UDC 621.3.013 https://doi.org/10.20998/2074-272X.2022.4.02

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The method of multi objective synthesis of nonlinear robust control by multimass electromechanical systems

Aim. *Development of the method of multi objective synthesis of nonlinear robust control by multimass electromechanical systems to satisfy various requirements for the operation of multi-mass systems in various modes.* ***Methodology.*** *The problem of multi objective synthesis of nonlinear robust control of multimass electromechanical systems is formulated and the possibility of satisfying various requirements for the operation of such systems in various modes based on the concept of functionally multiple membership of the state vector and the solution of the Hamilton-Jacobi-Isaacs equation is shown. A method for choosing weight matrices with the help the vector of purpose of nonlinear robust control is formed by solving a zero-sum vector antagonistic game has been substantiated and developed.* ***Results****. The results multi objective synthesis of nonlinear robust two-mass electromechanical servo systems in which differences requirements for the operation of such systems in various modes were satisfied are given. Based on the results of modeling and experimental studies it is established, that with the help of synthesized robust nonlinear controllers, it is possible to improve of quality indicators of two-mass electromechanical servo system in comparison with the system with standard regulators.* ***Originality.*** *For the first time the method of multi objective synthesis of nonlinear robust control by multimass electromechanical systems to satisfy various requirements for the operation of multimass systems in various modes is developed.* ***Practical value.*** *From the point of view of the practical implementation the possibility of solving the problem of multi objective synthesis of nonlinear robust control systems to satisfy various requirements for the operation of multimass electromechanical systems in various modes is shown.* References 32, figures 4.

Key words: **multimass electromechanical systems, nonlinear robust control, multi objective synthesis, Hamilton-Jacobi-Isaacs equation, computer simulation, experimental research.**

Мета*. Розробка методу багатокритеріального синтезу нелінійного робастного керування багатомасовими електромеханічними системами для задоволення різноманітних вимог до роботи багатомасових систем у різних режимах.* ***Методологія****. Сформульовано задачу багатокритеріального синтезу нелінійного робастного керування багатомасовими електромеханічними системами та показана можливість задоволення різноманітних вимог до роботи таких систем у різних режимах на основі концепції функціонально множинної належності вектора стану та рішення рівняння Гамільтона-Якобі-Айзекса. Обґрунтовано та розроблено метод вибору вагових матриць, за допомогою яких формується вектор мети нелінійного робастного керування, шляхом розв'язання векторної антагоністичної гри з нульовою сумою.* ***Результати****. Наведено результати багатокритеріального синтезу нелінійних робастних двомасових електромеханічних сервосистем керування, в яких були задоволені різноманітні вимоги до роботи таких систем у різних режимах. На основі результатів моделювання та експериментальних досліджень встановлено, що за допомогою синтезованих нелінійних робастних регуляторів можна підвищити якісні показники двомасової електромеханічної сервосистеми в порівнянні з системою зі стандартними регуляторами.* ***Оригінальність****. Вперше розроблено метод багатокритеріального синтезу нелінійного робастного керування багатомасовими електромеханічними системами для задоволення різноманітних вимог до роботи багатомасових систем у різних режимах.* ***Практичне значення****. З точки зору практичної реалізації показана можливість вирішення задачі багатокритеріального синтезу нелінійних робастних електромеханічних систем керування для задоволення різноманітних вимог до роботи таких систем у різних режимах.* Бібл. 32, рис. 4.

Ключові слова: **багатомасові електромеханічні системи, нелінійне робастне керування, багатокритеріальний синтез, рівняння Гамільтона-Якобі-Айзекса, комп'ютерне моделювання, експериментальні дослідження.**

Introduction. The central problem of modern theory and practice of automatic control is the creation of systems capable of providing high control accuracy under intense master and disturbing influences of a wide range of frequencies. Improving the accuracy of electromechanical control systems is often constrained by imperfect mechanical transmissions from the actuator to the working mechanism [1, 2]. This, first of all, manifests itself with an increase in the system bandwidth, when the frequencies of natural mechanical vibrations of the transmission, together with the actuator and the working mechanism, fall into the range of operating frequencies of the control systems. At the same time, it is necessary to take into account the presence of elastic elements between the shafts of the executive motor, the gearbox and the working mechanism, and instead of the single-mass model, the engine – the working mechanism, use two, three, and sometimes even a multi-mass model [3, 4]. The conditions of operation of electromechanical systems are also complicated by the presence of a nonlinear dependence of the moment (force) of friction on the speed of sliding of the working mechanism relative to the material being processed [5, 6]. This dependence often manifests itself in many modes of operation of electromechanical systems at low (creeping) speeds of movement of the working body. Moreover, for some mechanisms, this mode is working, and for others – emergency. The situation is even more aggravated when the presence of elastic elements is combined with the operation of the system on the falling section of the external friction characteristic, which can lead to the occurrence of sustained or even diverging mechanical vibrations [7, 8].

