bmatrix} \boldsymbol{V}_1^{abc} - \begin{bmatrix} \boldsymbol{I} & 0 & 0 \\ \boldsymbol{K}_w \boldsymbol{K}_v & \boldsymbol{I} & 0 \\ \boldsymbol{K}_w \boldsymbol{K}_v & \boldsymbol{I} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{V}_1^{abc} \\ \Delta \boldsymbol{V}_2^{Tr(abc)} \\ \Delta \boldsymbol{V}_3^{abc} \end{bmatrix}. \quad (31)$$

Equation (31) can be stated in the following contracted form:

$$\boldsymbol{V}_{bus} = \boldsymbol{B}_{1v} \boldsymbol{V}_1 - \boldsymbol{B}_v \Delta \boldsymbol{V}. \quad (32)$$

It is useful to note that $\boldsymbol{B}_{1v}$ is the first column of $\boldsymbol{B}_v$. For the bus voltages initialization, $\boldsymbol{V}_{bus}^{nl}$ is obtained by equalizing the (32) and (19). Its equation is below given:

$$\boldsymbol{V}_{bus}^{nl} = \boldsymbol{B}_{1v} \boldsymbol{V}_1. \quad (33)$$

$\boldsymbol{B}_I$ and $\boldsymbol{B}_v$ flowchart. For large distribution networks with n buses and m branches the flowchart for matrices $\boldsymbol{B}_v$ and $\boldsymbol{B}_I$ building is presented in Fig. 5. Branch data are stored in four vectors, $\boldsymbol{A}_s$ for sending-end buses, $\boldsymbol{A}_r$ for the receiving-end buses, $\boldsymbol{A}_I$ and $\boldsymbol{A}_v$ for current and voltage transformation coefficients k_I and k_v respectively if the branches contains transformer. It can be seen that if the branch type (h) to be added in $\boldsymbol{B}_I$ matrix is a line section, then, the column vector $\boldsymbol{B}_I$(:,s) is stored directly in $\boldsymbol{B}_I$(:,r). But in the case of transformer the $\boldsymbol{B}_I$(:,s) need to be multiplied by the current transformation coefficient k_I before to be stored in $\boldsymbol{B}_I$(:,r). In both cases $\boldsymbol{B}_I$(h,r) is set to 1. A similar procedure can be used for building $\boldsymbol{B}_v$ but, by changing columns to rows and taking voltage transformation coefficient k_v for branch containing transformer.

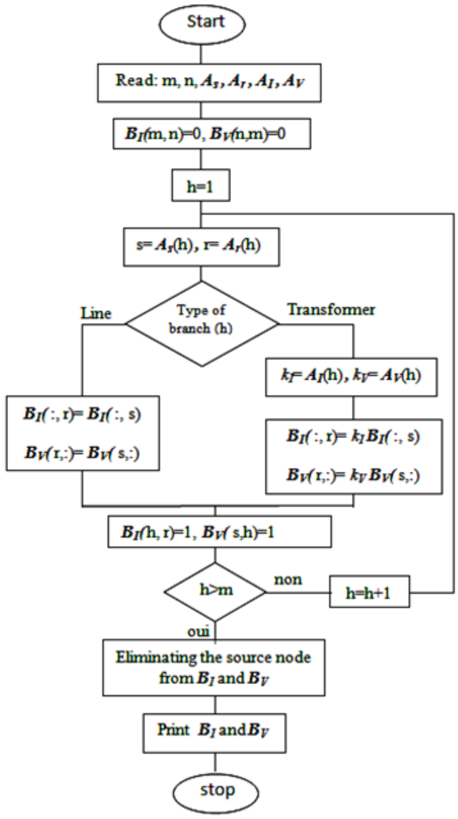

Fig. 5. Flowchart for $\boldsymbol{B}_I$ and $\boldsymbol{B}_v$ building for large distribution network

The proposed algorithm can easily be extended to a multiphase or multi-buses systems by expending the bus index i to vector (1×3). The corresponding 1 in $\boldsymbol{B}_I$ or in $\boldsymbol{B}_v$ matrices will be a 3×3 identity matrix and the corresponding k_I and k_v are respectively substituted by $\boldsymbol{K}_I$, $\boldsymbol{K}_L$ and $\boldsymbol{K}_v$, $\boldsymbol{K}_w$ matrices. If there are non-full phase components in the distribution system, the matrix columns and rows corresponding to missed phase will be eliminated.

