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Mathematical modeling of rheostat-reactor start of wound-rotor induction motors

Introduction. *Wound-rotor induction motors are less common compared squirrel-cage induction motors. However, they occupy a significant share among electric drives with difficult starting conditions. Their advantage is obtaining a high starting electromagnetic torque at lower values of starting currents.* ***Problem.*** *Due to the possibility of including different devices in the rotor circuit, it is possible to shape the starting characteristics according to the needs of the technological process. Due to a narrower range of applications of electric drives based on wound-rotor induction motors, they are less investigated. Selection of parameters of starting and regulating devices, included in the rotor circuit, is carried out by simplified methods, which do not satisfy modern requirements to regulated electric drives.* ***Goal.*** *The paper aims to develop mathematical models and methods for calculating the dynamic modes and static characteristics of the wound-rotor induction motor with a reactor in the rotor circuit.* ***Methodology.*** *In the developed algorithms, the mathematical model of the motor is presented by the differential equations made for electric circuits in a system of orthogonal coordinates that allows excluding angular coordinate from equations of electric equilibrium. The elements of the Jacobi matrix of equilibrium equations of motor circuits are eigenvalues, and mutual is the differential inductances of electrical circuits, which are determined based on the magnetization characteristics of the main magnetic flux and leakage fluxes of the rotor and stator circuits.* ***Results.*** *Mathematical models for the study of starting modes of wound rotor induction motor allow to calculate transients and static characteristics and, on their basis, to carry out design synthesis of starting reactors, which provide the law of change of electromagnetic torque during start-up operating conditions.* ***Originality.*** *The mathematical basis of the developed algorithms is the method of solving nonlinear systems of equations by Newton method in combination with the method of continuation by parameter. The developed mathematical models and software made on their basis have high speed that allows to carry out high-reliability calculation of starting modes taking into account saturation of a magnetic circuit of the motor.* ***Practical value.*** *The developed algorithms do not require significant computing resources, have high speed, and can be used both for the design synthesis of start-control devices and control of the electric drive in real time and to predict its course.* References 25, figures 4.*
Key words **induction motor, wound rotor, reactor start, mathematical model, static characteristics, transients, magnetic core saturation**.

Розроблено математичні моделі, методи і алгоритми аналізу пускових режимів і статичних характеристик асинхронних двигунів з фазним ротором. В розроблених алгоритмах математична модель двигуна подана диференціальними рівняннями, складеними для електричних контурів в системі ортогональних координат. Математичною основою розроблених алгоритмів розрахунку статичних характеристик є розв'язування нелінійних систем скінченних рівнянь електричної рівноваги методом Ньютона в поєднанні з методом продовження по параметру, а пускових режимів – числове інтегрування нелінійних систем диференціальних рівнянь електромеханічної рівноваги. Елементами матриці Якобі в розроблених алгоритмах є власні і взаємні диференціальні індуктивності електричних контурів, які визначаються на основі характеристик намагнічування основним магнітним потоком, а також потоками розсіювання контурів ротора і статора, що дає змогу здійснювати розрахунок з урахуванням насичення магнітопроводу двигуна. Розроблені програми і алгоритми мають високу швидкодію і дають змогу здійснювати проектний синтез пускових активних і реактивних опорів в колі ротора з метою забезпечення закону зміни електромагнітного моменту під час пуску, який відповідає роботі системи електроприводу в заданих технологічних умовах, а також здійснювати мікропроцесорне керування в динамічних режимах. Бібл. 25, рис. 4.*
Ключові слова: **асинхронний двигун, фазний ротор, реакторний пуск, математична модель, статичні характеристики, перехідні процеси, насичення магнітопроводу.**

Introduction. The most common among electric drives in industry, agriculture and household applications are squirrel-cage induction motors (IMs). Their main disadvantages are significant starting currents, which exceed 5-7 and even more times the nominal values, as well as a relatively small driving torque, which is insufficient for many technological processes. In addition, the direct connection of the motor to the network is accompanied by significant pulsations of the electromagnetic torque [1]. The development of start and regulating systems of automated control, as well as frequency converters has significantly expanded the scope of use of squirrel-cage IMs. However, the problem of starting the mechanisms of electric drives with difficult starting conditions needs further development and improvement.

