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SYNTHESIS OF THE DIGITAL REGULATOR OF THE MAIN CONTOUR OF THE THREE-CIRCUIT SYSTEM OF THE LINEAR ELECTRIC DRIVE OF THE WORKING BODY OF THE MECHANISM OF ONBOARD AVIATION EQUIPMENT

Goal *The purpose of the article is to further develop analytical methods for calculating and synthesizing power electronics systems with deep pulse width modulation (PWM). A three-circuit linear electric drive system for positioning the working body of the mechanism of onboard aircraft equipment, in which the linear electric motor is controlled from a pulse width converter (PWC), is considered. The power converter is included in the current loop. It has a noticeable effect on the level of current ripple, travel speed and positioning accuracy of the operating mechanism of a linear electric drive.* ***Methodology***. *To analyze the processes in the current loop, a discrete transfer function of a pulse-width converter for PWM in the final zone and «in the large» is obtained on the basis of the statistical linearization of the modulation characteristics of the multi-loop PWM model. The modulation characteristic of each circuit of the model is obtained as a result of the Fourier series expansion in Walsh functions of the output voltage of the PWM during the PWM process. Statistical linearization of modulation characteristics is performed based on Hermite polynomials.* ***Results.*** *During the analysis, discrete transfer functions of closed current loops, velocity and open loop position were obtained, for which a digital controller was synthesized in the form of a recursive filter.* ***Originality.*** *The parameters of the regulator links are found, which make it possible to complete the transient process in four PWC switching periods with an overshoot of no more than 6 %. The analysis of the speed-optimized positioning process of a linear electric drive based on the LED AT 605TU motor is carried out.* ***Practical significance***. *The purpose of the analysis was to establish the relationship between the switching period of the PWM and the value of the uncompensated constant, at which the pulsations of the positioning process are minimal while ensuring the minimum overshoot and maximum speed. It was found that the specified requirements are satisfied by the ratio between the switching period, PWC and uncompensated constant in the range of one or two.* References 12, figures 4.

*Key words****:*** **linear electric drive, discrete transfer function, pulse width modulation, positioning error, optimal regulator.**

В триконтурній системі лінійного електроприводу, робочий орган якого реалізує поступальне переміщення при виконанні команди бортового комп'ютера літального апарату, врахований вплив пульсацій широтно-імпульсного перетворювача постійної напруги на процес позиціонування. З умови кінцевої тривалості процесу позиціонування синтезовано цифровий регулятор головного контуру системи і запропонована його реалізація у вигляді рекурсивного цифрового фільтру. Бібл. 12, рис. 4.

Ключові слова: **лінійний електропривод, дискретна передавальна функція, широтно-імпульсна модуляція, помилка позиціонування, оптимальний регулятор.**

В трехконтурной системе линейного электропривода, рабочий орган которого реализует поступательное перемещение при выполнении команды бортового компьютера летательного аппарата, учтено влияние пульсаций широтно-импульсного преобразователя постоянного напряжения на процесс позиционирования. Из условия конечной длительности процесса позиционирования синтезирован цифровой регулятор главного контура системы и предложена его реализация в виде рекурсивного цифрового фильтра. Библ. 12, рис. 4.

Ключевые слова: **линейный электропривод, дискретная передаточная функция, широтно-импульсная модуляция, ошибка позиционирования, оптимальный регулятор.**

Introduction. Problem definition in general. In the context of the problem of creating an electric aircraft [1] there is an important scientific and practical task – the replacement of onboard hydraulic and pneumatic drives that control the linear movement of the working bodies of the respective mechanisms, with their electrical counterparts based on linear or stepper motors. The main requirement for them is to ensure accurate positioning after the completion of the translational movement without hesitation.

The accuracy characteristics of a linear electric drive are affected by load changes and control discreteness, which causes ripples of the motor supply voltage.

At the stage of designing accurate positioning systems there is a problem of taking into account the influence of these factors on the dynamic characteristics: speed, over-regulation, stability, with their subsequent optimization.

Analysis of basic research and publications and problem definition. In [2] the general principles of construction of systems of the automated electric drive on the basis of usual and linear electric motors of a direct current are revealed. High requirements for the accuracy characteristics of linear electric drives for aviation, space technology, precise technological processes, due to the presence of a power converter with pulse width modulation (PWM) of the output voltage, complicate the procedure of analysis and synthesis of their dynamic characteristics with a given quality.

Therefore, in the known works devoted to the development of systems for these areas, the main attention is paid to their practical design based on the analysis of mechanical parts operation modes [3], programming of control controllers based on fuzzy logic [4-6] using experimental data. For example, the programming of the training controller for the positioning system of the machine feed with numerical control is performed on the basis of experimental amplitude-frequency characteristics [7]. Electronic modelling is widely used to study processes in linear electric drives with different types of loads [8-10].

It can be noted that the methods of theoretical analysis and synthesis of systems with deep PWM have not yet received their further development. They are based on the account of only a constant component, or taking into account the discreteness of regulation «in small», when systems with PWM are equivalent to systems with amplitude pulse modulation.

