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A CALCULATION MODEL FOR DETERMINATION OF IMPEDANCE OF POWER HIGH VOLTAGE SINGLE-CORE CABLES WITH POLYMER INSULATION

Introduction. The wave parameters of power cables with polymer insulation differ significantly from the parameters of overhead lines and power transformers. As a result, there are more and more objects in electrical networks for which the occurrence of complex multi frequency transients, accompanied by dangerous overvoltages, should be expected. **Purpose.** To develop a computational model of the complex impedance of high-voltage single-core power cables of coaxial design required to determine the frequency dependencies of the active resistance and inductance of the conductive core and metal shield, taking into account the surface effect and proximity effect. **Methodology.** The method is based on solving a system of linear algebraic Kirchhoff equations (SLAE) for magnetically coupled contours. SLAE can be used to calculate conductors taking into account the skin effect and proximity effect. **Practical value.** The developed model is the basis for determining the characteristic impedance of high-voltage single-core power cables in a wide range of frequencies required to establish adequate criteria for evaluating the parameters of high-frequency effects critical for cross linked polyethylene insulation. References 16, table 1, figures 7.

Key words: power cables, complex resistance, magnetically coupled circuits, system of linear algebraic equations, conductor, screen, coefficient of irregularity of current distribution.

Запропонована чисельна розрахункова модель визначення активного опору та індуктивності струмопровідної жили й металевого екрану силових одножильних кабелів коаксіальної конструкції з урахуванням поверхневого ефекту та ефекту близькості в широкому діапазоні частоти. Виконано в залежності від частоти порівняння коефіцієнтів нерівномірності розподілу струму по перетину струмопровідної жили кабелю. Показано, що збільшення товщини мідного екрану призводить до збільшення коефіцієнта нерівномірності розподілу струму по перерізу екрану та активного опору кабелю для частоти 100 кГц при незмінному перерізі струмопровідної жили. Розроблена модель є основою для визначення характеристичного імпедансу силових високовольтних одножильних кабелів в широкому діапазоні частоти, необхідного для встановлення адекватних критеріїв оцінки параметрів високочастотних впливів, критичних для жиштої поліетиленової ізоляції. Бібл. 16, табл. 1, рис. 7.

Ключові слова: силові кабелі, комплексний опір, магнітоз'язані контури, система лінійних алгебраїчних рівнянь, струмопровід, екран, коефіцієнт нерівності розподілу струму.

Предложена численная расчетная модель определения активного сопротивления и индуктивности токопроводящей жилы и экрана силовых одножильных кабелей коаксиальной конструкции с учетом поверхностного эффекта и эффекта близости в широком диапазоне частоты. Выполнено в зависимости от частоты сравнение коэффициентов неравномерности распределения тока по сечению токопровода. Показано, что увеличение толщины медного экрана приводит к увеличению коэффициента неравномерности распределения тока по сечению экрана и активного сопротивления кабеля для частоты 100 кГц при неизменном сечении токопроводящей жилы. Разработанная модель является основой для определения характеристического импеданса силовых высоковольтных одножильных кабелей в широком диапазоне частоты, необходимого для установления адекватных критериев оценки параметров высокочастотных воздействий, критичных для жиштої полиэтиленовой изоляции. Библ. 16, табл. 1, рис. 7.

Ключевые слова: силовые кабели, комплексное сопротивление, магнитосвязанные контуры, система линейных алгебраических уравнений, токопровод, экран, коэффициент неравномерности распределения тока.

Introduction. High-voltage power cables with thermosetting polyethylene insulation are critical components of power systems. Many investments are made in their production to ensure reliable operation [1-3]. Power high-voltage cables with insulation based on cross-linked polyethylene with voltage of 6-500 kV are characterized by increased values of electrical capacitance and reduced values of impedance (characteristic impedance) compared to cables with paper-impregnated insulation [4-8]. As a result, there are more and more objects in electrical networks for which complex multi-frequency transients should be expected, which are accompanied by dangerous overvoltages and currents [5-8]. The values of overvoltage and the duration of the transient are determined by the length of the cable line and the impedance of the high-voltage power cable. The duration of transients is tens and hundreds of microseconds, which corresponds to the frequency of such processes from units to tens and hundreds of kilohertz [9]. High-frequency components of current and voltage accelerate, in particular, the development of water

treeings in polymer insulation and can cause the development of electrical treeings, i.e. loss of electrical insulation strength of high-voltage power cables [10, 11]. The development of adequate criteria for assessing the parameters of high-frequency effects critical for cross-linked polyethylene insulation of high-voltage cables is based on monitoring of the electrical insulation state during operation, including the results of temporal reflectometry, which requires data on the complex resistance of conductive core and screen to determine the characteristic impedance of the power cable [12].

