

CALCULATION AND DESIGN OF A ROBUST SPEED CONTROLLER OF A FREQUENCY-CONTROLLED INDUCTION ELECTRIC DRIVE

Purpose. The aim of the work is the calculation and design of a robust speed controller of a frequency-controlled induction electric drive with parametric uncertainty and the presence of interferences in the feedback channel. *Methodology.* The calculation and design of the controller was carried out in four stages. At the first stage, a linearized mathematical model of the control object with parametric uncertainty was constructed and the transfer function of the H_∞ -suboptimal controller was calculated in the Robust Control Toolbox using the mixed sensitivity method. At the second stage, the stability of the robust system and the accuracy of stabilization of the induction machine speed with random variations of the object's and controller's uncertain parameters within the specified boundaries were explored. At the third stage, the influence of interferences arising in the feedback channel on the speed of the electric motor was explored in the Simulink package. At the final stage, the transfer function of the H_∞ -suboptimal controller was decomposed into a continued fraction using the Euclidean algorithm. This fraction was used to build the electric scheme of the controller. *Results.* Computer modelling of the transfer function of H_∞ -suboptimal controller, the robust stabilization system for the speed of the frequency-controlled electric drive with random variations of the uncertain parameters of the object and the controller at specified boundaries, as well as with the presence of varying intensity interferences in the feedback channel, was carried out. The choice of variable parameters was carried out according to the Monte-Carlo method. The curves of transient processes of the induction machine speed with parametric uncertainty and at different ranges of interference are constructed, as well as a Bode diagram for an open system. By the scatter of the obtained curves of the transient processes, the accuracy of speed stabilization of the machine was determined, and according to the Bode diagram, stability reserves in the amplitude and the phase of the robust system were determined. They are within tolerances with comparatively large deviations of the varied parameters and the range of interferences. Based on the investigations, an electrical circuit of the H_∞ -suboptimal robust controller was developed. *Originality.* The mathematical model has been developed and the methodology for calculating and designing of H_∞ -suboptimal robust speed controller of the frequency-controlled system of an induction electric drive with random variations of the uncertain parameters of the object and the controller at determined boundaries and the presence of interferences in the feedback channel, ensuring the stability of the system with allowable reserves of the amplitude and the phase and high accuracy of speed stabilization of the machine within the tolerances of uncertain system parameters and interferences was proposed. *Practical value.* The obtained structure of the controller from analog elements makes it possible to carry out modernization of the electric drives frequency-controlled systems in operation with minimal financial costs. References 11, figures 7.

Key words: induction electric drive, frequency control, robust controller, electric circuit.

Мета. Метою роботи є розрахунок і проектування робастного регулятора швидкості системи частотного управління асинхронного електроприводу з параметричною невизначеністю та наявністю перешкод в каналі зворотного зв'язку. *Методологія.* Розрахунок і проектування регулятора проводився в чотири етапи. На першому етапі будувалася лінеаризована математична модель об'єкта управління з параметричною невизначеністю і розраховувалася в пакеті Robust Control Toolbox передавальна функція H_∞ -субоптимального регулятора за методом мішаної чутливості. На другому етапі досліджувалася стійкість робастної системи і точність стабілізації швидкості асинхронної машини при випадкових варіаціях невизначених параметрів об'єкта і регулятора в заданих межах. На третьому етапі вивчався в пакеті Simulink вплив перешкод, що виникають в каналі зворотного зв'язку, на швидкість електродвигуна. На заключному етапі виконувалося розв'язання передавальної функції H_∞ -субоптимального регулятора в ланцюгову дріб за алгоритмом Евкліда. Ця дріб використовувалася для побудови електричної схеми регулятора. *Результати.* Проведено комп'ютерне моделювання передавальної функції H_∞ -субоптимального регулятора, системи робастної стабілізації швидкості частотно-регульованого електроприводу при випадкових варіаціях невизначених параметрів об'єкта і регулятора в заданих межах, а також при наявності перешкод різної інтенсивності в каналі зворотного зв'язку. Вибір варіюваних параметрів здійснювався за методом Монте-Карло. Побудовано криві перехідних процесів швидкості асинхронної машини з параметричною невизначеністю і при розмахах перешкод, а також діаграма Бодє для розімкнутої системи. За розкладом отриманих кривих перехідних процесів визначалася точність стабілізації швидкості машини, а по діаграмі Бодє – запаси стійкості за амплітудою і фазою робастної системи. Вони знаходяться в межах допусків при порівняно великих відхиленнях варіюваних параметрів і розмахах перешкод. На базі проведених досліджень розроблено електричну схему H_∞ -субоптимального робастного регулятора. *Новизна.* Розроблена математична модель та запропонована методика розрахунку і проектування H_∞ -субоптимального робастного регулятора швидкості системи частотного управління асинхронного електроприводу при випадкових варіаціях невизначених параметрів об'єкта і регулятора в заданих межах і наявності перешкод в каналі зворотного зв'язку, яка забезпечує стійкість системи з запасами за амплітудою і фазою, що допускаються, та високу точність стабілізації швидкості машини в межах допусків невизначених параметрів системи і перешкод. *Практичне значення.* Отримана структура регулятора з аналогових елементів дає можливість проводити модернізацію систем частотного управління електроприводів, що знаходяться в експлуатації, з мінімальними фінансовими витратами. Бібл. 11, рис. 7.