In recent years, there has been a significant increase in interest in wound-rotor induction motors due to the expansion of the range of electric drives based on them, and therefore there is a need to study them, including mathematical modeling methods.

The starting properties of the induction electric drive can be significantly improved by using wound-rotor IMs [2], which, despite the higher cost compared with squirrel-cage rotor IMs, thanks to modern electronic control systems received a new impetus in development and application for lifting and transport mechanisms, conveyors, winches and other mechanisms with difficult starting conditions. Increasing the starting torque is achieved by connecting to the rotor winding of various

devices. Rheostat is most often used for this purpose, the active resistance of which can be changed discretely by switching on or shorting its sections [2]. Due to the increase in the active resistance of the rotor phases, the critical value of the electromagnetic torque does not change, and the critical slip increases with increasing resistance of the rheostat. Therefore, we can achieve the maximum value of the electromagnetic torque when sliding $s = 1.0$. This property is used to start the motor at torque of resistance exceeding the rated value of the starting torque. This reduces the starting current and increases $\cos\varphi$ [1, 2]. However, unsuccessful selection of the value of the additional resistance in the rotor circuit leads to a decrease in the starting electromagnetic torque. By changing the additional resistance in the rotor circuit, we can adjust the speed of rotation of the rotor down from the main one, although this method of regulation is uneconomical.

To ensure smooth acceleration during start, in addition to the active resistance, the inductance of a special design is sometimes included in the rotor circuit [3–8]. The inductive element can be connected both in series to the active one (Fig. 1) and in parallel. The reactor allows to provide smooth acceleration of the electric drive with a small number of sections of the rheostat, i.e. essentially acts as an automatic current regulator in the rotor and under certain conditions can ensure constant torque of the motor during start. As a result, the rotor current decreases more slowly than if there is only active resistance. In the case of a parallel connection between a resistor and an inductive coil at the beginning of the start, when the frequency of the current in the rotor is high, the current is mainly closed through the rheostat, which provides a sufficiently large starting torque. As the frequency decreases, the inductance decreases and the current is shorted through the inductive element.
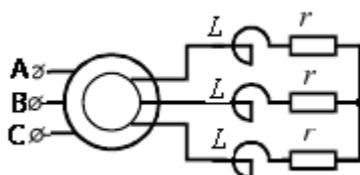

Fig. 1. Diagram of IM with a starting reactor
in the wound-rotor circuit

The choice of a rational way to connect the active and inductive elements and their parameters can be made by mathematical modeling, the reliability of the results of which is determined by the adequacy of the mathematical model. In addition, to automate the process of IM start with the help of programmable microcontrollers, it is necessary to have analysis software that has a sufficiently high speed, does not require a significant amount of computation. Therefore, the development of methods and algorithms for calculating the starting processes in wound-rotor IMs is an urgent task.

Analysis of recent research. The task of developing an algorithm for starting equipment operation requires determining the laws of change of starting currents and electromagnetic torque during start. To do this, it is necessary to have appropriate mathematical models for calculating static characteristics, as well as dynamic modes, in particular transients, taking into account the law of change of the torque of loading. They are based on mathematical models of motors which determine the accuracy of the results of mathematical modeling, as well as computational methods that serve as a mathematical basis for obtaining calculation results and on which the speed of developed software depends.

Most methods of analysis of steady and dynamic modes of operation of squirrel-cage IMs are developed using substitution circuits with constant parameters, which are mostly used in known mathematical application codes. This approach is also applied to the development of mathematical models based on wound-rotor IMs [3-14]. Although such motors do not need to take into account the displacement of currents in the rotor winding due to the absence of the skin effect, the influence of scattering fluxes on the course of processes is significant [9]. The analytical method of calculating static characteristics in the phase coordinate basis developed in [13] makes it possible to consider asymmetric and non-sinusoidal processes, but the inductive parameters are assumed to be constant.

The saturation of the magnetic cores of modern motors causes nonlinear dependencies of the flux linkages of circuits on currents, so mathematical models developed on the assumption of linearity of electromagnetic connections do not provide the ability to calculate dynamic modes with the required accuracy. The linearization of electromagnetic connections in the vicinity of the operating point [6] does not solve the problem, because in a real machine the saturation varies widely, and therefore it is a priori impossible to determine.