Various requirements are imposed on the designed multi-mass control systems during their operation in various modes. As a rule, certain restrictions are imposed on the quality of transient processes – the first coordination time, regulation time, overshoot, etc. are set. Usually, the maximum variance of the tracking error or stabilization during the development of random reference influences, or the compensation of random disturbing influences, is also specified, and in this case, naturally, the constraints on the state and control variables must be satisfied [9]. Another requirement for control systems is the limitation of mastering errors or compensation of disturbing influences in the form of harmonic signals. In this case, an input signal of one frequency, or several characteristic operating frequencies, can be set, and a range of operating frequencies can be set, in which certain conditions must be fulfilled. And, finally, for tracking systems of increased accuracy, the characteristic mode of operation is the development of low speeds or small displacements. For this mode, the roughness of movement is usually specified in the form of appropriate criteria. The reasons for the non-smooth movement of the working body at low speeds is the presence of nonlinearities such as dry friction in the executive motors and working bodies and elastic elements between the executive motor and the working body, which leads to stall vibrations of the moving parts of the executive motor and the working body, accompanied by stops and breakdowns of the moving parts relative to stop positions.

For such systems, in most practical cases, with the help of typical PID controllers, it is not possible to fulfill the technical requirements for the system, which necessitates the use of more complex controllers and modern methods of their synthesis [10–12]. One of the main requirements for multi-mass control systems is also the requirement for the robustness of the synthesized system, i.e. the ability of the system to maintain the technical requirements imposed on it when the parameters of the control object and external influences change within certain limits [13, 14].

The central problem of modern theory and practice of robust control is the creation of systems that can function effectively under conditions of uncertainty in the values of parameters, and possibly the structure of models of the control object, disturbing influences and measurement noises [15–17].

One of the rapidly developing approaches to the synthesis of robust control systems is the synthesis of controllers that minimize $H_\propto$ the norm of the vector of the goal of control [18, 19]. However, when designing real control systems, there are no requirements for $H_\propto$ the norm of the target vector, and the target vector of robust control itself is usually not specified.

In this case, the main difficulties in the practical application of modern control methods are associated not so much with the development of new control methods as with the informal choice of the vector of the goal of robust control or the criterion of the quality of optimal control.

The purpose of the work is to develop the method of multi objective synthesis of nonlinear robust control by multimass electromechanical systems to satisfy various requirements for the operation of multi-mass systems in various modes.

Problem statement. To solve the problem of multi objective synthesis of robust control, the concept of multi-functional membership on the elements of the state space has been developed. Let us consider the possibility of choosing such a quality criterion, under which it is possible to satisfy all the requirements for a system based on the concept of multi-functional membership.

Suppose that the original nonlinear system (1) can be described in the state space by a nonlinear differential equation of state in the following form:

$$\dot{x} = f(x,u,t),\ x(t_0) = x_0,\ t \ge t_0, \tag{1}$$

The classical optimal control problems solve the problem of control synthesis that minimizes the adopted performance criterion in form functional

$$J = \int_0^T f_0(\vec{x}(t),\vec{u}(t))dt. \tag{2}$$

The choice of the optimality criterion (2) characterizing the quality of control processes is a informal problem. As a rule, the criterion of optimality is conditional. When designing a system, it is necessary to select such an indicator of the quality of the system, which intuitively reflects the idea of what is good and what is bad for a given system. Therefore, the difficulties of designing an optimal system are actually reduced to the difficulties of forming such a criterion that would reflect the real requirements for the system. The semantic formulation of the optimization problem, as a rule, is a multi objective problem with constraints. Naturally, many methods for solving this problem are reduced to the formation of a one-criterion problem, when all the criteria and constraints with the help of the chosen compromise scheme are reduced into one indicator of the quality of the system. In conclusion, we note that the same value of the quality criterion in single-criterion optimization can correspond to transient processes that differ sharply in their form – oscillatory, aperiodic, and their quality indicators, such as regulation time, overshoot, differ by orders of magnitude. This is because in one criterion it is necessary to reflect both the quality of the dynamic characteristics of the systems and the energy consumption for control and constraints on the state variables of the system. Moreover, on the basis of the solution of the inverse problem of optimal control for any transient process, which is arbitrarily unsatisfactory in terms of the quality indicators, it is possible to choose such an indicator of the system quality, according to which it will be optimal. Therefore, it is the problem of choosing a quality criterion that is the main one, since the very solution of the optimization problem is not difficult.

The problem of choosing a quality criterion neither in optimal nor, even more so, in robust control remains unsolved to this day. The authors Letov and Kalman, as well as many researchers who tried to apply this theory to solving practical problems, also paid attention to the importance of the problem of choosing the quality functional in the problem of analytical design of regulators by integral quadratic quality criteria. However, to date, this problem has not been fully resolved.

In the concept of multi-functional membership on the elements of the state space it is assumed that all various requirements that are imposed for the operation of systems (1) in various modes for

$$u \in U(x,t), \tag{3}$$

where $U(x, t) \subset R^m$ – some given control vector set for each state vector x and $t \ge t_0$, to fulfill the following relation for the state vector:

$$x = x(t) \in Q(t),\ t \geq t_0, \tag{4}$$

$$Q(t) = \{x \in R^n : \psi(x,t) \leq 0\}, \tag{5}$$

where $\psi(x, t)$ – scalar function continuously differentiable in all its variables.

