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ON THE SIMILARITY OF PLANE PULSED MAGNETIC FIELDS CONTINUED FROM DIFFERENT COORDINATE AXES

Purpose. The purpose of this work is formulation of similarity conditions for plane magnetic fields at a sharp skin-effect continued in non-conducting and non-magnetic medium from different axes bounding plane surfaces of conductors. *Methodology.* Classic formulation of Cauchy problem for magnetic vector potential Laplace equations, mathematic physics methods and basics similarity theory are used. Two problems are considered: the problem of initial field continuation from one axis and the problem of similar field continuation from other axis on which magnetic flux density or electrical field strength is unknown. *Results.* Necessary and sufficient similarity conditions of plane pulsed or high-frequency magnetic fields continued from different axes of rectangular coordinates are formulated. For the given odd and even magnetic flux density distributions on axis of initial field corresponding the distributions on axis and solution of continued similar field problem are obtained. *Originality.* It is proved that for similarity of examined fields the proportion of corresponding vector field projections represented by dimensionless numbers in similar points of axes is necessary and sufficient. References 11, figures 4.

Key words: plane magnetic field, sharp skin-effect, Cauchy problem for Laplace equation, similarity theory.

Мета. Метою роботи є формулювання умов подібності плоскопаралельних магнітних полів при різкому поверхневому ефекті, що продовжуються в непровідне і немагнітне середовище з різних осей прямокутних координат, котрі обмежують плоскі поверхні провідників. *Методика.* Використано класичне формулювання задачі Коші для рівняння Лапласа відносно векторного потенціалу магнітного поля, методи математичної фізики та основні положення теорії подібності. Розглянуто дві відповідні задачі: задача продовження вихідного поля з однієї осі та задача продовження подібного поля з іншої осі, розподіл індукції магнітного поля на котрій є невідомим. *Результати.* Сформульовано необхідні та достатні умови подібності плоскопаралельних імпульсних або високочастотних магнітних полів, що продовжуються з різних осей прямокутних координат. Для заданих непарного та парного розподілів індукції на осі вихідного магнітного поля визначено відповідні розподіли на осі, а також розв'язані задачі продовження подібного поля. *Наукова новизна.* Доведено, що для подібності полів, які розглянуто, необхідна та достатня пропорційність представлених у критеріальній формі відповідних проекцій векторів цих полів у схожих точках осей. Бібл. 11, рис. 4.

Ключові слова: плоскопаралельне магнітне поле, різкий поверхневий ефект, задача Коші для рівняння Лапласа, теорія подібності.

Цель. Целью работы является формулировка условий подобия плоскопаралельных магнитных полей при резком поверхностном эффекте, которые продолжаются в непроводящую и немагнитную среду с различных осей прямоугольных координат, ограничивающих плоские поверхности проводников. *Методика.* Используются классическая формулировка задачи Коши для уравнения Лапласа относительно векторного потенциала магнитного поля, методы математической физики и основные положения теории подобия. Рассмотрены две соответствующие задачи: задача продолжения исходного поля с одной оси и задача продолжения подобного поля с другой оси, распределение индукции магнитного поля на которой неизвестно. *Результаты.* Сформулированы необходимые и достаточные условия подобия плоскопаралельных импульсных или высокочастотных магнитных полей, продолжаемых с различных осей прямоугольных координат. Для заданных нечетного и четного распределений индукции на оси исходного магнитного поля определены соответствующие распределения на оси, а также решены задачи продолжения подобного поля. *Научная новизна.* Доказано, что для подобия рассмотренных полей необходима и достаточна пропорциональность представленных в критеріальной форме соответствующих проекций векторов этих полей в сходственных точках осей. Библ. 11, рис. 4.

Ключевые слова: плоскопаралельное магнитное поле, резкий поверхностный эффект, задача Коши для уравнения Лапласа, теория подобия.