In recent years, there have been models that take into account the saturation by only the main magnetic flux [11-14]. This significantly increases the accuracy of the calculation, however, these models are not accurate enough for the analysis of dynamic processes [13, 16, 17], because, as noted in [9, 24] studies must take into account the effect of magnetic saturation not only on the main path of the magnetic flux, but also on the paths of scattering fluxes, the influence of which on the course of processes is decisive. In [15] an experimental procedure for determining the parameters of the machine was adopted, but it is too expensive and impossible to do at the design stage.

To form the necessary mechanical characteristics of the wound-rotor IM, programmable microcontroller starting systems are used [3, 23, 24], the programming of which requires appropriate preliminary research by methods of mathematical modeling. Advances in starter

switching, electronic processing, and microprocessor control require robust control algorithms based on appropriate controller software.

Reliable information about the course of processes during start can be obtained only with the help of highly developed mathematical models of IMs, which adequately take into account the saturation of the magnetic core. Since the methods based on the calculation of the magnetic field [20] due to bulkiness are not suitable for controlling the process of IM start in real time, the optimal in terms of accuracy and complexity are circuit methods [18, 21, 23], in which electromagnetic parameters are calculated based on characteristics of magnetization of the motor magnetic core [25]. Software developed on their basis does not require significant computing resources, allows to perform calculations in real time and uses them to automate [24] both starting and other dynamic modes.

The goal of the paper is the development of mathematical models, methods for calculating static characteristics and dynamic modes of wound-rotor induction motors with various parameters of the starting device in the rotor circuit.

Presentation of main material. An important issue in the development of mathematical models of electric drives is their complexity and speed, which is crucial in the case of their use to control the process in real time. Therefore, it is important to choose the coordinate system to describe the electromagnetic connections in the IM, which depends on both the amount of calculations and the accuracy of the calculation results.

Most of the practically important problems of mathematical modeling of processes in IMs can be solved using transformed coordinate systems, which are based on the theory of image vectors [1, 18]. The calculation algorithms described in the paper use a system of orthogonal axes x, y [25], which rotate at arbitrary speed. For symmetrical modes of operation of the IM, it has the lowest amount of calculations and a fairly high accuracy of the calculation results.

Electromagnetic processes in the wound-rotor IM in the x, y axes are described by a system of four differential equations (DEs) of electrical equilibrium

$$\begin{cases} \dfrac{d\psi_{sx}}{dt} = \omega_0\psi_{sy} - r_s i_{sx} + u_{sx}; \\[2mm] \dfrac{d\psi_{sy}}{dt} = -\omega_0\psi_{sx} - r_s i_{sy} + u_{sy}; \\[2mm] \dfrac{d\psi_{rx}}{dt} = s\omega_0\psi_{ry} - \left(r_r + r_p\right)i_{rx}; \\[2mm] \dfrac{d\psi_{ry}}{dt} = -s\omega_0\psi_{rx} - \left(r_r + r_p\right)i_{ry}, \end{cases} \quad (1)$$

where ψ_{sx}, ψ_{sy}, ψ_{rx}, ψ_{ry}, i_{sx}, i_{sy}, i_{rx}, i_{ry} are the flux linkages and currents of the converted circuits of the stator (index s) and rotor (index r); r_s, r_r are the active resistances of these circuits; r_p is the resistance of the

rheostat phase in the rotor circuit; ω_0 is the cyclic frequency of the supply voltage; s is the rotor sliding.

In equations (1), the parameters of the rotor winding are reduced to the stator winding according to the generally accepted method [19]. In addition, we will consider the image vector of the stator winding voltage $\vec{U}$ aligned with the x-axis, i.e. we take $u_{sx} = U_m$, and $u_{sy} = 0$.