Load flow algorithm. The proposed algorithm can be summarized in the following steps:

Step 1: Check the data and component modelling.

Step 2: Form $\boldsymbol{B}_I$ and $\boldsymbol{B}_v$ matrices built using procedures described in previous section.

Step 3: Assemble all branch impedance matrices in the full matrix $\boldsymbol{Z}$ as in (29).

Step 4: Determine the mDLF = $\boldsymbol{B}_v \boldsymbol{Z} \boldsymbol{B}_I$ matrix using (21).

Step 5: Initialize bus voltages using $\boldsymbol{V}_{bus}^{nl} = \boldsymbol{B}_{1v} \boldsymbol{V}_1$ given by (33).

Step 6: while the convergence rate is note reached. Solve iteratively the following equations, which, at $(k)^{th}$ iteration, are given by:

- Compute $\boldsymbol{I}_{Li}^{\varphi_1\varphi_2\varphi_3(k)}$ by (4) and (5) for a specified load and $\boldsymbol{V}_i^{(k)}$ at bus *i*;
- assemble all $\boldsymbol{I}_{Li}^{\varphi_1\varphi_2\varphi_3(k)}$ in a vector $\boldsymbol{I}_{bus}^{(k)}$ as in (23);
- calculate $\Delta\boldsymbol{V}_{bus}^{(k)}$ = mDLF $\boldsymbol{I}_{bus}^{(k)}$ given by (21);
- determine $\boldsymbol{V}_{bus}^{(k+1)} = \boldsymbol{V}_{bus}^{nl} - \Delta\boldsymbol{V}_{bus}^{(k)}$ using (19).

Step 7: End while.
Step 8: Write the results.
Step 9: End.

As convergence criterion at $(k+1)^{th}$ iteration, the following inequality, where ε is the convergence rate, fixed by user, is considered:

$$\max\left(\left|\Delta V_{bus}^{(k+1)} - \Delta V_{bus}^{(k)}\right|\right) \le \varepsilon \ . \qquad (34)$$

Test results. The load flow program was implemented using MATLAB. To validate the proposed method, the IEEE test networks stated by the Distribution Test Feeders Working Group, have been considered. Three test systems have been used in this work, it's about respectively the IEEE 4-bus, the IEEE 13-bus and the IEEE 37-bus networks.

The validation is first done for the IEEE 4-bus network the results of which are given in [23]. The obtained results have also been compared to those given by GridLAB-D for the 4-nodes, 13-nodes and 37-nodes IEEE power systems. GridLAB-D is a distribution software based on FBS method, well explained in [22, 24], using line and transformer models available in [19] and which are the same as those we had considered.

First test network. The proposed method has first been applied to the IEEE 4-bus test feeder shown by Fig. 6. Four practice winding connections of a step-down transformer with unbalanced loads were considered. Standard 30° connections are assumed for Wye-Delta and Delta-Wye banks. The line-to-line infinite bus-source voltages are equal to $[12.47\angle 0° \quad 12.47\angle{-120°} \quad 12.47\angle 120°]^T$ kV. The obtained voltages for each phase of buses 2, 3 and 4 are as given by Table 2 and Table 3 which show that our results are in agreement with those given by both IEEE [23] and GridLAB-D. It is to be noted that this version of GridLAB-D doesn't support the Wye-Delta and ground Wye-Delta configurations. As shown by (19) and (33), the voltages of the various nodes are calculated with respect to that of the ground taken as reference. Thereby one don't need to update the transformer primary voltage when there is a zero-sequence components.