Usually, some of the requirements for the system can be formulated in the form of a minimum or maximum (2). For example, it is desirable to ensure the minimum variance of the system error, the minimum control time, the minimum error in the harmonic signal processing, etc. Then the control goal can be formulated as a vector

$$y = \varphi(x(t),t) \in Q(t),\ t \geq t_0, \tag{6}$$

where $\varphi(x, t)$ – some given continuously differentiable $(n\times 1)$ – is a vector function, and the set

$$Q(t) = \{y \in R^n : \psi(y,t) \leq 0\}. \tag{7}$$

Note that specifying the set $Q(t)$ is a rather difficult task, and often can be formally unsolved problem. Probably the most versatile method of setting an area $Q(t)$ is to carry out simulation of a system with a specific control law. At the same time, the presence of a control law is necessary, since many quality indicators are presented not only to the executive motor, the plant, but also directly to the entire control system.

It is assumed that the goal of control, constraints on the state and control vector can be reduced to uniform constraints on the state vector of the system. To ensure the condition of membership of the state vector $x(t)$ the multitude $Q(t)$ in order to fulfill the constraints on the state vector and to ensure the condition that the control goal vector $y(t)$ the multitude $Q(t)$ and with minimization over the control vector in the synthesis of a robust control system for an object with uncertainties (parametric, structural, uncertainty of external influences, etc.) can be written in the form of the maximin inequality

$$\begin{gathered}\max_{x\in M(y,t)} \min_{u\in U(x,t)} \left(\nabla_y \psi, \nabla_x \varphi \cdot f(x,u,t)\right) + \cdots \\ \cdots + \left(\nabla_y \psi, \frac{\partial \varphi}{\partial t}\right) + \frac{\partial \psi}{\partial t} \leq 0,\end{gathered} \tag{8}$$

for each $y \in \mathrm{B}Q(t)$ and each $x \in M(y, t)$, $t \geq t_0$ where $\mathrm{B}Q(t) = \{x \in R^n : \psi(x, t) = 0\}$ is the boundary of the set $Q(t)$; $\nabla_y \psi$ is the gradient of the function $\psi(x, t)$; $\nabla_x \varphi$ – is the Jacobian of the function $\varphi(x, t)$; $(\nabla_x \psi, f(\cdot))$ – dot product of vectors $\nabla_x \psi, f(\cdot) \in R^n$; $M(y, t)$ – is a certain variety corresponding to $y \in \mathrm{B}Q(t)$ and determined according to the dependence

$$M(y,t) = \{x \in R^n : \varphi(x,t) = y\}, \tag{9}$$

$$Q(t) \subseteq \mathrm{B}_\varphi \text{ at } t \geq t_0. \tag{10}$$

These inequalities are valid for a robust control system for any structure of the control part of the system - software, with feedback, etc. We restrict ourselves to the law of control with feedback over the full state vector in the following form

The main difficulties of solving of the problem of multi objective synthesis of robust control based on functionally multiple membership are related to the difficulties of defining and calculating functions of the area of functional-set membership Q, defined by function $\psi(t)$. Note that obtaining analytical dependences of the functions $\psi(\vec{x},t)$ and $\varphi(\vec{x},t)$ can present significant difficulties, and often even impossible [20, 21]. However, to solve the problem, not the functions themselves are needed, but their gradient and Jacobian, which can be obtained by numerical methods.

Solution method. Consider the method of computation of this functions $\psi(\vec{x},t)$ and $\varphi(\vec{x},t)$ based on the modern theory of nonlinear robust control [22–24]. Consider the general case of a nonlinear system written in the following form

$$\dot{x} = F(x,\omega,u), \tag{11}$$

$$z = Z(x,u), \tag{12}$$

where ω is the vector of external uncontrolled disturbances.

Moreover, when synthesizing a robust control, this perturbation is considered to be independent and the worst-case condition is chosen for the control.

For this system, we write the Hamilton function in the following form

$$\begin{gathered}H(x,p,\omega,u) = p^T F(x,\omega,u) + \ldots \\ \ldots + \frac{1}{2}\|Z(x,u)\|^2 - \frac{1}{2}\|\omega\|^2.\end{gathered} \tag{13}$$

The Hamilton–Jacobi–Isaacs inequality for this nonlinear system takes the following form

$$\begin{gathered}H_*\left(x, V_x^T(x)\right) = V_x^T(x) F(x,\omega,u) + \ldots \\ \ldots + \frac{1}{2}\|Z(x,u)\|^2 - \frac{1}{2}\gamma^2\|\omega\|^2 \leq 0\end{gathered}. \tag{14}$$