To calculate the process of the IM start, it is necessary to supplement the DE system (1) with the equation of rotor dynamics:

$$\frac{ds}{dt} = -\frac{p_0}{J\omega_0}\left(\frac{3}{2}p_0\left(\psi_{sx}i_{sy} - \psi_{sy}i_{sx}\right) - M_c(t)\right), \quad (2)$$

where p_0 is the number of pole pairs of the IM; J is the reduced to the motor shaft moment of inertia of the electric drive system; M_c is the torque of loading on the motor shaft.

The DE system (1) together with (2) makes it possible to calculate the transient during the IM start. To do this, it is necessary to integrate it numerically under zero initial conditions, calculating at each step of integration the matrix of differential inductances as elements of the Jacobi matrix and the vector of flux linkages [25].

The flux linkage of each circuit consists of flux linkage with the main magnetic flux and with the scattering fluxes, and the flux linkage of the scattering of the stator circuits and the rotor circuits are mutually independent. Therefore, in order to take into account the inductance of the reactor in the rotor circuit, the equations for flux linkages of the rotor circuits can be represented as

$$\psi_{rx} = \psi_{\delta rx} + \left(L_{\sigma r} + L_p\right)\cdot i_{rx};$$
$$\psi_{ry} = \psi_{\delta ry} + \left(L_{\sigma r} + L_p\right)\cdot i_{ry},$$

where $\psi_{\delta rx}$, $\psi_{\delta ry}$ are the flux linkages of the respective rotor circuits due to the main magnetic flux; $L_{\sigma r}$ is the scattering inductance of the rotor circuits, which is determined from the magnetization characteristics of the scattering fluxes of the rotor winding, calculated on the basis of the geometry of the motor magnetic core [19]

$$\psi_{\sigma r} = \psi_{\sigma r}(i_r); \quad i_r = \sqrt{i_{rx}^2 + i_{ry}^2}, \quad (3)$$

where L_p is the inductance of the reactor in the rotor circuit, which is determined by the appropriate design formulas.

It is impossible to choose the parameters of the reactor and code the starting and regulating device based on the calculation of the transient. This problem requires the calculation and analysis of static characteristics, which can be calculated using equations (1) of electrical equilibrium. Having chosen on the basis of calculation of static characteristics resistive and inductive parameters of the starting device and the law of their change according to time dependence of the torque of loading $M_c = M_c(t)$, we carry out calculation of time dependencies of

coordinates by numerical integration of nonlinear DE system (1), (2).

Consider the algorithm for calculating static characteristics. In the steady-state mode of IM operation sliding s, the DE system of electromagnetic equilibrium (1) is reduced to a system of nonlinear algebraic equations, which in order to present the algorithm for calculating the steady-state are written in the form of the vector DE

$$\vec{Q}\left(\vec{Y}\left(\vec{I}_{xy}\right)\right) = \Omega_{xy}\vec{\Psi}_{xy} + R_{xy}\vec{I}_{xy} - \vec{U}_{xy}, \qquad (4)$$

where $\vec{\Psi}_{xy} = \left(\psi_{sx}, \psi_{sy}, \psi_{rx}, \psi_{ry}\right)^{*}$; $\vec{I}_{xy} = \left(i_{sx}, i_{sy}, i_{rx}, i_{ry}\right)^{*}$; $\vec{U}_{xy} = \left(U_m, 0, 0, 0\right)^{*}$ are the vectors of flux linkages, circuit currents and voltages applied to them;

$$R_{xy} = \begin{bmatrix} r_s & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 \\ 0 & 0 & r_r + r_p & 0 \\ 0 & 0 & 0 & r_r + r_p \end{bmatrix}$$

is the matrix of active resistances of circuits;

$$\Omega_{xy} = \begin{bmatrix} 0 & -\omega_0 & 0 & 0 \\ \omega_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -s\omega_0 \\ 0 & 0 & s\omega_0 & 0 \end{bmatrix}$$

is the auxiliary matrix in which ω_0 is the cyclic frequency of supply voltage.