Introduction. The shape of massive solenoids (inductors) and electrodes used in electrophysical technologies to obtain electromagnetic fields of a given distribution is found by solving the field continuation problem [1-3]. We restrict ourselves to considering plane pulsed or high-frequency magnetic fields, continued from one of the axes of rectangular coordinates (for example, the x -axis) [4]. The problem definition includes the distribution of a certain projection of the vector of the extended field specified on this axis. In practice, it may be necessary to solve the problem of continuation of a field with a similar distribution on the y -axis. In this case, it is obvious to use the results obtained for the x -axis. The main difficulty of this approach lies, first of all, in

insufficient theoretical substantiation, as a result of which the given field distribution on the y -axis turns out to be unknown.

The goal of the work is a formulation of conditions for the similarity of plane magnetic fields at a sharp skin-effect, which continue into a non-conductive and non-magnetic medium from different axes of rectangular coordinates that limit the flat surfaces of the conductors.

Conditions for the similarity of magnetic fields extended from flat surfaces of conductors. In a massive conductor with a flat boundary surface eddy currents are induced under the action of a pulsed or high-frequency magnetic field of an external inductor, the profile of

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which must be determined. The skin-effect is sharply manifested in the conductor. Let us accept the assumption of an ideal surface effect [3] and replace the conductor with an ideally superconducting half-space. We use three systems of Cartesian coordinates on the plane: the main (general) xOy and two auxiliary ones – x_1Oy_1 and x_2Oy_2 . Consider two corresponding problems of continuation of a plane magnetic field into non-magnetic non-conducting half-spaces $y_1 > 0$ and $x_2 > 0$ without sources (Fig. 1): from the x_1 axis (the first problem whose solution is known) and from the y_2 axis (the second problem). Half-spaces $y_1 < 0$ and $x_2 < 0$ are ideal superconductors.

Equation for the magnetic vector potential $A(x, y)$ of such fields has the form [5]

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = 0. \quad (1)$$

In (1) $x = x_1 \vee x_2$, $y = y_1 \vee y_2$, $A(x, y) = A_1(x_1, y_1) \vee A_2(x_2, y_2)$. Boundary conditions on the x_1 -axis –

$$A_1(x_1, 0) = 0, \quad \left. \frac{\partial A_1}{\partial y_1} \right|_{y_1=0} = B_{1x}(x_1, 0), \quad (2, 3)$$

on the y_2 -axis –

$$A_2(0, y_2) = 0, \quad \left. \frac{\partial A_2}{\partial x_2} \right|_{x_2=0} = -B_{2y}(0, y_2), \quad (4, 5)$$

where $B_{1x}(x_1, 0)$ and $B_{2y}(0, y_2)$ are the projections of the magnetic flux density $\vec{B}_1(x_1, y_1)$, $y_1 \geq 0$ and $\vec{B}_2(x_2, y_2)$, $x_2 \geq 0$ on the x_1, y_2 axes.

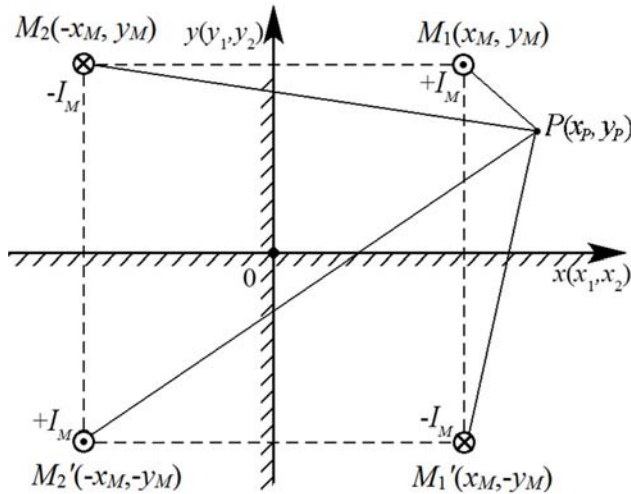


Fig. 1. Systems of coordinates and parallel axes with currents creating specified distributions of magnetic flux density (8)

Note that $B_{1x}(x_1, 0)$ is a given function, and the projection $B_{2y}(0, y_2)$ is to be determined. Comparing the formulations of the two considered problems (1)-(3) and (1), (4), (5), we note that they have geometrically similar solution domains (half-spaces $y_1 > 0$ and $x_2 > 0$, Fig. 1) with the same physical properties, contain an equation of the same type and similar boundary conditions (2), (4) on the x_1 and y_2 axes from which the fields continue. The described conditions are necessary, but they are not enough for the similarity: the boundary conditions (3), (5) remain.