The normative and technical documentation gives the values of the resistance of the current-carrying core at DC at temperature of 20 °C and the inductance of the power single-core high-voltage cable of coaxial design depending on the spatial location in a three-phase cable line (triangle or plane) [2]. To determine the active resistance of the core during the flow of AC with frequency of 50 Hz, mathematical expressions are given

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that approximate the dependence of resistance due to the skin effect and the proximity effect of adjacent single-core cables [2, 13].

In [14] the analytical model of calculation of longitudinal complex resistance taking into account the proximity effect for a current-carrying core and inductance of a core and the metal screen without taking into account influence of frequency of a single-core power cable is presented. In [15], the intrinsic inductance of the metal screen is determined under the assumption of uniform distribution of the induced current in the screen of a single-core power cable.

Thus, obtaining the frequency dependencies of the active resistance and inductance of current-carrying cores and metal shields of high-voltage single-core power cables, taking into account the skin effect and proximity effect is an important and urgent problem, which until recently is insufficiently developed.

The goal of the research is to develop a computational model of complex resistance of high-voltage single-core power cables of coaxial design, necessary to determine the frequency dependencies of active resistance and inductance of conductive core and metal screen taking into account the skin effect and the proximity effect.

Numerical determination of the active resistance and inductance of the current line, taking into account the skin effect and the proximity effect. Divide the conductor into a number of parallel branches (Fig. 1) – current filaments. Each i -th branch has an active resistance R_i and an inductance L_i , and due to the magnetic field is connected to the j -th branch. The mutual inductance between the i -th and the j -th branches is denoted as M_{ij} .

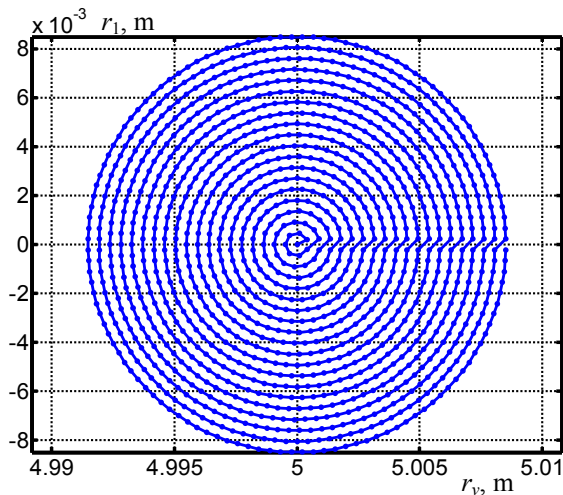


Fig. 1. Layout of nodes along the cross section of the current line. The first node is in the center of the current line, the others are located radially. The line connecting the nodes determines the numbering order – counterclockwise

Suppose that in the i -th branch there is a source of variable electromotive force EMF E_i of frequency f . Then for the closed contour of this branch we write the Kirchhoff equation:

$$(R_i + j\omega L_i) \cdot \dot{I}_i + \sum_{j \neq i}^{N_1} (j\omega M_{ij} \cdot \dot{I}_j) = \dot{E}_i, \quad (1)$$

where the first term $(R_i + j\omega L_i) \cdot \dot{I}_i$ determines the voltage drop on the active resistance and the intrinsic inductance of the i -th branch, and all other components $\sum_{j \neq i}^{N_1} (j\omega M_{ij} \cdot \dot{I}_j)$ are the voltage drops on the mutual inductance of the i -th branch with all other branches.

Equation (1) contains N_1 terms with unknown currents of parallel branches: $I_1, I_2, \dots, I_b, \dots, I_j, \dots, I_N$. For each of N_1 branches we write an equation similar to (1). We obtain a system of linear algebraic Kirchhoff equations (SLAE):

$$\begin{cases} (R_1 + j\omega L_1) \cdot \dot{I}_1 + \sum_{j \neq 1}^{N_1} (j\omega M_{1j} \cdot \dot{I}_j) = \dot{E}_1; \\ (R_2 + j\omega L_2) \cdot \dot{I}_2 + \sum_{j \neq 2}^{N_1} (j\omega M_{2j} \cdot \dot{I}_j) = \dot{E}_2; \\ \dots \\ (R_{N_1} + j\omega L_{N_1}) \cdot \dot{I}_{N_1} + \sum_{j \neq N_1}^{N_1} (j\omega M_{N_1j} \cdot \dot{I}_j) = \dot{E}_{N_1}. \end{cases} \quad (2)$$