Ключові слова: електропривод асинхронний, частотне управління, робастний регулятор, електрична схема.

Цель. Целью работы является расчет и проектирование робастного регулятора скорости частотно-регулируемого асинхронного электропривода с параметрической неопределенностью и наличием помех в канале обратной связи.

Методология. Расчет и проектирование регулятора проводился в четыре этапа. На первом этапе строилась линеаризованная математическая модель объекта управления с параметрической неопределенностью и рассматривалась в пакете Robust Control Toolbox передаточная функция H_∞ -субоптимального регулятора по методу смешанной чувствительности. На втором этапе исследовалась устойчивость робастной системы и точность стабилизации скорости асинхронной машины при случайных вариациях неопределенных параметров объекта и регулятора в заданных границах. На третьем этапе изучалось в пакете Simulink влияние помех, возникающих в канале обратной связи, на скорость электродвигателя. На заключительном этапе выполнялось разложение передаточной функции H_∞ -субоптимального регулятора в цепную дробь по алгоритму Евклида. Эта дробь использовалась для построения электрической схемы регулятора. **Результаты.** Проведено компьютерное моделирование передаточной функции H_∞ -субоптимального регулятора, системы робастной стабилизации скорости частотно-регулируемого электропривода при случайных вариациях неопределенных параметров объекта и регулятора в заданных границах, а также при наличии помех различной интенсивности в канале обратной связи. Выбор варьируемых параметров осуществлялся по методу Монте-Карло. Построены кривые переходных процессов скорости асинхронной машины с параметрической неопределенностью и при размахах помех, а также диаграмма Бode для разомкнутой системы. По разбросу полученных кривых переходных процессов определялась точность стабилизации скорости машины, а по диаграмме Бode – запасы устойчивости по амплитуде и фазе робастной системы. Они находятся в пределах допусков при сравнительно больших отклонениях варьируемых параметров и размахах помех. На базе проведенных исследований разработана электрическая схема H_∞ -субоптимального робастного регулятора. **Новизна.** Разработана математическая модель и предложена методика расчета и проектирования H_∞ -субоптимального робастного регулятора скорости системы частотного управления асинхронного электропривода при случайных вариациях неопределенных параметров объекта и регулятора в заданных границах и наличии помех в канале обратной связи, обеспечивающая устойчивость системы с допустимыми запасами по амплитуде и фазе и высокую точность стабилизации скорости машины в пределах допусков неопределенных параметров системы и помех. **Практическое значение.** Полученная структура регулятора из аналоговых элементов дает возможность проводить модернизацию систем частотного управления электроприводов, находящихся в эксплуатации, с минимальными финансовыми затратами. Библиограф. 11, рис. 7.

Ключевые слова: электропривод асинхронный, частотное управление, робастный регулятор, электрическая схема.

Introduction. In frequency-controlled induction electric drives operating in conditions of uncertainty, the task of robust stabilization of the motor speed with a given accuracy is essential. Several methods are known [1-5], which are most often used at different times by domestic and foreign scientists to solve this problem. Of these, the method of synthesizing the stabilizing H_∞ -suboptimal robust regulator is most widely used. In [6-9], on the basis of this method, a scientific research methodology, a calculation procedure, and an electrical circuit of a stabilizing H_∞ -controller of the rotor flux linkage control system for random variations of undefined parameters at specified boundaries and interference in the feedback channel are developed.

In the present work, this methodology is used to construct a mathematical model as well as a method for calculating and designing the electrical circuit of the H_∞ -suboptimal robust speed controller of the frequency control system of an induction electric drive.

The goal of the work is the calculation and design of a robust speed controller for the frequency control system of an induction electric drive with parametric uncertainty and the presence of interference in the feedback channel.