Then the feedback in the form $u = \alpha_u(x, V_x^T(x))$, where $\alpha_u(x, p)$ is determined from the following system of Hamilton–Jacobi–Isaacs differential equations

$$\frac{\partial H}{\partial \omega}\left(x, p, \alpha_\omega(x,p), \alpha_u(x,p)\right) = 0, \tag{15}$$

$$\frac{\partial H}{\partial u}\left(x, p, \alpha_\omega(x,p), \alpha_u(x,p)\right) = 0, \tag{16}$$

$$\alpha_\omega(0,0) = 0, \quad \alpha_u(0,0) = 0. \tag{17}$$

These equalities are necessary conditions for the extremum of the Hamilton function, as in the control vector u, and by the vector of external disturbances ω. Moreover, it is necessary to find the minimum norm of the target vector by the control vector and the maximum of this norm by the vector of external disturbances, i.e. solve the maximin extreme problem. Note that these conditions are necessary conditions for optimizing a dynamic game in which the first player, the controller, minimizes the goal vector, and the second player, external disturbances, maximizes the same goal vector.

This approach can be interpreted as a zero-sum differential game of two players in which one player minimizes the accepted quality criterion for control u, and the other player maximizes this criterion with respect to the vector of external variables ω. In this case, the minimization strategy for control u

$$u^* = \alpha_u(x) = -g_u^T(x) V_x^T(x), \tag{18}$$

and the strategy of maximization along the vector of external influences ω

$$\omega(x)=\alpha_{\omega}(x)=\frac{1}{\gamma^{2}}g_{\omega}^{T}(x)V_{x}^{T}(x). \qquad (19)$$

In this case, external influences ω, found from the condition for the worst case, at which the energy of the target vector is maximized.

In contrast to optimal control under robust control, the role of the integrand $f_0(x, u)$ in (2) playing target vector norm $||z(x, u)||^2$ in (12), and, in addition, the norm of the vector of external influences is introduced into the Hamilton function $||\omega||^2$ is introduced into the Hamilton function (13) and this external influence during the synthesis of the system is considered independent and can be determined from the condition of maximum «harmfulness» (worst – case disturbance) – maximum deviation of the target vector norm.

Naturally, the dynamics of the synthesized robust system is largely determined by the goal function $z(x, u)$, and all system requirements must be satisfied by the appropriate selection of this function of the goal. Setting the functional-multiple membership of the state vector in the form of a set Q in the form of inequality $\psi(x, t)\leq 0$ in (7) together with setting the vector of the control goal $y(t) = \varphi(x, t)$ in (6) is equivalent to specifying the integrand f_0 in the integral criterion (2) for optimal control or goal function $Z(x, \omega, t)$ in robust control (12). Moreover, by choosing these functions, in fact, it is necessary to satisfy all the requirements for the system.

Thus based on the concept of functional – multiple membership of the state vector and the solution of the Hamilton-Jacobi-Isaacs equation it shown that it is possible to satisfy all the requirements for the system by choosing the target vector of nonlinear robust control, when multi objective synthesis of nonlinear robust control by multimass electromechanical systems is calculated.

The method of computation of the goal vector of nonlinear robust control. Let us now consider a method for calculating the goal vector $z(x, u)$ of a robust control (12) in a multi objective synthesis of a nonlinear robust control. Let us introduce the vector J of quality indicators that apply to the operation of the system in various operating modes. The components of this vector, in particular, can be: the transient times are usually specified when certain input signals are applied: the accuracy of working off the specified minimum speed value the uneven movement of the working body at the minimum speed: minimum value of the stabilization dispersion of a given random change in the reference action is usually required under the action of random disturbing influences caused, for example, by a random change in the road profile.

In addition, we introduce a vector G of limitations, the components of which can be the limiting values of voltages, currents, rates of change of currents, moments, elastic moments, rates of change of moments (jerks), speeds of various elements of a multi-mass electromechanical system, their positions, etc.

We also introduce the vector S of uncertainties in the parameters of the initial system and external influences. The components of this wind, in particular, can be changes in the moments of inertia of the working body.

The dynamic characteristics of the synthesized nonlinear robust system are determined by the mole of the control object, external master and disturbing influences, and, of course, the parameters of the synthesized nonlinear robust controllers. The control system designer can only change the robust control target vector. Let us perform a parameterization of the function, with the help of which the goal vector of the nonlinear robust control is set and introduce the vector Z of these desired parameters.

Then, using the given value of the Z vector of these desired parameters, the vector J of the values of quality indicators that are imposed on the system operation, and the vector G of the restrictions when the system is operating in various operating modes and for various setting and disturbing and for various values of the vector S of the uncertainty of the initial system parameters and external influences.

Then the problem of multi objective synthesis of non-linear robust control can be formulated as the zero-sum vector antagonistic game [25, 26].

In this game, the first player is the vector Z parameterization of the function, with the help of which the goal vector of the nonlinear robust control is set, and it strategy is to minimize the game payoff vector. The second player is the vector S of uncertainties in the parameters of the initial system and external influences, and it strategy is to maximize the same game payoff vector J. This approach is the standard approach in the robust control synthesis for the «worst» case.

To correctly calculation of solution of this vector antagonistic game from the set of Pareto-optimal solutions, binary preference relations of local performance criteria B are used.