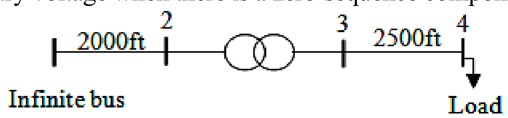

Fig. 6. IEEE4 bus test feeder

Table 2. IEEE 4-bus test feeder voltages comparison (IEEE results)

Connection	Node ID	V^{φ_1} [Volt/°deg]		V^{φ_2} [Volt/°deg]		V^{φ_3} [Volt/°deg]	
		IEEE	Proposed method	IEEE	Proposed method	IEEE	Proposed method
gY-gY	2	7164/–0.1	7163.72/–0.14	7110/–120.2	7110.47/–120.18	7082/119.3	7082.05/119.26
	3	2305/–2.3	2305.53/–2.26	2255/–123.6	2254.71/–123.62	2203/114.18	2202.91/114.79
	4	2175/–4.1	2175.02/–4.12	1930/–126.8	1929.82/–126.79	1833/102.8	1832.86/102.85
D-gY	2	12350/29.6	12350.22/29.60	12314/–90.4	12313.83/–90.39	12333/149.8	12332.68/149.75
	3	2290/–32.4	2290.34/–32.39	2261/–153.8	2261.65/–153.81	2214/85.2	2214.05/85.18
	4	2157/–34.2	2156.90/–34.24	1936/–157.0	1936.16/–157.03	1849/73.4	1849.59/73.39
Y-D	2	7112/–0.2	7111.14/–0.20	7144/–120.4	7143.62/–120.43	7112/119.5	7111.11/119.54
	3	3896/–2.8	3896.39/–2.82	3972/–123.8	3972.17/123.82	3874/115.7	3875.16/115.70
	4	3425/–5.8	3425.54/–5.76	3646/–130.3	3646.38/–130.27	3298/108.6	3297.76/108.58
gY-D	2	7113/–0.2	7111.1/–0.2	3896/–2.8	3896.4/–2.82	3425/–5.8	3425.5/–5.76
	3	7144/–120.4	7143.6/–120.4	3972/–123.8	3972.2/–123.82	3646/–130.3	3646.4/–130.28
	4	7111/119.5	7111.1/119.54	3875/115.7	3875.2/115.7	3298/108.6	3297.8/108.58
D-D	2	12341/29.8	12341.02/29.81	12370/–90.5	12370.28/–90.48	12302/149.5	12301.78/149.55
	3	3902/27.2	3901.86/27.20	3972/–93.9	3972.54/–93.91	3871/145.7	3871.49/145.74
	4	3431/24.3	3430.79/24.28	3647/–100.4	3647.53/–100.36	3294/138.6	3293.82/138.62

$\varphi = \{\varphi_1 \quad \varphi_2 \quad \varphi_3\} = \{\{ag, bg, cg\}$ or $\{ab, bc, ca\}\}$; *a*, *b*, *c*: phases, *g*: ground

Table 3. IEEE 4-bus test feeder voltages comparison (GridLAB-D results)

Connection	Node ID	V^{φ_1} [Volt/°deg]		V^{φ_2} [Volt/°deg]		V^{φ_3} [Volt/°deg]	
		Proposed method	GridLab-D	Proposed method	GridLab-D	Proposed method	GridLab-D
gY-gY	2	7163.72/–0.14	7163.7/–0.14	7110.47/–120.18	7110.5/–120.18	7082.05/119.26	7082/119.26
	3	2305.53/–2.26	2305.5/–2.26	2254.71/–123.62	2254.7/–123.62	2202.91/114.79	2202.8/114.79
	4	2175.02/–4.12	2175/–4.12	1929.82/–126.79	1929.8/–126.8	1832.86/102.85	1832.7/102.85
D-gY	2	12350.22/29.60	12350/29.6	12313.83/–90.39	12314/–90.4	12332.68/149.75	12333/149.8
	3	2290.34/–32.39	2290.3/–32.39	2261.65/–153.81	2261.65/–135.8	2214.05/85.18	2213.9/85.2
	4	2156.90/–34.24	2156.9/–34.24	1936.16/–157.03	1936.1/–157.0	1849.59/73.39	1849.4/73.4
Y-D	2	7111.14/–0.20	CHTC	7143.62/–120.43	CHTC	7111.11/119.54	CHTC
	3	3896.39/–2.82	CHTC	3972.17/123.82	CHTC	3875.16/115.70	CHTC
	4	3425.54/–5.76	CHTC	3646.38/–130.27	CHTC	3297.76/108.58	CHTC
gY-D	2	7111.1/–0.2	CHTC	3896.4/–2.82	CHTC	3425.5/–5.76	CHTC
	3	7143.6/–120.4	CHTC	3972.2/–123.82	CHTC	3646.4/–130.28	CHTC
	4	7111.1/119.54	CHTC	3875.2/115.7	CHTC	3297.8/108.58	CHTC
D-D	2	12341.02/29.81	12341/29.8	12370.28/–90.48	12370.3/–90.5	12301.78/149.55	12301.7/149.5
	3	3901.86/27.20	3901.8/27.2	3972.54/–93.91	3972.5/–93.9	3871.49/145.74	3871.5/145.7
	4	3430.79/24.28	3430.7/24.3	3647.53/–100.36	3647.5/–100.4	3293.82/138.62	3293.8/138.6