These methods do not allow to take into account the influence of pulsations of the supply voltage of a linear motor, which are a consequence of deep PWM, in the synthesis of dynamic characteristics of the positioning system with a given quality. There is an obvious problem that needs to be solved.

The goal of the work is to synthesize the digital regulator of the main circuit of the linear electric drive of the working body of the onboard aviation equipment mechanism, which provides a transient process of finite duration taking into account the effect of pulsation of the converter with PWM, which allows to increase positioning accuracy.

Presentation of the main material. Diagram shown in Fig. 1, consists of three circuits: current, speed, position.

The current circuit includes a current regulator, a link of current formation and a link of uncompensated time constant, which is determined by time constants of filters for smoothing current ripples. Their transfer functions, respectively:

$$K_{CC}(p) = K_{CS}\frac{1+p\cdot T_E}{p\cdot T_C};$$

$$K_{FC}(p) = \frac{1}{R_y(1+p\cdot T_E)};$$

$$K_{CT}(p) = \frac{1}{1+p\cdot\sigma},$$

where K_{CS} is the transfer coefficient of the proportional component; T_C is the integration constant of the current loop; T_E is the electric constant of the motor armature; R_y is the active resistance of the armature; σ is the uncompensated constant.

The source of current ripple is a pulse-width converter (PWC), which is powered by the onboard network of the aircraft. In Fig. 1 PWC is represented by a set of pulse element (PE) with gain K_{IE}, which is equal to one, and a forming element (FE), CS is a control system with gain K_{CK}. The influence of pulsations on the quality of the transient process of the positioning system in general can be taken into account using the transfer function of the PWC, which is a link in the current circuit.

In [11], as a result of the analysis of the voltage spectrum at the output of the PWC in the basis of orthogonal discrete Walsh functions, modulation characteristics were obtained that reflect the dependencies of the amplitudes of the spectral components on the duty cycle of regulation.

The transfer function of the PWC is found in the form of a vector. Its dimension is determined by the number of Walsh functions taken into account, which depends on the cutoff frequency of the system and the approximation error.

Component of the *i*-th vector of the transfer function of the link with PWM (forming element):

$$K^i{}_{pws}(q) = mK_i^{Wal}\frac{\exp\left(-\frac{i}{m}q\right)-\exp\left(-\frac{i+1}{m}\right)}{q}, \quad (1)$$

where m is the number of approximating functions; K_i^{Wal} = 1, if the pulse of the unit amplitude of a rectangular shape is modulated; $q = p\cdot T$, where T is the switching period of the PWC; i = 0, 1, 2,...m–1 is the number of components of the vector of the transfer function with the range of change of duty cycle $i/m \le \gamma \le i+1/m$.

According to the results of statistical linearization of modulation characteristics based on Hermite polynomials, in [11] the transfer function of the PWM link for $0 \le \gamma \le 1$ was obtained, which, taking into account the four Walsh functions, has the following form:

$$K_{pws}(q) = \frac{K_{1C} - K_{kC}\sum_{k=2}^{5} e^{-0,25q(k-1)}}{q}, \quad (2)$$

where K_{1C} = 1.086; K_{2C} = 0.114; K_{3C} = 0.280; K_{4C} = 0.246; K_{5C} = 0.446 are the statistical linearization coefficients that correspond to the PWM of the pulse of a rectangular shape of unit amplitude on a unit period.

From (2) it is seen that due to the statistical linearization of the four modulation characteristics, the PWM «in large» is replaced by the equivalent amplitude-pulse modulation of the four-stage pulse. The amplitudes of the stages are determined by the corresponding coefficients of statistical linearization.

The speed circuit contains the link of formation of counter-EMF with transfer function

$$K_{FE}(p) = \frac{R}{p\cdot T_M},$$

where T_M is the electromechanical constant; and also the speed formation link with transmission factor $1/K_E$, where K_E is the coefficient of counter-EMF of the motor armature.

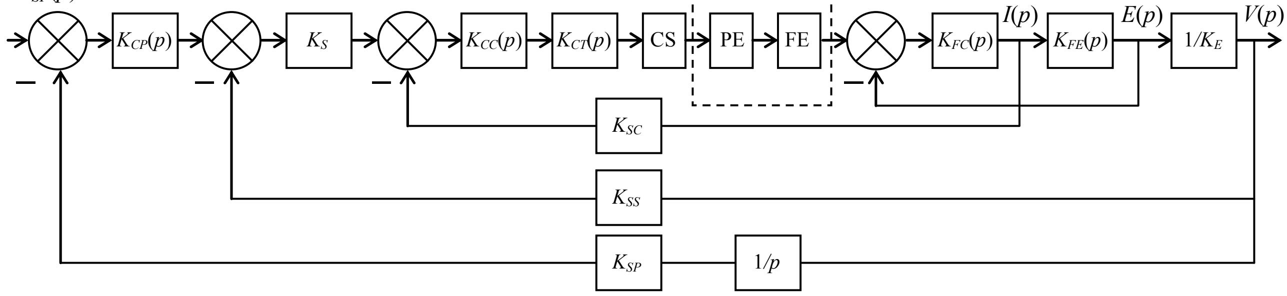

Fig. 1. Block diagram

The system circuits include sensors of: K_{SC} – current, K_{SS} – speed, K_{SP} – position, as well as the proportional regulator of the speed circuit with a gain K_S. Current and speed regulators are set to the modular optimum.