Since the flux linkage vector $\vec{\Psi}_{xy}$ is determined by a set of circuit currents, unknown in the system of finite equations (4) is the current vector $\vec{I}_{xy}$, which can be used to determine the flux linkage, electromagnetic torque, etc. Since equation (4) includes the coordinates U_m, s, r_p, L_p, we can assume that the vector of currents is a function of these coordinates

$$\vec{I}_{xy} = \vec{I}_{xy}\left(\vec{U}_{xy}, s, r_p, L_p\right).$$

Equation (4) makes it possible to investigate the influence of each of these coordinates on the value of the current vector $\vec{I}_{xy}$, i.e. to calculate the multidimensional static characteristic as the dependence of the current vector components on a given coordinate. To do this, we must change this coordinate within the specified limits as a parameter, leaving the others unchanged. Obviously, here there is a problem of solving nonlinear systems of finite equations, and since system (4) is nonlinear due to the saturation of the magnetic core, it can be solved by one of the numerical methods, in particular, Newton method.

According to Newton iterative method, the (k+1)-th approximation of the vector $\vec{Y}$ is determined by the formula

$$\vec{Y}^{(k+1)} = \vec{Y}^{(k)} + \Delta\vec{Y}^{(k)}, \qquad (5)$$

where $\Delta\vec{Y}^{k}$ is the increment of the vector $\vec{Y}^{k}$, which at each step is determined from a linear system of equations

$$J\Delta\vec{Y}^{(k)} = -\vec{Q}\left(\vec{Y}^{(k)}\right), \qquad (6)$$

in which $\vec{Q}\left(\vec{Y}^{(k)}\right)$ is the value of the residual vector $\vec{Q}$ at $\vec{Y} = \vec{Y}^{(k)}$; J is the Jacobi matrix of vector function (4).

Due to the fact that the flux linkages of the IM circuits consist of flux linkages due to the main magnetic flux ψ_δ, and scattering flux linkages ψ_σ together with the flux linkage ψ_p of the reactor

$$\begin{cases} \omega_0\psi_{sx} = \omega_0\psi_{\delta x} + x_{\sigma s}i_{sx}; \\ \omega_0\psi_{sy} = \omega_0\psi_{\delta y} + x_{\sigma s}i_{sy}; \\ \omega_0\psi_{rx} = \omega_0\psi_{\delta x} + \left(x_{\sigma r} + x_p\right)i_{rx}; \\ \omega_0\psi_{ry} = \omega_0\psi_{\delta y} + \left(x_{\sigma r} + x_p\right)i_{ry}; \end{cases} \qquad (7)$$

the Jacobi matrix is determined by the formula

$$J = X_\delta + X_\sigma + R_{xy}, \qquad (8)$$

where

$$X_\delta = \begin{bmatrix} x_{sysx} & x_{sysy} & x_{syrx} & x_{syry} \\ -x_{sxsx} & -x_{sxsy} & -x_{sxrx} & -x_{sxry} \\ sx_{rysx} & sx_{rysy} & sx_{ryrx} & sx_{ryry} \\ -sx_{rysx} & -sx_{rxsy} & -sx_{rxrx} & -sx_{rxry} \end{bmatrix};$$

$$\vec{X}_\sigma = \begin{bmatrix} x_{\sigma s} & 0 & 0 & 0 \\ 0 & -x_{\sigma s} & 0 & 0 \\ 0 & 0 & s\left(x_{\sigma r} + x_{\sigma p}\right) & 0 \\ 0 & 0 & 0 & -s\left(x_{\sigma r} + x_{\sigma p}\right) \end{bmatrix}.$$

As can be seen from (8), the elements of the Jacobi matrix are the own and mutual differential inductances of the IM circuits. They are determined according to [25]. In addition, to calculate the electromagnetic torque, it is necessary to determine the flux linkage of the circuits in accordance with the selected coordinate system.

Newton iterative method has quadratic convergence, but requires an initial approximation, which lies around attraction. The following algorithm is used to determine it.

Given the value of sliding $s = 1.0$ and the parameters r_p, L_p equal to zero, we increase in 5–10 steps in proportion to the parameter ε ($0 < \varepsilon \leq 0$) from zero to the nominal value of the applied voltage $U = \varepsilon U_m$. This makes it possible to ensure the convergence of the iterative process at every step. The resulting value of the components of the vector $\vec{I}_{xy}$ serves as the initial conditions for calculating the static characteristics. Given a number of sliding values s of the rotor, we can obtain a multidimensional static characteristic in the form of the dependence of the coordinates on the sliding. However, the calculation of any static characteristic can be done by a differential method. To do this, we differentiate equation (4) by one of the coordinates ($\lambda = s$, r_p, L_p) as a

parameter of the required characteristic. As a result, we obtain the DE of the argument λ

$$J \frac{d\vec{I}_{xy}}{d\lambda} = \frac{\partial \vec{Q}}{\partial \lambda}, \qquad (9)$$

in which the Jacobi matrix is the same as in (6).