Write SLAE (2) in matrix form:

$$\begin{pmatrix} (R_1 + j\omega L_1) & j\omega M_{12} & \dots & j\omega M_{1,N} \\ j\omega M_{21} & (R_2 + j\omega L_2) & \dots & j\omega M_{2,N} \\ \dots & \dots & \dots & \dots \\ j\omega M_{N,1} & j\omega M_{N,2} & \dots & (R_N + j\omega L_N) \end{pmatrix} \cdot \begin{pmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dots \\ \dot{I}_N \end{pmatrix} = \begin{pmatrix} \dot{E}_1 \\ \dot{E}_2 \\ \dots \\ \dot{E}_N \end{pmatrix}. \quad (3)$$

On the diagonal of the matrix of SLAE coefficients (3) there are the intrinsic resistances of the branches (active and inductive), outside the diagonal – the mutual inductive resistances of the branches.

In some cases, the values of EMF acting in the branches of the current line are the same – \dot{E}_s . Then SLAE (3) takes the form:

$$\begin{pmatrix} (R_1 + j\omega L_1) & j\omega M_{12} & \dots & j\omega M_{1,N} \\ j\omega M_{21} & (R_2 + j\omega L_2) & \dots & j\omega M_{2,N} \\ \dots & \dots & \dots & \dots \\ j\omega M_{N,1} & j\omega M_{N,2} & \dots & (R_N + j\omega L_N) \end{pmatrix} \cdot \begin{pmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dots \\ \dot{I}_N \end{pmatrix} = \begin{pmatrix} \dot{E}_s \\ \dot{E}_s \\ \dots \\ \dot{E}_s \end{pmatrix}. \quad (4)$$

SLAE (4) allows to determine the parameters of the complex resistance of current lines, taking into account the skin effect and the proximity effect.

The mutual inductance M_{12} between two coaxial ring contours with currents is calculated based on the analytical formula [16]:

$$M_{12} = 2\mu\mu_0\sqrt{r_1 \cdot r_2} \cdot \frac{1}{k} \cdot \left[\left(1 - \frac{k^2}{2} \right) K(k) - E(k) \right], \quad (5)$$

where r_1, r_2 are the radii of the current lines (for single-phase power cable of coaxial design r_1 is the radius of the current-carrying core; r_2 is the inner radius of the metal cable screen, respectively); $k = \sqrt{\frac{4r_1r_2}{(r_1+r_2)^2 + G^2}}$; where G

is the distance between the planes of the current lines along the axis of symmetry; $K(k)$ and $E(k)$ are functions of complete elliptic integrals of the first and second kind:

$$K(k) = \int_0^{\pi/2} \frac{d\beta}{\sqrt{1-k^2 \sin^2 \beta}}; \quad E(k) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \beta} \cdot d\beta.$$

The intrinsic inductance of a circular conductor of massive cross-section is calculated as the mutual inductance between two filamentary conductors of the same radius, located at a distance G from each other.

For a circular section of radius r , the geometric mean distance of the area of the circle from itself G is equal to:

$$G = \frac{r}{\sqrt{e}}, \quad (6)$$

where $e = 2.71828\dots$ is the basis of natural logarithms.

As a result of solution (4), the currents in the branches I and the total current I_Σ of all parallel branches of the current line are determined.

After finding the total current of all parallel branches, the total complex resistance, its active R and reactive X_L components, the equivalent inductance L of the current line are determined:

$$\dot{Z} = \frac{\dot{E}_s}{I_\Sigma \cdot 2\pi r_v}, \quad R = \text{Re}(\dot{Z}), \quad X_L = \text{Im}(\dot{Z}), \quad L = \frac{X_L}{\omega}, \quad (7)$$

where r_v is the bending radius of the current line.

Results of numerical modelling. The axis of symmetry of the annular current line (Fig. 1) is on the left at a distance Z from the center of the current line. Passing the nodes in each layer begins on the outside of the annular current line, then passes on the inside and returns to the outside. The current tends to pass along the path of the smallest length (on the inner side of the current line), due to which its density is higher there (Fig. 1).

Figures 2–4 show the effect of frequency on the distribution of currents I in branches N (Fig. 2), active resistance R_c (Fig. 3), equivalent inductance L_c (Fig. 4) of a copper conductive core of a power cable with a cross section of 240 mm^2 (Fig. 2,a; curves 1 in Fig. 3, 4) and 800 mm^2 (Fig. 2,b; curves 2 in Fig. 3, 4), respectively.