The structure and parameters of the digital controller of the main circuit (position) are subsequently obtained as a result of its optimization by the criterion of speed taking into account the ripples of the PWC.

Discrete transfer functions of positioning system circuits. Let's turn the bloc diagram (Fig. 1) into the diagram (Fig. 2) in which feedback on counter-EMF of the motor is not taken into account since $T_M >> T_C$ [11].

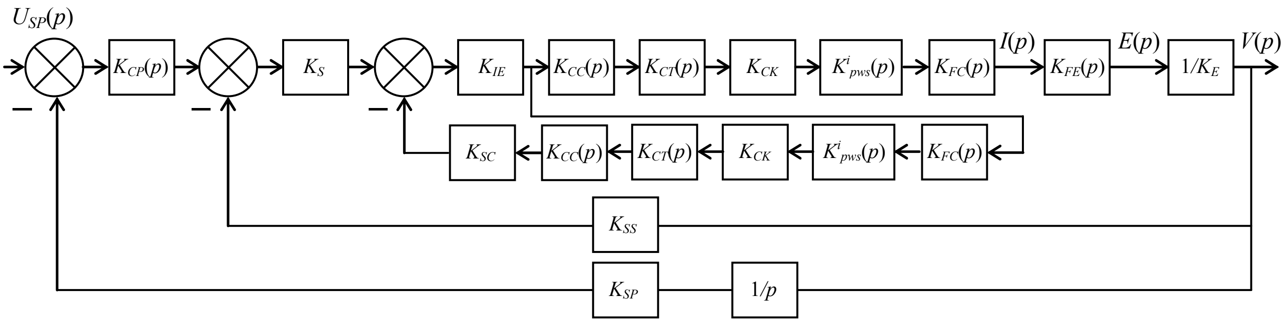

Fig. 2. Transformed diagram

a) Current circuit. Assume that the range of changes in the duty cycle of the regulation taking into account the four Walsh functions (m = 4) is in the zone i = 0, i.e. $0 \le \gamma \le 0.25$. Then, taking into account (1), the transfer function of the feedback circuit of the current loop is a modified z-transform:

$$W^*_{fb}(z,\varepsilon) = Z_M\left\{K_{sc}K_{cs}K_{CK}\frac{1+pT_E}{pT_C}\times\right.$$
$$\left.\times\frac{4\left(1-e^{-0,25pT}\right)}{p\left(1+p\,\sigma\right)}\,\frac{1}{R_y\left(1+pT_E\right)}\right\}.$$

As a result of the modified z-transform with the replacement of $p = q/T$, we obtain:

$$W^*_{fb}(z,\varepsilon) = K_{fb}\frac{F^*_{1c}(z,\varepsilon)}{\left(z-1\right)\left(z-e^{-\beta}\right)}, \tag{3}$$

for range $0 \le \varepsilon \le 0.25$, where $K_{fb} = \dfrac{4K_{sc}K_{CK}T^2K_{cs}}{R_yT_C\sigma}$,

$$F^*_{1c}(z,\varepsilon) = A_1\left[\varepsilon\left(z-1\right)+0,25\right]\left(z-e^{-\beta}\right)+$$
$$+\left(z-1\right)\left[A_2\left(z-e^{-\beta}\right)+A_3e^{-\beta\varepsilon}\left(z-e^{-0,75\beta}\right)\right],$$

$$A_1 = \frac{1}{\beta},\quad A_2 = -\frac{1}{\beta^2},\quad A_3 = \frac{1}{\beta^2},\quad \beta = \frac{T}{\sigma}.$$

For range $0.25 \le \varepsilon \le 1$

$$W^*_{fb}(z,\varepsilon) = K_{fb}\frac{F^*_{2c}(z,\varepsilon)}{\left(z-1\right)\left(z-e^{-\beta}\right)}, \tag{4}$$

where

$$F^*_{2c}(z,\varepsilon)=z\left[0,25A_1\left(z-e^{-\beta}\right)+A_3e^{-\beta\varepsilon}\left(z-e^{0,25\beta}\right)\left(z-1\right)\right].$$

The left value of (4) is equal to the right value of the transfer function (3), i.e.:

$$W^*_{fb}(z,-0) = z^{-1}\lim_{\varepsilon\to 1}W^*_{fb}(z,\varepsilon) = W^*_{fb}(z,0) =$$
$$= K_{fb}\frac{0,25A_1\left(z-e^{-\beta}\right)+A_3e^{-\beta}\left(1-e^{0,25\beta}\right)\left(z-1\right)}{\left(z-1\right)\left(z-e^{-\beta}\right)}. \tag{5}$$

This indicates that at the time of quantization, the transfer function does not contain jumps.