Let us assume that the considered magnetic fields are similar. Then it follows from (1) – (5) that the similar coordinates [6-8] in the field continuation domains are x_1 and y_2 , y_1 and x_2 , and the corresponding functions are $A_1(x_1, y_1)$ and $A_2(x_2, y_2)$. Therefore, analogous quantities characterizing supposedly similar fields should be

$$\frac{\partial A_1}{\partial x_1} \text{ and } \frac{\partial A_2}{\partial y_2}, \quad \frac{\partial A_1}{\partial y_1} \text{ and } \frac{\partial A_2}{\partial x_2}, \text{ or}$$

$$-B_{1y}(x_1, y_1) \text{ and } B_{2x}(x_2, y_2), B_{1x}(x_1, y_1) \text{ and } -B_{2y}(x_2, y_2). \quad (6, 7)$$

Let P be the observation point of the field with coordinates $x_P = x_{1P} \vee x_{2P}$, $y_P = y_{1P} \vee y_{2P}$ (Fig. 1). Then the similar coordinates of the point P will be x_{1P} and y_{2P} , y_{1P} and x_{2P} .

Taking into account the main provisions of the similarity theory [6-8], in addition to the noted necessary conditions, we can assert the following: for the similarity of two compared magnetic fields, it is necessary and sufficient that the values of the presented in criterial form magnetic flux density projections $B_{1x}(x_1, 0)$ and $B_{2y}(0, y_2)$ at similar points of the axes, from which these fields continue, should be proportional.

This condition allows to find $B_{2y}(0, y_2)$ and thus obtain a complete formulation of the second problem.

Magnetic flux density distribution on the y_2 axis for a similar magnetic field. Let in the first problem the given distributions of the magnetic flux density on the x_1 -axis can be represented by the formula

$$B_{1x}(x_1, 0) = \frac{\mu_0 I_{1M}}{\pi} \left\{ y_{1M} \left[\frac{1}{(x_1 - x_{1M})^2 + y_{1M}^2} \mp \frac{1}{(x_1 + x_{1M})^2 + y_{1M}^2} \right] \right\}, \quad (8)$$

where μ_0 is the magnetic constant, I_{1M} , x_{1M} , y_{1M} are the distribution parameters.

The function in curly brackets of formula (8) is odd or even, depending on the minus or plus sign between the terms in square brackets. In both cases, it has the well-known sine or cosine Fourier transform. The multiplier before of the considered function is constant therefore $B_{1x}(x_1, 0)$ also has such transform. The physical meaning of (8) is the magnetic flux density created on the x_1 -axis by a system of four parallel, symmetrically located axes with currents $\pm I_M = \pm I_{1M}$, two of which (M_1' and M_2') replace the influence of the lower ideally superconducting half-space [9, 10] (Fig. 1). The parameters $x_M = x_{1M}$, $y_M = y_{1M}$ determine the position of the axes at the points M_1, M_1', M_2, M_2' of the x_1Oy_1 plane. Currents $+I_M$ have positive directions, and $-I_M$ have negative ones, indicated by a dot or a cross, respectively. For the currents in Fig. 1 we obtain an odd magnetic flux density distribution. If the currents in the upper half-space have the same (for example, positive) direction (while the currents in the lower half-space are also directed in the same way, but opposite to the first ones), we have an even distribution of magnetic flux density.