With increasing frequency, the displacement of current on the surface of the current line increases. As a result, the effective resistance of the current line increases (Fig. 3).

Figure 5 shows a layout of the location of the nodes at the intersection of the conductive core and the screen of the power single-core cable of coaxial structure. The results of numerical calculation of frequency-dependent active resistance and inductance L of the power cable with voltage of 110 kV with a cross section of a copper conductive core of 240 mm^2 and a copper screen with thickness of 1 mm are shown in Fig. 6, 7, respectively. The active resistance of the cable R consists of the resistance of the conductive core R_c and the metal screen R_s : $R = R_c + R_s$. The equivalent inductance of the cable L includes the equivalent inductance of the conductive core L_c , the equivalent screen inductance L_s and the mutual inductance between core and screen.

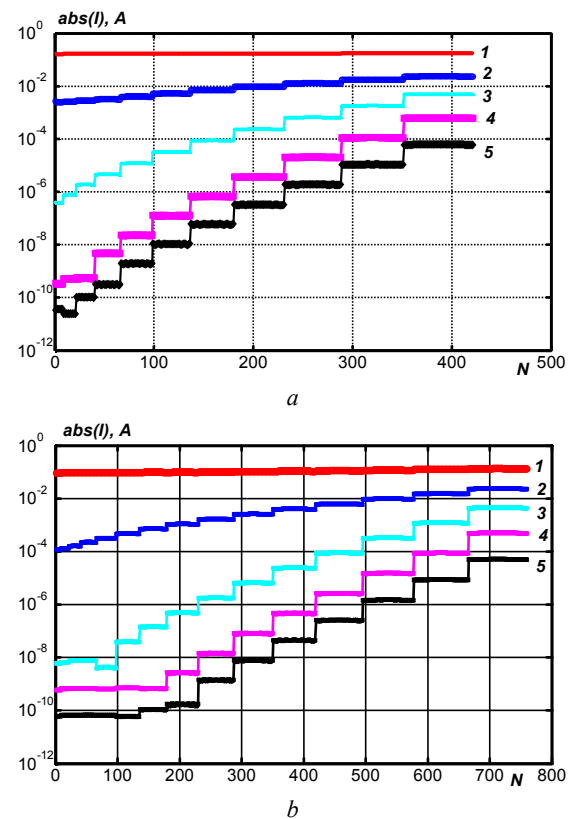


Fig. 2. Influence of frequency on current distribution on branches N of the power current line (curve 1 – frequency 50 Hz, curve 2 – 1 kHz, curve 3 – 10 kHz, curve 4 – 100 kHz, curve 5 – 1 MHz)

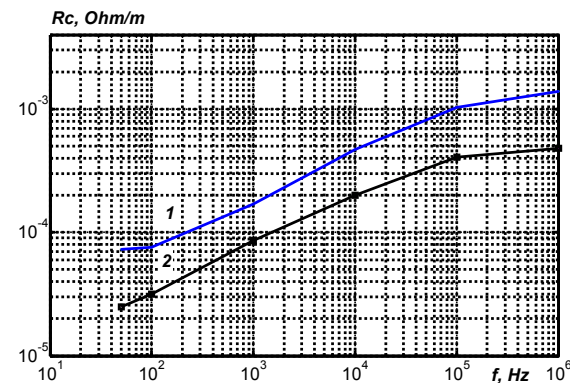


Fig. 3. Frequency dependencies of active resistance of copper power current line

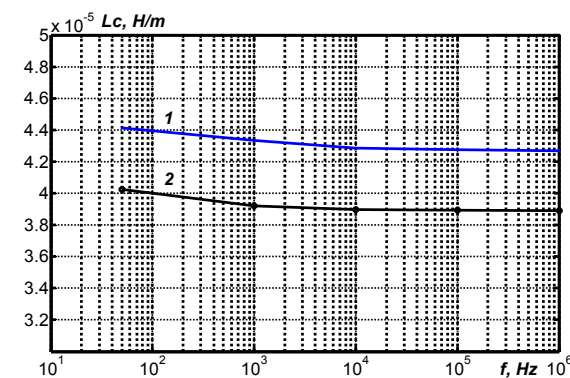


Fig. 4. Frequency dependencies of equivalent inductance of copper power current line