The transfer function of a closed current circuit for the diagram (Fig. 2)

$$W^*_{fbc}(z,0) = \frac{1}{1+W^*_{fb}(z,0)},$$

Taking into account (5) we have:

$$W^*_{fbc}(z,0)=$$
$$=\frac{\left(z-1\right)\left(z-e^{-\beta}\right)}{\left(z-1\right)\left(z-e^{-\beta}\right)+K_{fb}\left[0,25A_1\left(z-e^{-\beta}\right)+A_3e^{-\beta}\left(1-e^{0,25\beta}\right)\left(z-1\right)\right]}. \tag{6}$$

b) Speed circuit. The transfer function of the part of the open speed circuit, which is obtained as a result of the transformation of the diagram (Fig. 1) into the diagram (Fig. 2), is obtained in the process of the following modified z-transform:

$$W^*_{cs}(z,\varepsilon) = Z_M\left\{K_{cs}\frac{1+pT_E}{pT_C}\cdot\frac{K_{CK}}{1+p\,\sigma}\times\right.$$
$$\left.\times\frac{4\left(1-e^{-0,25pT}\right)}{pK_ER_y\left(1+pT_E\right)}\,\frac{R_y}{pT_M}\right\}.$$

Let's make a replacement $p = q/T$ and obtain:

$$W^*_{cs}(z,\varepsilon) = Z_M\left\{K_{0s}\left(\frac{B_3}{q^3}+\frac{B_2}{q^2}+\frac{B_1}{q}+\frac{B_0}{q+\beta}\right)\times\right.$$
$$\left.\times\left(1-e^{-0,25q}\right)\right\},$$

where

$$K_{0s}=\frac{4K_{cs}K_{CK}T^3}{T_CK_ET_M\sigma},\ B_1=\frac{1}{\beta^3},\ B_2=-\frac{1}{\beta^2},\ B_3=\frac{1}{\beta},\ B_0=-\frac{1}{\beta^3}.$$

As a result of the modified z-transform for range $0 \le \varepsilon \le 0.25$ we have:

$$W^*_{cs1}(z,\varepsilon) = K_{0s}\frac{F^*_{ps1}(z,\varepsilon)}{2(z-1)^2\left(z-e^{-\beta}\right)}, \qquad (7)$$

and for range $0.25 \le \varepsilon \le 1$ we have:

$$W^*_{cs2}(z,\varepsilon) = K_{0s}\frac{F^*_{ps2}(z,\varepsilon)}{2(z-1)^2\left(z-e^{-\beta}\right)}, \qquad (8)$$

where

$$F^*_{ps1}(z,\varepsilon) = B_3\left[z^2\varepsilon^2 + z\left(0,44+0,5\varepsilon-2\varepsilon^2\right)-0,94\right]\times$$
$$\times\left(z-e^{-\beta}\right)+2B_2\left[z\varepsilon-\varepsilon-0,75\right](z-1)\left(z-e^{-\beta}\right)+$$
$$+2B_0e^{-\beta\varepsilon}\left(z-e^{-0,75\beta}\right)(z-1)^2+2B_1(z-1)^2\left(z-e^{-\beta}\right);$$

$$F^*_{ps2}(z,\varepsilon) = B_3\left[0,5z+z(z-1)(0,5\varepsilon-0,0625)\right]\times$$
$$\times\left(z-e^{-0,25\beta}\right)+B_2(z-1)\left(z-e^{-\beta}\right)z\cdot 0,5+$$
$$+2B_0ze^{-\beta\varepsilon}\left(1-e^{0,25\beta}\right)(z-1)^2.$$

From the transfer function (8) we have its left value:

$$W^*_{sc2}(z,-0) = z^{-1}\lim_{\varepsilon\to 1}W^*_{sc2}(z,\varepsilon) =$$
$$= K_{0s}\left[\frac{B_3(0,44z+0,22)\left(z-e^{-\beta}\right)}{2(z-1)^2\left(z-e^{-\beta}\right)}+\right. \qquad (9)$$
$$\left.+\frac{B_2(z-1)\left(z-e^{-\beta}\right)+2B_0e^{-\beta}\left(1-e^{0,25\beta}\right)(z-1)^2}{2(z-1)^2\left(z-e^{-\beta}\right)}\right].$$