Let us find such distributions $B_{2y}(0, y_2)$ on the y_2 -axis that satisfy the necessary and sufficient similarity condition formulated above. To do this, we first establish

similar parameters [6-8] of the distributions $B_{1x}(x_1, 0)$ and $B_{2y}(0, y_2)$: I_{1M} and I_{2M} , x_{1M} and y_{2M} , y_{1M} and x_{2M} , where I_{2M} , x_{2M} and y_{2M} are the parameters of the distributions $B_{2y}(0, y_2)$ unknown so far. Then, using the correspondence of similar values (7), we replace in formula (8) the coordinates and parameters with similar coordinates and parameters of the second problem. We obtain:

$$B_{2y}(0, y_2) = -\frac{\mu_0 I_{2M}}{\pi} \left\{ x_{2M} \left[\frac{1}{(y_2 - y_{2M})^2 + x_{2M}^2} \mp \frac{1}{(y_2 + y_{2M})^2 + x_{2M}^2} \right] \right\} \quad (9)$$

We represent (8), (9) in dimensionless form (in criterial form) using two systems of basic quantities: l_{1b} and l_{2b} – length, I_{1b} and I_{2b} – current, B_{1b} and B_{2b} – magnetic flux density (basic values for formula (8) have number 1 in the subscript, for (9) – number 2). Dimensionless quantities are obtained by dividing the corresponding dimensional ones by the basic ones and marked with asterisks. After transformations formulas (8), (9) take the following form:

$$B_{1x}^*(x_1^*, 0) = \frac{1}{\pi} I_{1M}^* y_{1M}^* \left[\frac{1}{(x_1^* - x_{1M}^*)^2 + y_{1M}^{*2}} \mp \frac{1}{(x_1^* + x_{1M}^*)^2 + y_{1M}^{*2}} \right], \quad -\infty < x_1^* < \infty; \quad (10)$$

$$B_{2y}^*(0, y_2^*) = -\frac{1}{\pi} I_{2M}^* x_{2M}^* \left[\frac{1}{(y_2^* - y_{2M}^*)^2 + x_{2M}^{*2}} \mp \frac{1}{(y_2^* + y_{2M}^*)^2 + x_{2M}^{*2}} \right], \quad -\infty < y_2^* < \infty. \quad (11)$$

Comparing (10) and (11), we see that for

$$x_{1M}^* = y_{2M}^*, \quad y_{1M}^* = x_{2M}^* \quad (12)$$

at similar points on the x_1 and y_2 axes with coordinates $x_1^* = y_2^*$, the values $B_{1x}^*(x_1^*, 0)$ are proportional to the values $B_{2y}^*(0, y_2^*)$. Consequently, the necessary and sufficient similarity condition is satisfied, and the sought distributions $B_{2y}(0, y_2)$ for such a magnetic field have the form (9). If in addition to (12) to accept

$$I_{1M}^* = I_{2M}^*, \quad (13)$$

then the absolute values of the compared magnetic flux density values will be equal, although this is not necessary for similarity.

All quantities included in conditions (12), (13) are similarity criteria. We choose the basic values l_{1b} , l_{2b} in such a way that conditions (12) are satisfied. In the general case, I_{1b} and I_{2b} can be any, but, if necessary, we find them taking into account condition (13). When determined $B_{1x}^*(x_1^*, 0)$ and $B_{2y}^*(0, y_2^*)$, we accepted $B_{1b} = \mu_0 I_{1b}/l_{1b}$, $B_{2b} = \mu_0 I_{2b}/l_{2b}$.

The physical meaning of distributions (9) is similar to that described for (8): the magnetic flux density created on the y_2 axis by four parallel axes with currents $\pm I_M = \pm I_{2M}$ (the axes are located at points whose coordinates $\pm x_M$ and $\pm y_M$ are determined by the parameters $\pm x_{2M}$ and $\pm y_{2M}$, Fig. 1).