$\varphi = \{\varphi_1 \quad \varphi_2 \quad \varphi_3\} = \{\{ag, bg, cg\}$ or $\{ab, bc, ca\}\}$; *a*, *b*, *c*: phases, *g*: ground; CHTC – **c**annot **h**andle **t**his **c**onfiguration

Second test network. The second network considered is the IEEE 13-bus test system which originally contains variety of components such as cables and lines with various configurations and only one transformer at node 633. As shown by Fig. 7, this test system has been modified by excluding the regulator at substation and the distributed load on line 632-671. A second transformer has been added to the line 671-680. The results given by the proposed method have been compared to those obtained using GridLAB-D program. The results in Table 4 validate the proposed method and demonstrate its good level of accuracy.

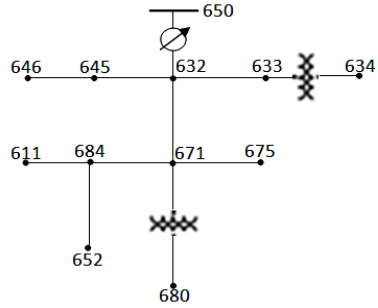

Fig. 7. IEEE 13-bus test feeders

Table 4. Voltages results of IEEE 13-bus test feeder

Node ID	V^{φ_1} [Volt/°deg]		V^{φ_2} [Volt/°deg]		V^{φ_3} [Volt/°deg]	
	Proposed method	GridLab-D	Proposed method	GridLab-D	Proposed method	GridLab-D
650	2401.8/0.0	2401.8/0.0	2401.8/–120	2401.8/–120	2401.8/120	2401.8/120
632	2286.2/–2.06	2286.2/–2.06	2335.7/–122.09	2335.7/–122.09	2306.9/118.29	2306.9/118.29
671	2201.5/–4.98	2201.6/–4.98	2341.7/–122.68	2341.7/–122.68	2200.6/116.91	2200.6/116.91
680	256.36/–34.02	256.36/–34.02	259.07/–152.76	259.07/–152.76	262.62/86.09	262.62/86.09
633	2277.8/–2.15	2278.4/–2.14	2330.2/–122.13	2330.8/–122.14	2301.3/118.27	2300.2/118.29
634	255.67/–2.93	255.73/–2.92	263.4/–122.65	263.48/–122.67	260/117.74	259.86/117.75
645	–	–	2301.3/–122.28	2301.2/–122.28	2315.2/118.18	2315.1/118.18
646	–	–	2290.4/–122.35	2290.3/–122.36	2318/118.15	2317.8/118.15
675	2181.6/–5.15	2181.6/–5.15	2344.4/–122.78	2344.4/–122.78	2191.7/117.02	2191.7/117.02
684	2197.4/–4.99	2197.5/–4.99	–	–	2195.7/116.8	2195.7/116.8
611	–	–	–	–	2190.9/116.64	2190.9/116.63
652	2180.5/–4.99	2180.3/–5.01	–	–	–	–
– missed phases						

Third test network. The third test feeder is the IEEE 37-bus network where voltage regulator and distributed load are discarded. As shown by Fig. 8, this network includes four down-step transformers located at nodes 702, 703, 709 and 737.