Open speed circuit transfer function for $0 \le \varepsilon \le 0.25$:

$$W^*_{ss1}(z,\varepsilon) = W^*_{fbc}(z,0)\cdot W^*_{cs1}(z,\varepsilon),$$

and for $0.25 \le \varepsilon \le 1$:

$$W^*_{ss2}(z,\varepsilon) = W^*_{fbc}(z,0)\cdot W^*_{cs2}(z,\varepsilon).$$

Closed circuit speed transfer function for $0 \le \varepsilon \le 0.25$:

$$W^*_{fbs1}(z,\varepsilon) = \frac{W^*_{ss1}(z,\varepsilon)}{1+W^*_{fbc}(z,0)\cdot W^*_{cs2}(z,-0)},$$

and for $0.25 \le \varepsilon \le 1$:

$$W^*_{fbs2}(z,\varepsilon) = \frac{W^*_{ss2}(z,\varepsilon)}{1+W^*_{fbc}(z,0)\cdot W^*_{cs2}(z,-0)}.$$

c) Position circuit. The transfer function of the open position circuit:

$$W^*_{fbp1}(z,\varepsilon) = W^*_{fbs1}(z,\varepsilon)\cdot\frac{K_{sp}z}{z-1}, \quad 0 \le \varepsilon \le 0.25;$$

$$W^*_{fbp2}(z,\varepsilon) = W^*_{fbs2}(z,\varepsilon)\cdot\frac{K_{sp}z}{z-1}, \quad 0.25 \le \varepsilon \le 1.$$

Taking into account (6)-(9) after the corresponding transformations we have the transfer functions of the position circuit in the open state:

$$W^*_{fbp1}(z,\varepsilon) = K_0\times$$
$$\times\frac{z^4a_{13}(\varepsilon)+z^3a_{12}(\varepsilon)+z^2a_{11}(\varepsilon)+za_{10}(\varepsilon)}{(z-1)\left[z^3b_3(1)+z^2b_2(1)+zb_1(1)+b_0(1)\right]}, \qquad (10)$$

for $0 \le \varepsilon \le 0.25$;

$$W^*_{fbp2}(z,\varepsilon) = K_0\times$$
$$\times\frac{z^4a_{23}(\varepsilon)+z^3a_{22}(\varepsilon)+z^2a_{21}(\varepsilon)+za_{20}(\varepsilon)}{(z-1)\left[z^3b_3(1)+z^2b_2(1)+zb_1(1)+b_0(1)\right]}, \qquad (11)$$

where $0,25 \le \varepsilon \le 1$,

$$K_0 = K_{0i}K_{sp};$$

$$a_{13}(\varepsilon) = B_3\varepsilon^2+2B_2\varepsilon+2B_1+2B_0e^{-\beta\varepsilon};$$

$$a_{12}(\varepsilon) = B_3\left(0,44+0,5\varepsilon-2\varepsilon^2-\varepsilon^2\cdot e^{-\beta}\right)+$$
$$+2B_2\left[0,25-\varepsilon-\varepsilon\left(1+e^{-\beta}\right)\right]-2B_1\left(2+e^{-\beta}\right)-$$
$$-2B_0\left(2+e^{-0,75\beta}\right)e^{-\beta\varepsilon};$$

$$a_{11}(\varepsilon) = B_3\left[0,06+\varepsilon^2-0,5\varepsilon-e^{-\beta}\left(0,44+0,5\varepsilon-2\varepsilon^2\right)\right]+$$
$$+2B_2\left[\varepsilon\left(1+e^{-\beta}\right)-0,25\left(1+e^{-\beta}\right)+\varepsilon\cdot e^{-\beta}\right]+$$
$$+2B_1\left(1+2e^{-\beta}\right)+2B_0\left(1+2e^{-0,75\beta}\right)e^{-\beta\varepsilon};$$

$$a_{10}(\varepsilon) = B_3\left[-e^{-\beta}\left(0,06+\varepsilon^2-0,5\varepsilon\right)\right]+$$
$$+2B_2\left(-\varepsilon e^{-\beta}+0,25e^{-\beta}\right)-2B_1e^{-\beta}-2B_0e^{-\beta(\varepsilon+0,75)};$$

$$b_3(1) = 2\left(1+K_{0c}A_3e^{-\beta}\right);$$

$$b_2(1) = K_{0c}\left(0,5A_1-6A_3e^{-\beta}\right)+K_{0s}\left(0,45B_3+B_2\right)-2e^{-\beta};$$

$$b_1(1) = 2\left(1+2e^{-\beta}\right)+$$
$$+K_{0s}\left[B_3\left(0,05-0,45e^{-\beta}\right)-B_2\left(1+e^{-\beta}\right)\right]+$$
$$+K_{0c}\left[6A_3e^{-\beta}-0,5A_1\left(1+e^{-\beta}\right)\right];$$

$$b_0(1) = K_{0c}\left(0,5A_1e^{-\beta}-2A_3e^{-\beta}\right)-$$
$$-K_{0s}\left(0,05B_3e^{-\beta}-B_2e^{-\beta}\right)-2e^{-\beta};$$

$$a_{23}(\varepsilon) = B_3(0,5\varepsilon-0,22)+2B_0e^{-\beta\varepsilon}\left(1-e^{-0,25\beta}\right)+B_2\cdot 0,5;$$

$$a_{22}(\varepsilon) = B_3\left[0,0625-0,5\left(1+e^{-\beta}\right)\varepsilon+0,0625e^{-\beta}\right]-$$
$$-B_2\left(1+e^{-\beta}\right)-4B_0e^{-\beta\varepsilon}\left(1-e^{0,25\beta}\right);$$

$$a_{21}(\varepsilon) = 2B_0e^{-\beta\varepsilon}\left(1-e^{-0,25\beta}\right)+$$
$$+0,5B_2e^{-\beta}-B_3(0,5625-0,5\varepsilon);$$

$$a_{20}(\varepsilon) = 0.$$

Synthesis of digital position circuit controller. The purpose of the synthesis is to ensure the final duration of the transient process in the positioning circuit with minimal over-regulation. To do this, we use the most general, the second polynomial equation of synthesis [11].