Figure 2 shows the symmetric parts of the odd (a) and even (b) distributions of the magnetic flux density on the axes at $x_1 \geq 0$ and $y_2 \geq 0$, calculated by (10), (11). Accepted: $l_{1b} = l_{2b}$, $I_{1b} = I_{2b}$, $I_{1M} = I_{2M}$; для curves 1 – $x_{1M}^* = 0,1$, $y_{1M}^* = 0,1$; 2 – 0,25, 0,1; 3 – 0,15, 0,2; 4 – 0,25, 0,2. The values x_{2M}^* and y_{2M}^* are determined using relations (12). The coincidence of the distributions $B_{1x}^*(x_1^*, 0)$ and $-B_{2y}^*(0, y_2^*)$ illustrates the necessary and sufficient condition for the similarity of magnetic fields.

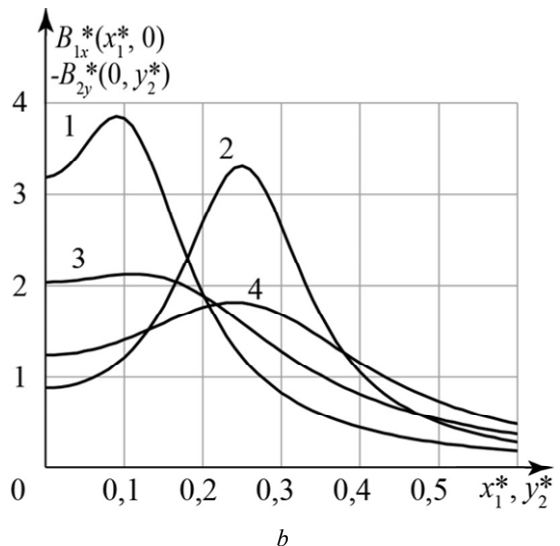
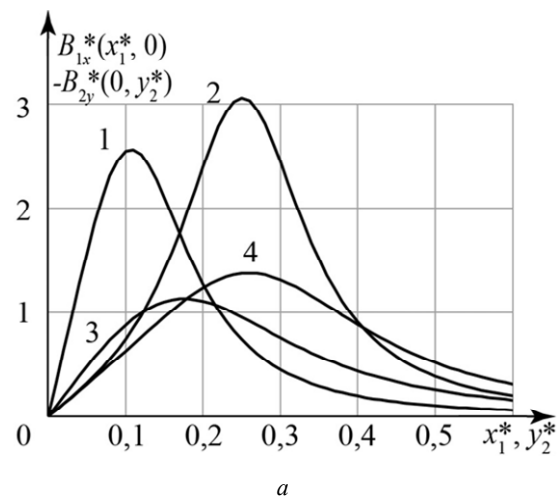


Fig. 2. Odd (a) and even (b) distributions of the magnetic flux density on the axes x_1, y_2

Continuation of similar magnetic fields by solving the first problem. The solutions of the first problem (1)-(3), taking into account (8), obtained by the method of particular solutions that continuously depend on the parameter, have the following form:

$$A_1(x, y) = \frac{2\mu_0 I_M}{\pi} \int_0^{\infty} e^{-y_M \lambda} \left\{ \begin{array}{l} \sin(x_M \lambda) \sin(x \lambda) \\ \cos(x_M \lambda) \cos(x \lambda) \end{array} \right\} \times \\ \times \lambda^{-1} \operatorname{sh}(y \lambda) d\lambda, \quad -\infty < x < \infty, \quad 0 < y < y_M. \quad (14)$$

The first line of the multiplier of the integrand in curly braces (14) refers to the odd distribution $B_{1x}(x_1, 0)$, the second – to the even one.