In conclusion, we note that the computation of the pay game vector J, the constraint vector G, and the vector B of binary preference relations is algorithmic in nature and requires large computational resources. First, to calculate the nonlinear robust control it is need to solve the Hamilton-Jacobi-Isaacs equation.

Then, in order to calculate the values of payoff game vector J, the constraint vector G, and the vector B of binary preference relations it is necessary to simulate the initial non-linear system closed by synthesized nonlinear robust controllers for given system operation modes and for given driving and perturbing influences at given values of the nonlinear vector certainty of the parameters of the original system.

The calculation of the solution of this vector antagonistic game from set of Pareto-optimal solutions based on stochastic multiagent optimization [27, 28]. To date, a large number of particle swarm optimization algorithms have been developed – PSO algorithms based on the idea of collective particle swarm intelligence, such as gbest PSO and lbest PSO algorithms. The use of stochastic multi-agent optimization methods to solve vector antagonistic game today causes some difficulties and this area continues to develop intensively. To solve the initial vector antagonistic game with constraints, we construct an algorithm for stochastic multiagent optimization based on a set of swarms of particles, the number of which is equal to the number of components of the payoff vector game, In a standard particle swarm optimization algorithm, the change in particle velocities is performed according to linear laws. To increase the speed of finding a global solution, special nonlinear algorithms for stochastic multi-agent optimization have recently become widespread [29–32].

Simulation results. As an example, consider the results of modeling an electromechanical servo system synthesized in the course of multi objective synthesis. There are elastic elements between the motor shaft and the working body in the system under consideration, therefore the mathematical model is adopted in the form of a two-mass electromechanical system.

There are also nonlinear elements in the control system. This, first of all, concerns the presence of dry friction both in the executive engine and in the control object drive in the horizontal guidance channel and in the control object drive in the vertical guidance channel. In addition, the system has nonlinear characteristics of the elastic elements between the actuating motors and drive mechanisms due to backlash-selecting springs. Let us consider the influence of these elements on the dynamic characteristics of the system.

In this case, we will consider the dynamic characteristics of the system for three values of the moments of inertia of the working mechanism – the nominal value and those that differ from the nominal value by a factor of two up and down.

One of the intense criteria imposed on the synthesized system is the requirement for the quality of transient processes in the mode of working out small angles.

As an example, in Figure 1 are shown the transients of state variables: *a*) the angle of the plant; *b*) the speed of the plant; *c*) moment of elasticity; and *d*) the speed of the motor in this mode of operation.

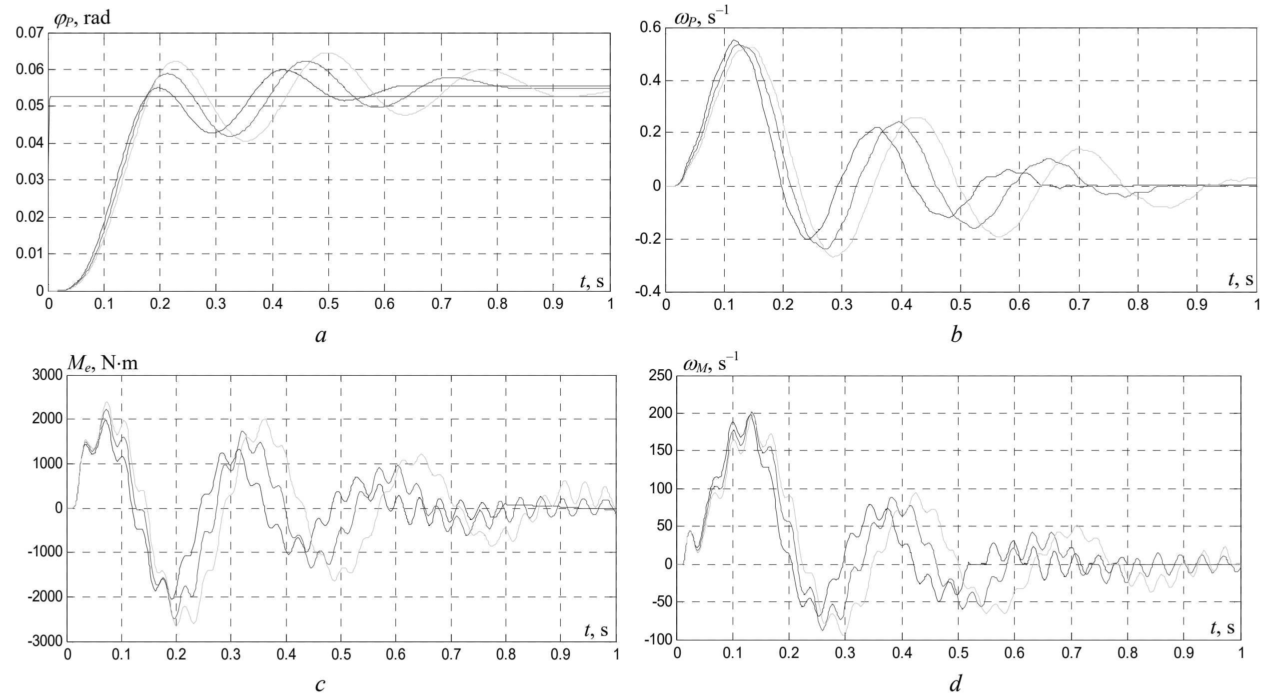


Fig. 1. Transient processes of state variables of the electromechanical servo system in the mode of working out small angles

The time of the first coordination in the synthesized robust control system is significantly less compared to the time of the first coordination in the existing system, which is usually about one second, and, therefore, with the help of synthesized robust controllers for improved mathematical models, it is possible to reduce the time of transients by 1.5–2 times compared to a system with typical regulators.