Equation (9) for different independent coordinates of the static characteristic differs only in the vector of the right parts. In particular, for the coordinates s, r_p, x_p they have the form

$$\frac{\partial \vec{Q}}{\partial s} = \begin{bmatrix} 0 \\ 0 \\ \omega_0 \psi_{ry} \\ -\omega_0 \psi_{rx} \end{bmatrix}; \quad \frac{\partial \vec{Q}}{\partial r_p} = \begin{bmatrix} 0 \\ 0 \\ -i_{rx} \\ -i_{ry} \end{bmatrix}; \quad \frac{\partial \vec{Q}}{\partial x_p} = \begin{bmatrix} 0 \\ 0 \\ -i_{rx} \\ -i_{ry} \end{bmatrix}.$$

As a result of integrating the nonlinear DE system (9) by one of the numerical methods on s we obtain a multidimensional characteristic in the form of dependencies of the set of coordinates of the vector $\vec{I}_{xy}$ on the chosen independent coordinate, using which we obtain dependencies of flux linkages, electromagnetic torque and so on.

Figures 2–4 show examples of calculation of the static characteristics of the wound-rotor IM ($P_N = 250$ kW, $U = 380$ V, $I = 263$ A, $n_N = 1000$ rpm).
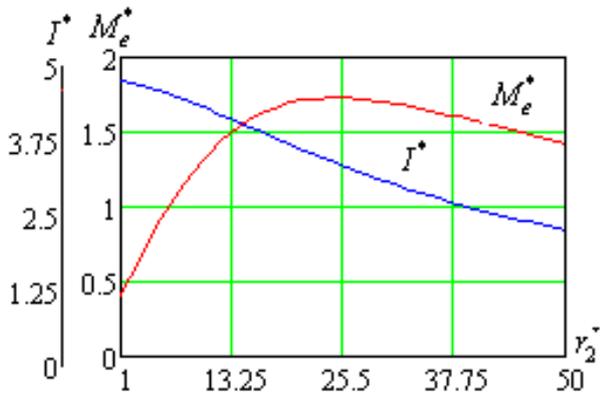

Fig. 2. Dependencies of current (I^*) and electromagnetic torque (M_e^*) in relative units on the active resistance in the rotor circuit when sliding $s = 1.0$
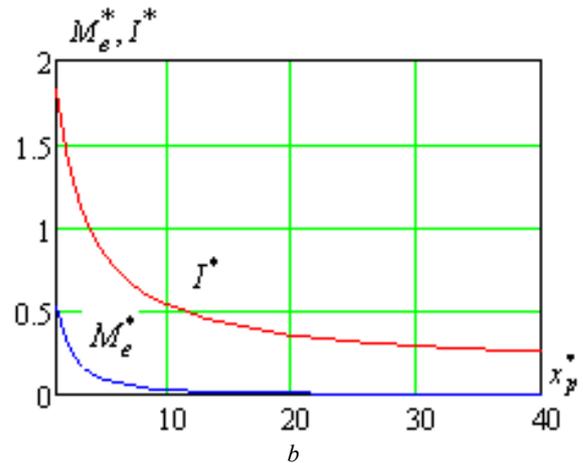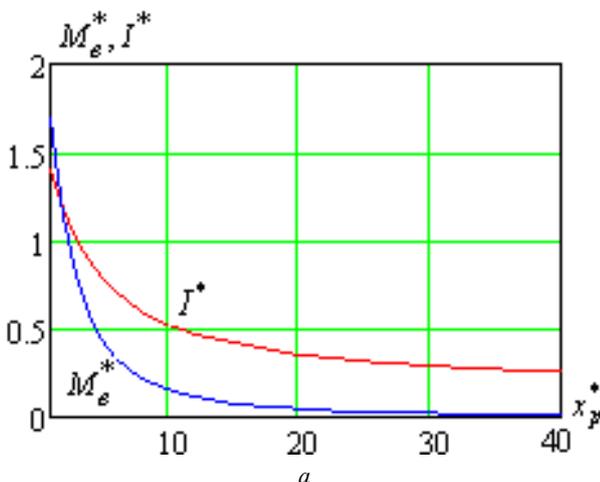