The solutions of the second problem (1), (4), (5) taking into account (9) are found by replacing coordinates and parameters in (14) by similar values of a similar field. We obtain:

$$A_2(x, y) = \frac{2\mu_0 I_M}{\pi} \int_0^{\infty} e^{-x_M \lambda} \left\{ \begin{array}{l} \sin(y_M \lambda) \sin(y \lambda) \\ \cos(y_M \lambda) \cos(y \lambda) \end{array} \right\} \times \\ \times \lambda^{-1} \operatorname{sh}(x \lambda) d\lambda, \quad -\infty < y < \infty, \quad 0 < x < x_M. \quad (15)$$

In (14), (15)) $I_M = I_{1M} \vee I_{2M}$, $x_M = x_{1M} \vee x_{2M}$, $y_M = y_{1M} \vee y_{2M}$. The constraints $y < y_M$ and $x < x_M$ are due to the convergence of improper integrals [4]. The correctness of the described method for determining such a magnetic field and, in particular, (15) is confirmed by the coincidence of the latter with the solution of the second problem by the same method as the first one.

Another method for solving the first problem is to use the Green function for an axis with a unit current located in a non-magnetic and non-conductive medium parallel to the surface of an ideally superconducting half-space. For the odd distribution $B_{1x}(x_1, 0)$ (8) we have [10]:

$$A(P) = \frac{\mu_0 I_M}{\pi} \ln \left[\frac{r_{M_1'P} r_{M_2P}}{r_{M_1P} r_{M_2'P}} \right], \quad (16)$$

where r_{M_1P} , $r_{M_1'P}$, r_{M_2P} , $r_{M_2'P}$ is the distance between points P and, accordingly, M_1 , M_1' , M_2 , M_2' (Fig. 1).

Using the known relationship between the magnetic flux density and the vector potential of the magnetic field [5] and (16) to calculate the projections, we obtain the following formulas:

$$B_{1x}(P) = -\frac{\mu_0 I_M}{2\pi} \left[(y_P - y_M) \left(\frac{1}{r_{M_1P}^2} - \frac{1}{r_{M_2P}^2} \right) + \right. \\ \left. + (y_P + y_M) \left(\frac{1}{r_{M_2'P}^2} - \frac{1}{r_{M_1'P}^2} \right) \right]; \quad (17)$$

$$B_{1y}(P) = \frac{\mu_0 I_M}{2\pi} \left[(x_P - x_M) \left(\frac{1}{r_{M_1P}^2} - \frac{1}{r_{M_1'P}^2} \right) + \right. \\ \left. + (x_P + x_M) \left(\frac{1}{r_{M_2P}^2} - \frac{1}{r_{M_2'P}^2} \right) \right]. \quad (18)$$

Note that in (16) – (18) it is assumed that the observation point P is located in the upper half-space $y > 0$ (in a particular case, on the x -axis). Let us find $B_{2x}(P)$ and $B_{2y}(P)$ for a similar magnetic field in the

region $x > 0$ (in a particular case on the y -axis), replacing coordinates and parameters in (17), (18) with similar values. We obtain surprising, at first glance, results: the formulas for determining $B_{2x}(P)$ and $B_{2y}(P)$ formally coincide with (17), (18). The reason is that a system of four axes with currents, which creates a magnetic field in the region $y > 0$ of the first problem (for more details, in the physical sense of (8)), simultaneously creates a similar magnetic field in the region $x > 0$. Here, the axes located in points M_2 and M_2' (Fig. 1), replace the influence of an ideally superconducting half-space $x < 0$. Therefore, formula (16) is also a solution of the second problem for a similar magnetic field in the region $x > 0$ in the case of an odd distribution $B_{2y}(0, y_2)$.

When using the Green function in the case of even distributions $B_{1x}(x_1, 0)$ and $B_{2y}(0, y_2)$, it is necessary to change the directions of currents in two axes to the opposite with respect to those adopted in Fig. 1: for the original field – in the M_2 and M_2' axes, for a similar field – in the M_1' and M_2' axes. In contrast to odd distributions of the magnetic flux density on the axes, the vector potential $A(P)$ is described by two different formulas. We obtain them from formula (16), having changed places r_{M_2P} and $r_{M_2'P}$ for the original field, $r_{M_1'P}$ and $r_{M_2'P}$ for a similar field:

$$A_1(P) = \frac{\mu_0 I_{1M}}{\pi} \ln \left[\frac{r_{M_1'P} r_{M_2'P}}{r_{M_1P} r_{M_2P}} \right], \quad (19)$$

$$A_2(P) = \frac{\mu_0 I_{2M}}{\pi} \ln \left[\frac{r_{M_2P} r_{M_2'P}}{r_{M_1P} r_{M_1'P}} \right]. \quad (20)$$