We present the transfer functions of the open position circuit as follows:

$$W^*_{fbp1}(z,\varepsilon)=\frac{P^*_1(z,\varepsilon)}{(z-1)Q^*_1(z)} \text{ for } 0\le\varepsilon\le 0.25;$$

$$W^*_{fbp2}(z,\varepsilon)=\frac{P^*_2(z,\varepsilon)}{(z-1)Q^*_1(z)} \text{ for } 0.25\le\varepsilon\le 1,$$

where

$$P^*_1(z,\varepsilon)= =K_0\left[a_{13}(\varepsilon)z^4+a_{12}(\varepsilon)z^3+a_{11}(\varepsilon)z^2+a_{10}(\varepsilon)z\right], \quad (12)$$

$$P^*_2(z,\varepsilon)=K_0\left[a_{23}(\varepsilon)z^4+a_{22}(\varepsilon)z^3+a_{21}(\varepsilon)z^2\right], \quad (13)$$

$$Q^*_1(z)=z^3b_3(1)+z^2b_2(1)+zb_1(1)+b_0(1). \quad (14)$$

Minimum duration of the transient process:

$$S_{\min}=l_Q+r-r_0,$$

where l_Q are the degrees of denominators (10), (11), r_0 = 1 is the own astatism of the position circuit, r = 1 is its astatism according to the results of the synthesis procedure. Therefore, $S_{\min}$ = 4.

For a stable continuous part of the positioning circuit at ranges 0 ≤ ε ≤ 0.25 and 0.25 ≤ ε ≤ 1 the following relation is true:

$$P^*(z,0)\cdot M^*(z,0)+(z-1)^r N^*(z,0)=z^4, \quad (15)$$

where $M^*(z,0)=C_0$ is the polynomial of the degree $l_M = r - 1 = 0$,

$$N^*(z,0)=d_4z^4+d_3z^3+d_2z^2+d_1z+d_0 \quad (16)$$

is the polynomial of the degree $l_N \ge l_P$, where l_p = 4 is the degree of $P_1^*,(z,e)$.

As a result (15) takes the form:

$$C_0K_0\left[a_{13}(0)z^4+a_{12}(0)z^3+a_{11}(0)z^2+a_{10}(0)z\right]+ +(z-1)\left(d_4z^4+d_3z^3+d_2z^2+d_1z+d_0\right)=z^4. \quad (17)$$

We equate the coefficients of the same degrees z, and from (17) obtain:

$$C_0=\frac{1}{K_0A_n(0)},$$

where

$$A_n(0)=\sum_0^3 a_{1n}(0), \quad d_0=0, \quad d_1=\frac{a_{10}(0)}{A_n(0)},$$

$$d_2=\frac{a_{10}(0)+a_{11}(0)}{A_n(0)}, \quad d_3=\frac{\sum_0^2 a_{1n}(0)}{A_n(0)}, \quad d_4=0.$$

As a result we have:

$$N^*(z,0)=z^3\frac{\sum_0^2 a_{1n}(0)}{A_n(0)}+z^2\frac{\sum_0^1 a_{1n}(0)}{A_n(0)}+z\frac{a_{10}(0)}{A_n(0)}. \quad (18)$$

The transfer function of the series-activated optimal controller:

$$K^*_{ocp}(z,0)=\frac{Q^*_1(z)\cdot M^*(z,0)}{(z-1)^{r-r_0}\cdot N^*(z,0)}.$$

Taking into account (14), (16), (18) we obtain:

$$K_{ocp}{}^*(z,0)=\frac{z^3b_3(1)+z^2b_2(1)+zb_1(1)+b_0(1)}{K_0\left[z^3\sum_0^2 a_{1n}(0)+z^2\sum_0^1 a_{1n}(0)+za_{10}(0)\right]}. \quad (19)$$

Divide the numerator and denominator of (19) by $K_0\sum_0^2 a_{1n}(0)$ and obtain:

$$K_{ocp}{}^*(z,0)=\frac{z^{-1}\mu_0+\mu_1z^{-2}+\mu_3z^{-3}+\mu_4z^{-4}}{1+\alpha_1z^{-1}+\alpha_2z^{-2}}= =\frac{\Delta U_{OUT}{}^*[z,0]}{\Delta U_{IN}{}^*[z,0]}, \quad (20)$$

where

$$\mu_0=\frac{b_3(1)}{K_0\sum_0^2 a_{1n}(0)}, \quad \mu_1=\frac{b_2(1)}{K_0\sum_0^2 a_{1n}(0)}, \quad \mu_2=\frac{b_1(1)}{K_0\sum_0^2 a_{1n}(0)},$$

$$\mu_3=\frac{b_0(1)}{K_0\sum_0^2 a_{1n}(0)}, \quad \alpha_1=\frac{\sum_0^1 a_{1n}(0)}{\sum_0^2 a_{1n}(0)}, \quad \alpha_2=\frac{a_{10}(0)}{\sum_0^2 a_{1n}(0)}.$$