Research methods and results. Figure 1 is a structural diagram of the mechanical characteristic of the control object linearized within the working area in the input-output signal space [10, 11]. It contains the transfer functions of a frequency converter with the transmission coefficient K_{fc} and the time constant T_{fc} and a squirrel-cage induction motor. An induction motor is represented by a first-order aperiodic link with the stiffness module β and the electromagnetic time constant T_e and an integrating link with inertia moment J , taking into account the inertia moment of the actuator reduced to the rotor axis. The load torque (static moment of resistance) will be

considered constant and applied to the rotor in steady state. Therefore, its increment M_{rs} at the working point of the static mechanical characteristic is neglected. The increment of the electromagnetic torque M of the motor at the same point is taken equal to this torque [10, 11].

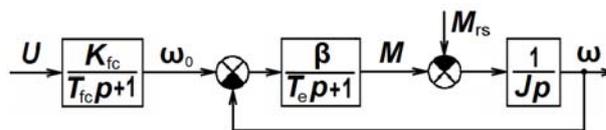


Fig. 1. Structural diagram of an induction electric drive

We pass from the structural diagram of the object to the equations of state in normal operator form:

$$\begin{aligned} p\omega &= \frac{1}{J}M; \\ pM &= -\frac{1}{T_e}M + \frac{\beta}{T_e}(\omega_0 - \omega); \\ p\omega_0 &= -\frac{1}{T_{fc}}\omega_0 + \frac{K_{fc}}{T_{fc}}U, \end{aligned} \quad (1)$$

where

$$\beta = \frac{2M_{cr}}{\omega_{0n}s_{cr}}; \quad T_e = \frac{1}{z_p\omega_{0n}s_{cr}}; \quad (2)$$

p is the Laplace operator; U is the control action; ω , ω_0 are the angular speed of the rotor and the rotating magnetic field relative to the stator, respectively; M , M_{cr} are, respectively, the electromagnetic and critical torques of the motor; s_{cr} is the critical slip; z_p is the number of pole pairs; n is the index of nominal values.

We introduce dimensionless quantities

$$x_1 = \frac{\omega}{\omega_n}; \quad x_2 = \frac{M}{M_n}; \quad x_3 = \frac{\omega_0}{\omega_{0n}}; \quad u = \frac{U}{U_n}. \quad (3)$$

$$B_2^T = \begin{bmatrix} 0 & 0 & \frac{1}{T_{fc}} \end{bmatrix}; C_2 = [1 \ 0 \ 0];$$

$$D_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 2z_p M_{crn} p \beta & 0 \\ 0 & 0 & -p \beta & 0 \\ 0 & 0 & 0 & -p J \end{bmatrix}; D_{12}^T = [1 \ 0 \ 0 \ 0];$$

$$D_{21} = [0 \ 0 \ 0 \ 0]; D_{22} = [0];$$

$x = (x_1, x_2, x_3)^T$ is the phase vector; y is the one-dimensional output vector along which feedback is closed; $z = (z_1, z_2, z_3, z_4)^T$, $w = (w_1, w_2, w_3, w_4)^T$ are the input and output uncertainty vectors, which are related by the matrix expression $w(p) = \Delta(p)z(p)$ in which the uncertainty matrix $\Delta(p)$ has a diagonal form.

The resulting system of equations (6) allows, together with the weighting functions proposed in [4] for quality control of a robust system, to calculate the transfer function of the H_∞ -suboptimal controller for a nominal object in the Robust Control Toolbox package using the mixed sensitivity method. For an induction electric drive with MDXMA 100-32 motor with power of 3 kW, parameters $z_p = 2$; $M_n = 20.2$ N·m; $M_{crn} = 48.5$ N·m; $J_n = 0.013$ kg·m²; $\omega_n = 148.178$ rad/s; $\omega_{0n} = 157.08$ rad/s; $\beta_n = 1.908$ N·m/(rad/s) and a frequency converter with the transmission coefficient $K_{fc} = 1.06$ rad/(V·s) and the time constant $T_{fc} = 10^{-4}$ s, the calculated transfer function of the H_∞ -controller turned out to be equal to:

$$K(p) = \frac{b_1 p^2 + b_2 p + b_3}{a_1 p^3 + a_2 p^2 + a_3 p + a_4}, \quad (7)$$

where $a_1 = 1$; $a_2 = 1.524 \cdot 10^5$; $a_3 = 1.261 \cdot 10^6$; $a_4 = 4.729 \cdot 10^6$; $b_1 = 3.53 \cdot 10^5$; $b_2 = 7.385 \cdot 10^6$; $b_3 = 5.681 \cdot 10^8$.