Formulas for calculating magnetic flux density projections $B_{1x}(P)$ and $B_{1y}(P)$, $B_{2x}(P)$ and $B_{2y}(P)$ differ from (17), (18) in opposite signs before the fractions $1/r_{M_2P}^2$ and $1/r_{M_2'P}^2$, $1/r_{M_1'P}^2$ and $1/r_{M_2'P}^2$. The correctness of the transforms is confirmed by the correspondence of the obtained formulas to the relations (6), (7).

Figures 3, 4 show the magnetic field lines of the initial and similar magnetic fields $A(x, y) = \text{const}$, calculated by (16), (19), (20) for the distributions of the magnetic flux density 2 in Fig. 2, a, b. It is accepted that $A^* = A/A_b$, $A_b = \mu_0 I_M$, $A_b = A_{1b} \vee A_{2b}$, $A_{1b} = A_{2b}$, $l_{1b} = l_{2b}$. For magnetic field lines 1, 5 – $A_1^* = A_2^* = 0,05$, 2, 6 – 0,1, 3, 7 – 0,15, 4, 8 – 0,2.

We see that the corresponding field lines of the considered magnetic fields are geometrically similar, which confirms the correctness of the obtained results. The field lines shown in Fig. 4, a, b, limit the profiles of current-conducting inductors to create pulsed or high-frequency magnetic fields of given distributions on the axis y_2 .

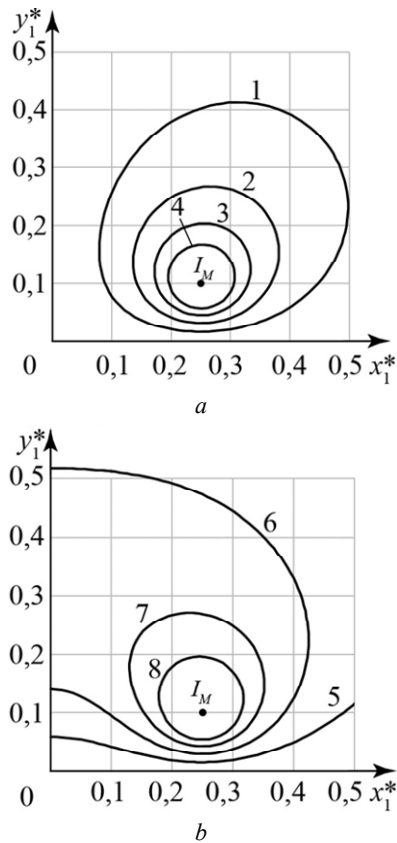


Fig. 3. Magnetic field lines at odd (a) and even (b) distributions of the magnetic flux density on the axis x_1

The results obtained for the magnetic field can be used to determine the profiles of one or more long parallel uniformly charged electrodes, with the help of which an electrostatic field of a given distribution is to be created on the flat surface of the conductor. For this we use the electrostatic analogy of plane electrostatic and magnetic fields of conductors with a sharp skin-effect (J.D. Cockroft, 1929, [4]), according to which the distributions of the taken with a minus sign projection of the electric field strength $E_{1y}(x_1, 0)$ and $B_{1x}(x_1, 0)$ correspond to one another.