One of the intense criteria imposed on the synthesized system is the requirement for the accuracy of compensation for random disturbances acting on the control object during its operation. As an example, in Figure 2 are shown the implementation of random processes of state variables of an electromechanical servo system under random external influences

In the Fig. 2 are shown the implementation of random processes of state variables *a*) changes in the angle of plant; *b*) the derivative of the plant; *c*) the moment of stabilization of the plant; and *d*) the derivative of the moment of stabilization of the plant under random external influences.

With the help of synthesized robust nonlinear controllers for improved mathematical models, it is possible to reduce the variance of the error in compensating for random disturbance acting on the control object by 1.7–2.3 times compared to a system with typical controllers. Note that this requirement largely determines the potential accuracy of the synthesized electromechanical tracking system.

Experimental research. A stand of a two-mass electromechanical servo system was developed for experimental research. The mechanical part of the stand is made on the basis of two identical micro-motors of a direct current like DPT-25-H2. The motor shafts are connected by an elastic transmission. With the help of the second motor, a load is created on the first motor. Sensors are located on the motor shafts, which are used to measure the angles of rotation and angular velocities of the first and second motor.

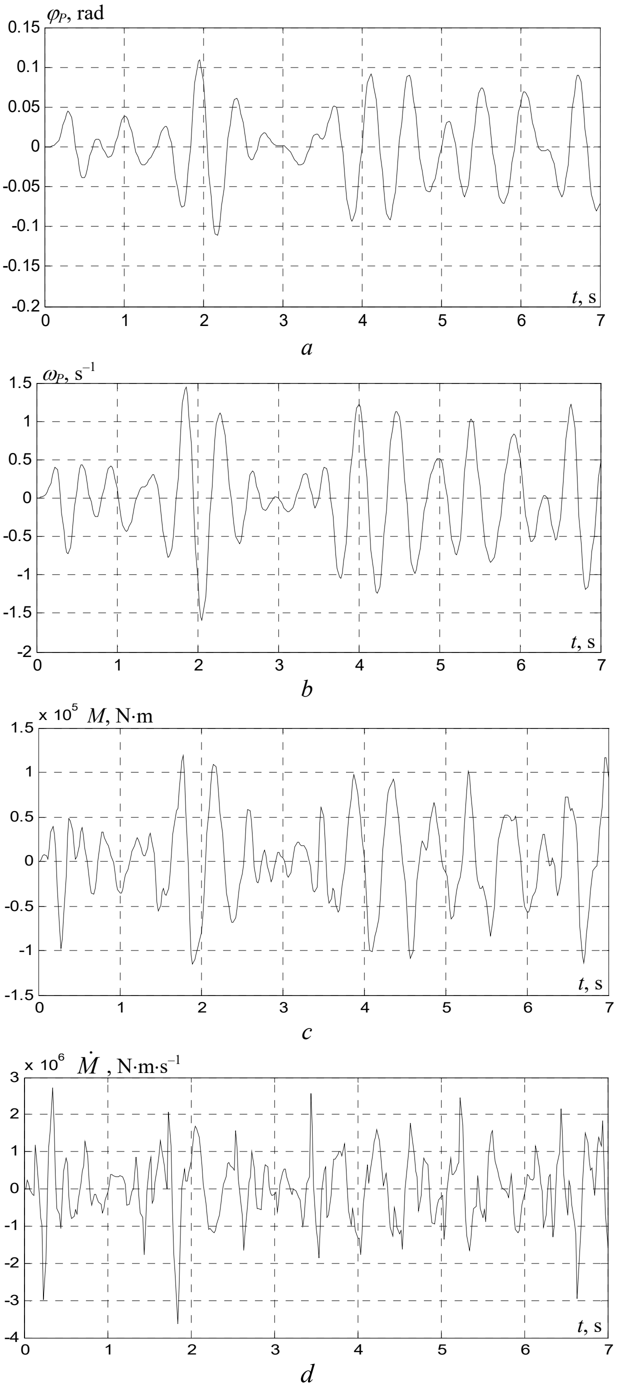
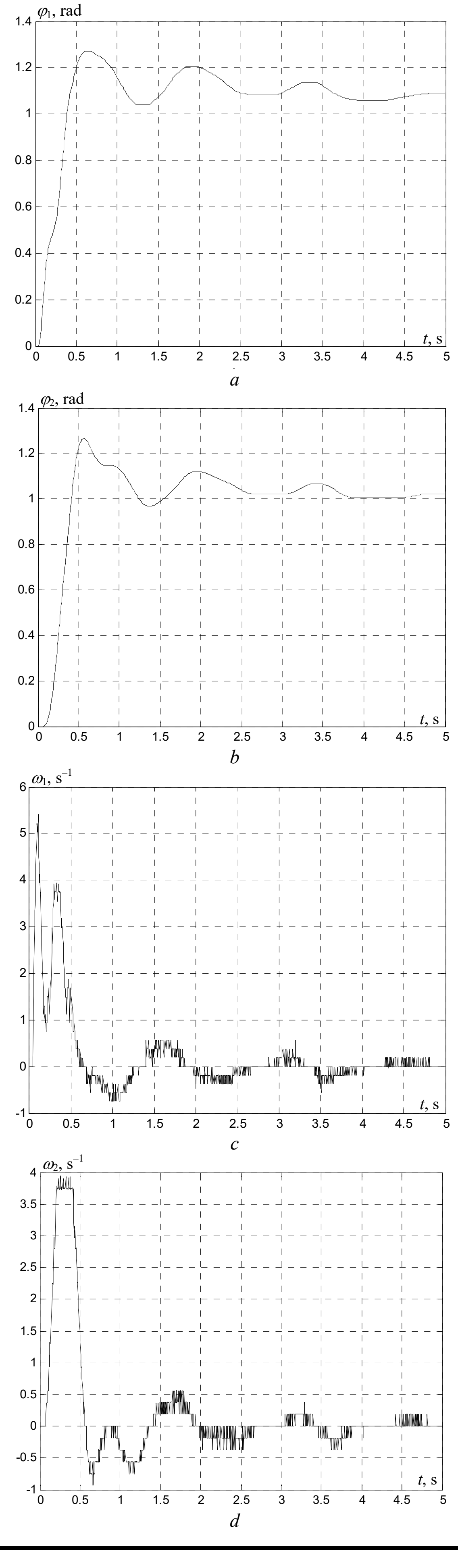
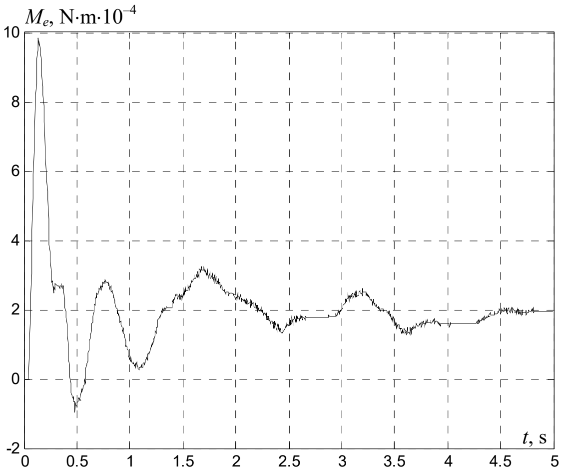