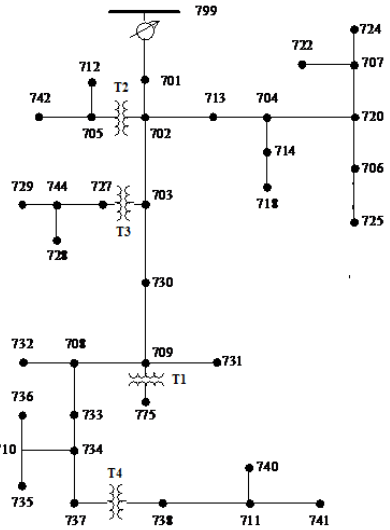

Fig. 8. IEEE 37-bus test feeders

Voltage profiles given by both proposed method and GridLAB-D are shown by Fig. 9,*a*,*b*. Figure 9,*a* shows voltage-profiles for nodes located at transformers primary-sides, the line-to-line voltage magnitudes of which is between 4.55 kV and 4.58 kV. Figure 9,*b*, on the other hand, gives voltage-profiles for nodes located at transformers secondary-sides whose line-to-line voltage magnitudes are between 360 V and 480 V. These figures show that the voltage profile obtained by the proposed method agree with that given by GridLAB-D.

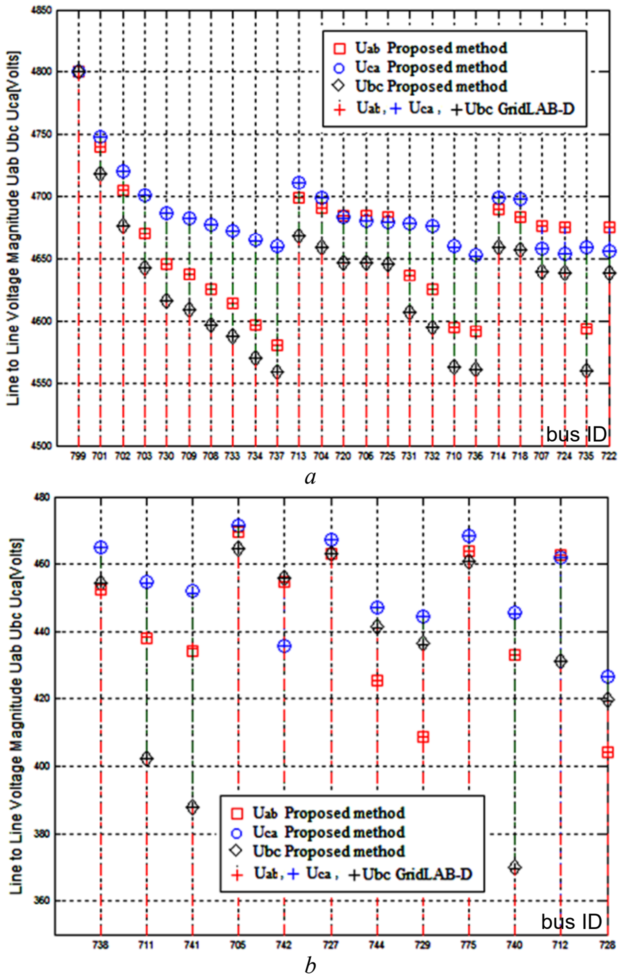

Fig. 9. Proposed method and GridLAB-D voltage profiles for IEEE 37-bus: *a* – voltages of nodes situated before transformers; *b* – voltages of nodes situated at transformers secondary side

Conclusions. In this paper, the well-known matrices bus injection to branch current and branch current to bus voltage have been modified and led to new matrices. The latter support all practical transformer models and configuration types. No assumptions are made using these new matrices. One can use either real values or per-unit system for the network parameters. Based on an elaborate flowchart of topological matrix construction, the proposed power flow method is validated by comparing the results obtained with those given by the GridLAB-D program for three IEEE test systems. It has been shown that the proposed method is efficient, can handle different distribution components and can be extended to large and complex balanced and unbalanced distribution networks.

Conflict of interest. The authors declare no conflict of interest.

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