Appendix. The use of two methods for solving field continuation problems allows not only checking the results, but also obtaining formulas for calculating complex improper integrals that are absent in the reference literature [11]. For example, comparing formulas (14) and (19) for the initial field, we have two improper integrals:

$$\int_0^{\infty} e^{-y_M \lambda} \lambda^{-1} \left\{ \begin{array}{l} \sin(x_M \lambda) \sin(x \lambda) \\ \cos(x_M \lambda) \cos(x \lambda) \end{array} \right\} \text{sh}(y \lambda) d\lambda =$$

$$= \frac{1}{8} \ln \left\{ \frac{(x - x_M)^2 + (y + y_M)^2}{(x - x_M)^2 + (y - y_M)^2} \times \right.$$

$$\left. \times \left[\frac{(x + x_M)^2 + (y - y_M)^2}{(x + x_M)^2 + (y + y_M)^2} \right] \right\}. \quad (21)$$

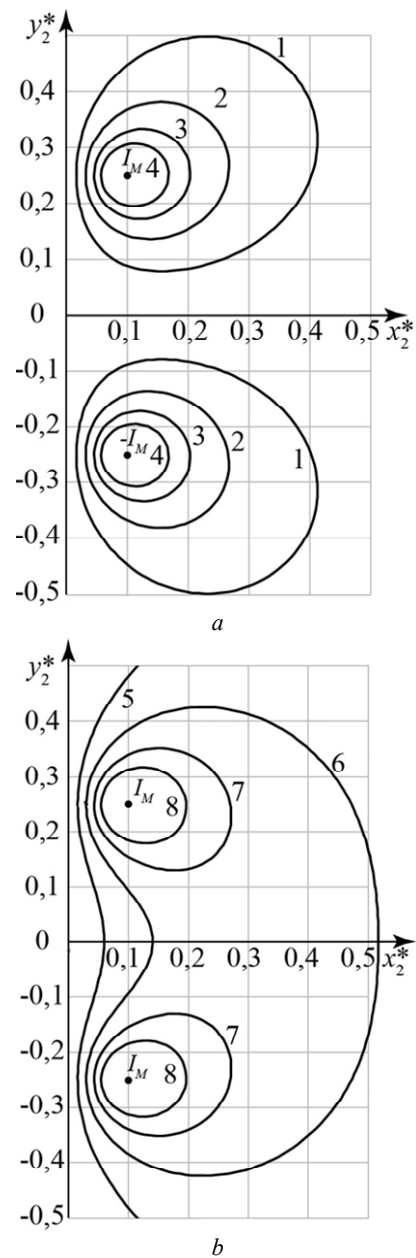


Fig. 4. Magnetic field lines at odd (a) and even (b) distributions of the magnetic flux density on the axis x_2

The limits of x and y change are the same as in (14). In the described way, one can obtain several more formulas for calculating improper integrals using (17), (18), as well as the corresponding formulas for a similar field. Comparison of (15) with (20) leads to a number of improper integrals. For instance:

$$\int_0^{\infty} e^{-x_M \lambda} \lambda^{-1} \left\{ \begin{array}{l} \sin(y_M \lambda) \sin(y \lambda) \\ \cos(y_M \lambda) \cos(y \lambda) \end{array} \right\} \text{sh}(x \lambda) d\lambda =$$

$$= \frac{1}{8} \ln \left\{ \frac{(x + x_M)^2 + (y - y_M)^2}{(x - x_M)^2 + (y - y_M)^2} \times \right.$$

$$\left. \times \left[\frac{(x - x_M)^2 + (y + y_M)^2}{(x + x_M)^2 + (y + y_M)^2} \right] \right\}. \quad (22)$$

We see that formula (22) differ from (21) only by similar values of a similar magnetic field. In addition, it is necessary to take into account also other limits of variation of x and y (see formula (15)).

Conclusions.

1. For the similarity of plane pulsed or high-frequency magnetic fields continued into a non-magnetic and non-conductive medium from different axes of Cartesian coordinates that bound the flat surfaces of the conductors, it is necessary and sufficient that the values of the corresponding projections of the magnetic flux density presented in the criterial form at similar points of the axes are proportional. This condition makes it possible to find the distribution of the magnetic flux density on the axis from which the similar field continues.

2. Solutions to the problems of the continuation of similar magnetic fields can be obtained from the known solutions of the problems of the continuation of the initial fields by replacing the coordinates and parameters in them with the corresponding similar quantities.

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