From (20) we found that:

$$\Delta U_{OUT}{}^*[z,0]=\Delta U_{IN}{}^*[z,0]\sum_1^3\mu_kz^{-k}-\Delta U_{OUT}{}^*[z,0]\sum_1^2\alpha_kz^{-k}.$$

The obtained z-image $\Delta U_{OUT}{}^*[z, 0]$ corresponds to the original of the difference equation:

$$\Delta U_{OUT}{}^*[nT]= =\sum_1^3\mu_k\Delta U_{IN}{}^*\left[(n-k)T\right]-\sum_1^2\alpha_k\Delta U_{OUT}{}^*\left[(n-k)T\right]. \quad (21)$$

The difference equation (21) is solved by a digital recursive filter (Fig. 3), which contains four delay links for one period of switching of the PWC and amplifiers in the forward and reverse transmissions with gains μ_k, α_k. Implementation of a digital filter is possible on the basis of a programmable microcontroller.

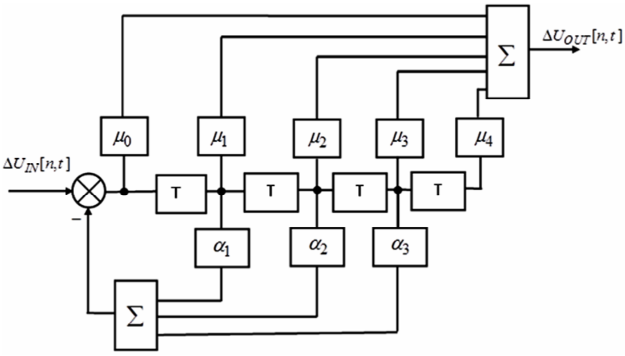

Fig. 3. Block diagram of the position regulator

Transient process analysis in an optimized positioning system. When the optimal synthesized controller is included in the position circuit, the optimal transfer functions of the closed positioning system for ranges of the current parameter ε values:

$$W^*_{1,2fbp}(z,\varepsilon) = P^*_{1,2}(z,\varepsilon)\frac{M^*(z,0)}{z^4}.$$

Taking into account (12), for $0 \le \varepsilon \le 0.25$ we have:

$$W^*_{1fbp} = \frac{a_{13}(\varepsilon)z^4 + a_{12}(\varepsilon)z^3 + a_{11}(\varepsilon)z^2 + a_{10}(\varepsilon)z}{A_n(0)\cdot z^4}. \quad (22)$$

Similarly, taking into account (13), it is possible to obtain the optimal transfer function of the closed positioning system for $0.25 \le \varepsilon \le 1$.

Image of the transient characteristics of the positioning system:

$$H_{1,2}^*[z,\varepsilon] = \frac{z}{z-1}W^*_{fbp}(z,\varepsilon).$$

Taking into account (22) we have:

$$H^*_1(z,\varepsilon) = \frac{1}{A_n(0)}\left[a_{13}(\varepsilon)\frac{z}{z-1} + \right.$$

$$\left. + a_{12}(\varepsilon)\frac{1}{z-1} + a_{11}(\varepsilon)\frac{1}{z(z-1)} + a_{10}(\varepsilon)\frac{1}{z^2(z-1)}\right].$$

The image of the transient characteristic for the values $0 \le \varepsilon \le 0.25$ corresponds to the original:

$$H^*_1(n,\varepsilon) = \frac{1}{A_n(0)}\left[a_{13}(\varepsilon) + \right.$$
$$\left. + a_{12}(\varepsilon)[n-1] + a_{11}(\varepsilon)[n-2] + a_{10}(\varepsilon)[n-3]\right]. \quad (23)$$

Similarly, it is possible to obtain the transition characteristic $H^*_2(n, \varepsilon)$ for the values $0.25 \le \varepsilon \le 1$.

From expression (23) it is seen that in the system optimized by the criterion of speed of the system the transient positioning process is completed in four periods of switching of the PWC. The optimization process begins with a delay of zero period, during which the system is open due to the fact that the feedback signal appears with a delay of one period.

For the positioning system, which is made on the basis of the linear motor LED AT605TU, the transient characteristics for different values of $\beta = T/\sigma$ are calculated. Linear motor parameters: $T_E = 5\cdot10^{-3}$ s, $T_M = 0.1$ s, $R_y = 3$ Ω, $K_E = 10.38$ V·s/m. Transmission coefficients of sensors of circuits of positioning system: $K_{SC} = 15$ V/A, $K_{SS} = 20$ V·s/m, $K_{SP} = 200$ V/m.