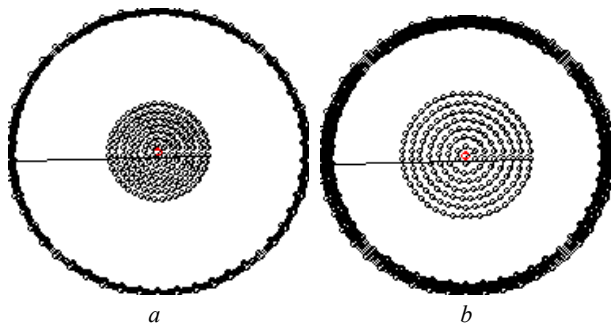


Fig. 5. Layout of the nodes at the intersection of the conductive core and the screen with thickness of 1 mm (a) and 3 mm (b)

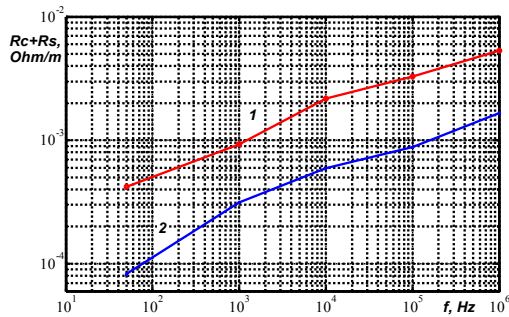


Fig. 6. Frequency dependence of the active resistance of a power cable of single-core design of voltage of 110 kV

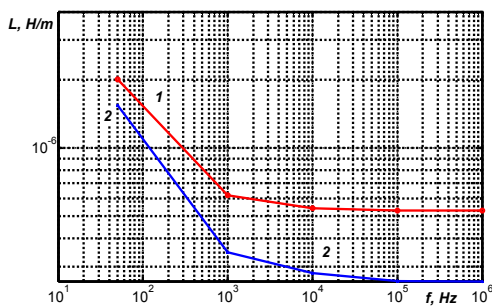


Fig. 7. Frequency dependence of the equivalent inductance of a power cable of single-core design of voltage of 110 kV

Increase the thickness of the copper screen 3 times (from 1 to 3 mm) leads to an increase in the coefficient of non-uniformity of current distribution across the screen by 1.25 and 4.48 times with constant value of the coefficient of non-uniformity of current distribution across the copper core section for frequency of 1 kHz and 100 kHz, respectively. For frequency of 1 kHz, the active resistance of a cable with a screen of smaller thickness is 1.4 times the active resistance of a cable with a screen of thicker thickness.

Checking the reliability of the results of numerical calculations is performed on the basis of comparing the coefficients of non-uniformity of the current distribution due to the skin effect and the proximity effect, along the cross section of the current line.

The first coefficient K_f is calculated through the ratio of heat output at AC and DC. The power of heat dissipation at AC is determined by the total power of heat dissipation of parallel branches taking into account the redistribution of current along the cross section of the conductive core. The power of heat dissipation at DC is determined by the uniform distribution of current across the cross section of the conductive core.

The second coefficient K_R is calculated through the ratio of the resistance of the current line at AC and DC.

Table 1 shows a comparative analysis of the coefficients of non-uniformity of current distribution at the intersection of conductive copper cores. The results of the calculations are identical.

Table 1
Comparative analysis of the coefficients of non-uniformity of current distribution across the current line section

Frequency, Hz	Cross section of the copper current line 240 mm ²		Cross section of the copper current line 800 mm ²	
	K_f	K_R	K_f	K_R
50	1,0156	1,0156	1,1546	1,1545
100	1,0605	1,0605	1,4598	1,4598
1000	2,3581	2,3581	3,9658	3,9655
10000	6,5865	6,5865	9,2692	9,2686
100000	14,3079	14,3079	19,4337	19,4337
1000000	19,4332	19,4332	22,2853	22,2836

Conclusions.

For the first time, a computational numerical model for determining the complex resistance of high-voltage single-core power cables with polymer insulation is proposed, which allows to determine the active resistance and inductance of both conductive core and copper screen, taking into account the skin effect and proximity effect.

The developed model is the basis for determining the characteristic impedance of high-voltage single-core power cables in a wide range of frequencies required to establish adequate criteria for evaluating the parameters of high-frequency effects critical for cross-linked polyethylene insulation.

Based on the proposed computational numerical model, an appropriate technique can be developed to determine the frequency dependencies of the active resistance and inductance of a single-core power cable of coaxial design, which can be the basis for improving the accuracy of mathematical modelling of transients in cable lines arising at single-phase short circuits to the earth and other switchings in electric networks at bilateral grounding of the screen.

Conflict of interest. The authors declare that they have no conflicts of interest.

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