Fig. 2. Implementation of random processes of state variables of an electromechanical servo system under random external influences

Let us now consider the results of experimental studies of the stand with robust controllers. As an example in Fig. 3 are shown the experimental transient processes of state variables: the angles of rotation of the first φ_1 (*a*) and the second φ_2 (*b*) motor; rotation speeds of the first ω_1 (*c*) and the second ω_2 (*d*) motor and moment of elasticity M_e (*e*), when the system is working out a given angle of rotation φ_{in} = 1 rad.

Experimental studies have shown that the use of robust control of a stand of two mass electromechanical systems synthesized during multi objective synthesis reduces the time of the first negotiation of the transition process at the shaft angle of the second engine by more than 1.7 times compared to the system with standard regulators.





e

Fig. 3. Experimental transient processes of the of state variables when the system is working out a given angle of rotation $\varphi_{in} = 1$ rad

In the experimental transient process of the stand rotation angle, there are nonlinear sections due to the presence of friction moments in the stand. The experimental transients of state variables of motor rotor speeds and voltages on motor armature circuits obtained on the stand contain high-frequency components, while model transients of the same state variables change more smoothly.

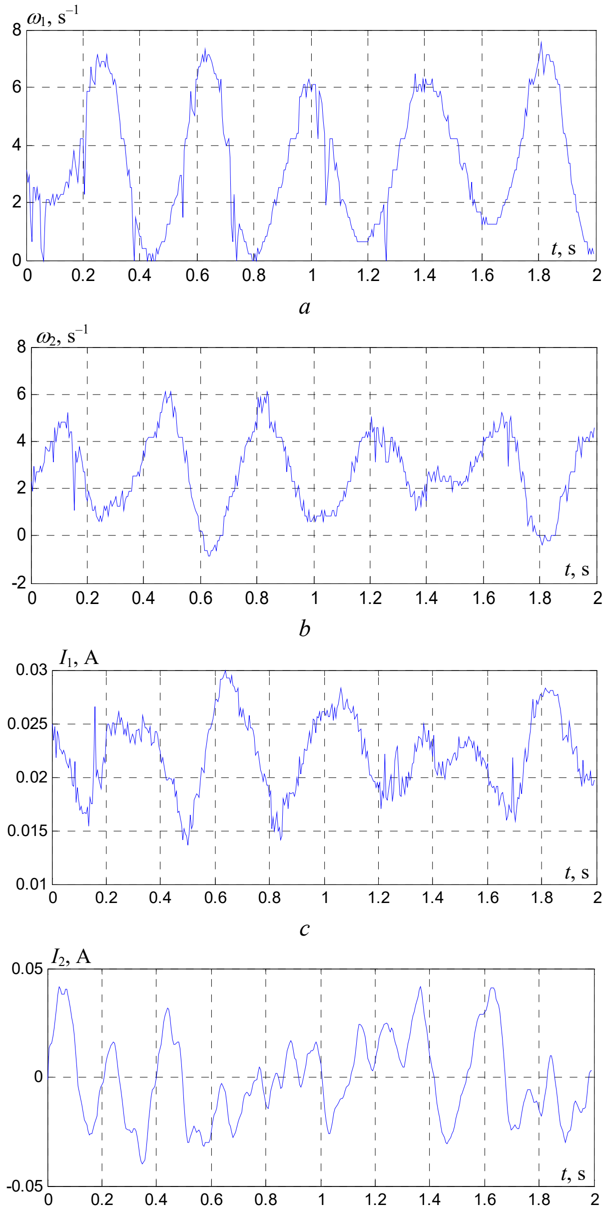
Note that the quality of transients is significantly influenced by the characteristics of nonlinearities of actuators and it is they who determine the potential accuracy of the system with synthesized optimal regulators.

As an example, in Figure 4 are shown the implementation of random processes of the stand of a two-mass electromechanical system in the mode of stabilization of the rotation speed of the shaft of the second motor under the action of random changes in the moment of resistance created by the second motor.

In Fig. 4 are shown the following state variables: rotational speeds of the first *a*) and second *b*) motors and currents of anchor circuits of the first *c*) and second *d*) motors.

Experimental studies have shown that the use of robust control of a stand of two mass electromechanical systems synthesized during multi objective synthesis reduces the error of adjusting the speed of rotation of the shaft of the second motor by more than 1.2 times, as well as reduces the control error the angle of rotation of the shaft of the second motor more than 2 times in comparison with the system with standard regulators at random change of the moment of resistance formed by means of the second motor.

Notice, that improving the control accuracy of the system with robust controllers is accompanied by more intense work of the actuator motor. In particular, the armature current of the first motor in a system with a robust regulator has significantly higher-frequency components and a larger amplitude c of rotation of the shaft of the second motor more than 2 times in comparison with the system with standard regulators.



a

b

c

d

Fig. 4. Implementations of random processes of variables of the state of the stand of a two-mass electromechanical system with a random change in the moment of resistance

Conclusions.

1. For the first time the method of multi objective synthesis of nonlinear robust control by multimass electromechanical systems to satisfy various requirements for the operation of multi-mass systems in various modes is developed.
2. Based on the concept of functional – multiple membership of the state vector and the solution of the Hamilton-Jacobi-Isaacs equation it is shown that it is possible to satisfy all the requirements for the system by choosing the target vector of nonlinear robust control. The problem of multi objective synthesis of nonlinear robust control of multimass electromechanical systems is formulated by solving a zero-sum vector antagonistic game.
3. The computation of the game payoff vector, the constraint vector, and the vector of binary preference relations is algorithmic in nature and requires large computational resources. To calculate the non-linear

robust control it is need to solve the Hamilton-Jacobi-Isaacs equation. Then as a result of modeling a closed system, the vector of the values of the quality indicators that are imposed on the system operation, and the vector of the restrictions, when the system is operating in various operating modes and for various setting and disturbing and for various values of the vector of the initial uncertainty of the system parameters and external influences are calculated.

4. The results of multi objective synthesis of nonlinear robust control by servo two-mass electromechanical systems in which differences requirements for the operation of such systems in various modes were satisfied are given. The results of modeling and experimental studies of transients of two-mass servo electromechanical tracking system and realizations of state variables of this system under random external influences are presented.

5. Based on the results of modeling and experimental studies it is established, that with the help of synthesized robust nonlinear controllers, it is possible to reduce the error of adjusting the speed of rotation of the shaft of the second motor by more than 1.2 times, as well as reduces the control error the angle of rotation of the shaft of the second motor more than 2 times and to reduce the variance of the error in compensating for random disturbance acting on the plant by 1.7–2.3 times in comparison with the system with standard regulators.

Conflict of interest. The authors declare that they have no conflicts of interest.

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Received 10.04.2022
Accepted 21.05.2022
Published 20.07.2022

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How to cite this article:
Kuznetsov B.I., Nikitina T.B., Bovdui I.V., Voloshko O.V., Kolomiets V.V., Kobylianskyi B.B. The method of multi objective synthesis of nonlinear robust control by multimass electromechanical systems. *Electrical Engineering & Electromechanics*, 2022, no. 4, pp. 12-20. doi: https://doi.org/10.20998/2074-272X.2022.4.02