Fig. 3. Dependencies of relative values of current (I^*) and electromagnetic torque (M_e^*) at $s = 1.0$ on relative value $x_p = x_r / x_{2\sigma}'$ of inductance in the rotor circuit and different values of the multiplicity of the active reactance of the reactor:
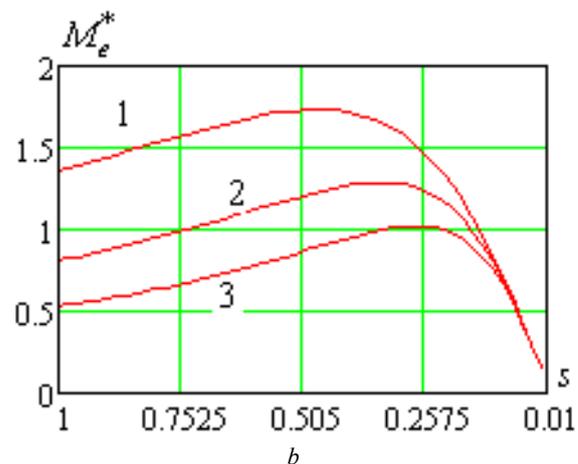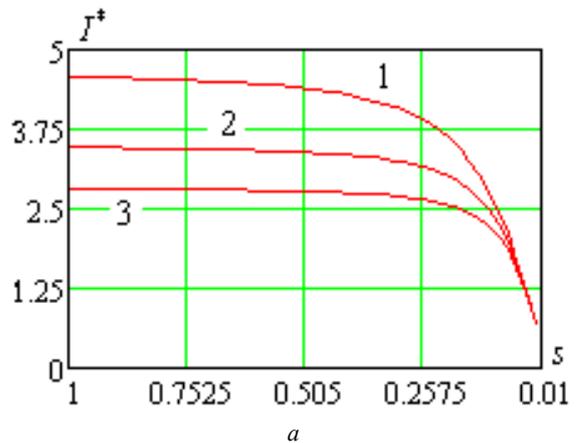a) – 1.0; b) – 10.0

Fig. 4. Static starting characteristics with three values of inductance in the rotor circuit and two relative values of active resistances:
a) – 3.9; b) – 11.7

Presented curves serve only as an illustration of the possibility of developed calculation algorithms. It is obvious that, having chosen one of the parameters of the starting and regulating device, it is necessary to calculate

the mechanical characteristic, and each value of active resistance corresponds to its mechanical characteristic, which in turn depends on the inductance of the reactor. Since the resistance of the inductive coil depends on the frequency of the current in the rotor, which is variable during start, its correctly selected parameters have a positive effect on the value of the starting current, automatically reducing its value.

Conclusions.

Unlike squirrel-cage induction motors, less attention is paid to wound-rotor motors in the technical literature, although the wound rotor allows for more diverse mechanical characteristics, which is important for electric drives with difficult starting conditions.

The calculation methods developed in the paper allow utilization of mathematical modeling methods to analyze static starting characteristics and transients of wound-rotor IMs with different laws of regulation of starting device parameters in rotor winding in order to provide the necessary law of electromagnetic torque change.

The calculation code is based on a mathematical model of the IM, which uses the real characteristics of the magnetization by the main magnetic flux and the scattering fluxes of the stator and rotor windings, which allows to adequately take into account the saturation of the magnetic core which ensures the accuracy of the calculation results.

The described methods of calculation of modes and characteristics in orthogonal coordinate axes x, y allow to perform modeling with a minimum amount of calculations and, accordingly, the cost of CPU time, which allows to use them to control the electric drive system in dynamic modes in real time.

Conflict of interest. The authors of the paper state that there is no conflict of interest.

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