The parameters of current and speed regulators are obtained from the condition of setting these circuits to the modular optimum: $K_{CS} = \frac{T_C R_y}{2K_{sc}\sigma}$, $K_S = \frac{K_E T_M}{4R_y K_{ss}\sigma}$.

To minimize current ripple, in [12], it is shown $T_C \ge 2\ \sigma$. In further calculations $\sigma = 10^{-4}$ s, which determined the following gains: $K_{CC} = 0.2$; $K_{CS} = 43$; $K_{OC} = 2$; $K_{OS} = 0.17\cdot10^{-2}$; $K_O = 0.34$.

According to (23) for different values of β transient characteristics of the positioning system are calculated. The results are presented in Fig. 4.

For $\varepsilon = 0.25$, the steady-state values of the deviation of the transient characteristic at the switching period $H^*_c[0.25]$ are calculated. Their difference with the steady-state values of the transient characteristic at the time of operation of the pulse element determines the maximum relative values of the pulsations of the stabilized parameter, i.e. $\Delta^*_c = H^*_c[0{,}25]-1$.

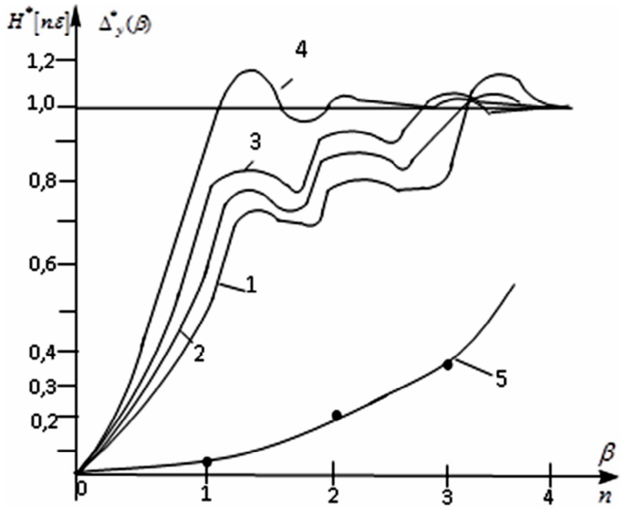

Fig. 4. Transient characteristics of the positioning system: curves 1-4 – for $\beta = 0.5$; 1; 2; 4, curve 5 – the dependence of the maximum pulsations on β in the steady state – $\Delta^*_{max}(\beta)$

The results of the calculation of $\Delta^*_c(\beta)$ are presented in Fig. 4 by curve 5. Obviously, a decrease in β leads to a decrease in ripple, but this reduces the speed of the transient process and increases the overregulation: curve 1 in Fig. 4.

Excessive increase in β leads to an increase in ripples, which negatively affects also the nature of the transient process. From the curves presented in Fig. 4, it is seen that the compromise between the quality of the transient process and the value of the ripples corresponds to $\beta = 1\div2$.

For one such compromise value, $\beta = 1$, the parameters of the links of the optimal position circuit regulator are calculated: $\mu_0 = 32.3$; $\mu_1 = -37.9$; $\mu_2 = 72.6$; $\mu_3 = -17$; $\alpha_1 = 0.5$; $\alpha_2 = 0.1$.

The obtained values of the parameters of the optimal speed digital position controller allow to realize the transient characteristic 2 (Fig. 4), for four intervals of switching of the PWC at relative values of ripples at the level of 0.04.

Curves 2, 3 in Fig. 4 shows that the transient processes that correspond to the recommended values $\beta = 1\div2$, accompanied by a slight over-regulation, which can be eliminated by increasing their duration, which is possible by increasing the degree of the polynomial (16).

Conclusions and prospects for development.

1. The transfer functions of PWC which allow to estimate ripples of the parameter stabilizing at deep regulation in transient and constant modes are proposed.
2. For the final range of change of duty cycle in the process of PWM the regulator of a position circuit is synthesized and its implementation in the form of the digital recursive filter which allows to complete transient process for four periods of switching of the PWC at the minimum overregulation is proposed.
3. It is established that the compromise between the quality indicators of the transient process and the

minimum of ripples of the stabilized parameter (position) corresponds to the values $\beta = 1 \div 2$.

The obtained results of estimating the influence of β on the nature of the transient process and the value of the ripples of the positioning system correspond to the transfer function (1), which is valid for a limited range of regulation. The transfer function (2) reflects the equivalent between the depth of the PWM and the amplitude pulse modulation of the multistage pulse. Based on it, it is possible to establish the pattern of ripple changes in the entire PWM range and, taking this into account, adjust the positioning circuit controller to the final duration of the process, which requires separate consideration.

Obtaining of the transfer function of the link with PWM «in large» taking into account the nonlinearities of the modulation characteristics of the model is possible. To do this, it is necessary to use a multidimensional *z*-transform and Volterra series to separate the linear and nonlinear components of the response of the link to the perturbation.

Conflict of interest. The authors declare no conflict of interest.

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