Using the MATLAB commands, we attach the robust controller (7) and the unit feedback encompassing the « H_∞ -controller-object» system to the object (4) programmatically. Using the Monte Carlo method [4], we study the accuracy of stabilization of the angular speed of the machine and the stability of the resulting system with random variations of the uncertain parameters of the object K_{fc} , M_{cr} in the range of $\pm 15\%$, β in the range of $\pm 30\%$, J in the range of $\pm 25\%$ and the coefficients $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ of the regulator (7) in the range of $\pm 15\%$.

Figure 4 presents 20 generated transient curves of the angular speed of the rotor of the induction motor with a single spasmodic change in the signal at the input of the system and a random selection of the uncertain parameters of the object and controller from the given ranges by the Monte Carlo method.

As expected, presented in Fig. 4 curves do not go beyond the boundaries of the 3% tube.

To study the stability of the system, we apply the method of logarithmic frequency characteristics with a random selection by the Monte Carlo method of the indefinite parameters of the object and controller in the given ranges.

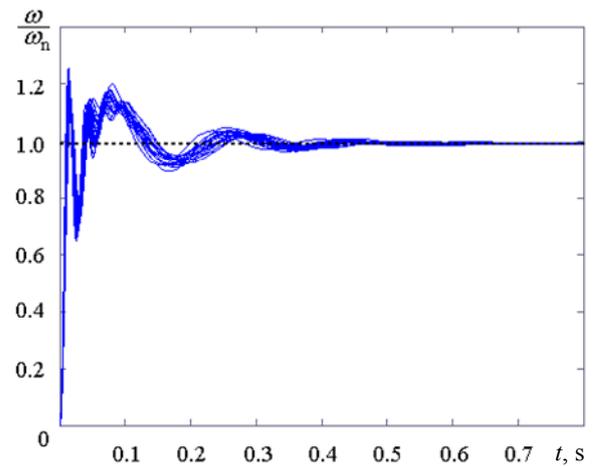


Fig. 4. Transients of the rotor angular speed

Figure 5 shows a Bode diagram with 20 generated curves of amplitude $L(\omega)$ and with 20 curves of phase $\varphi(\omega)$ frequency response curves for the same parameters what were used to calculate the curves shown in Fig. 4.

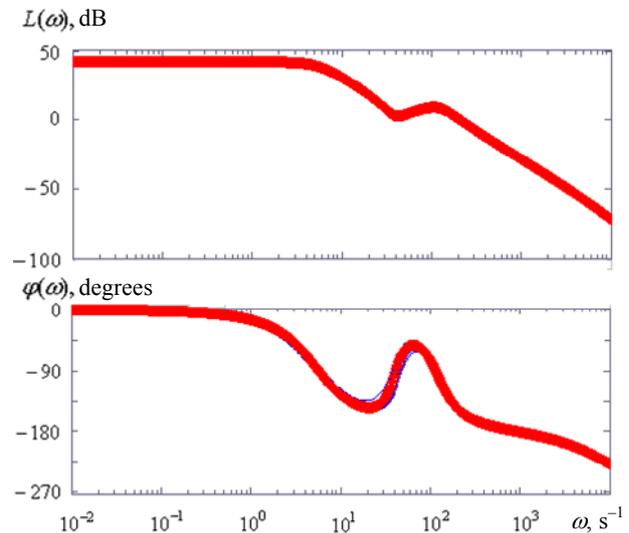


Fig. 5. Open system's Bode diagram

From the amplitude $L(\omega)$ and phase $\varphi(\omega)$ characteristics presented in this diagram, it can be seen that the system is stable, since the amplitude characteristic crosses the abscissa axis earlier than the phase characteristic, finally decaying, goes over the angle value of -180° . In this case, the calculated value of the stability margin in amplitude is 23.12 dB, and in phase -31.75° for the nominal values of the parameters of the object and the regulator with scatter of random curves not exceeding 4 dB for amplitude and 15° for phase frequency characteristics.

We proceed to the construction of the electrical circuit of the H_∞ -suboptimal robust controller.

We decompose the transfer function (7) into a continued fraction according to the Euclidean algorithm:

$$\frac{1}{\frac{28,33}{10^7} p + \frac{1}{2,32 + \frac{34,1}{10^3} p + \frac{1}{-0,252 + \frac{1}{-\frac{31,78}{10^3} p + \frac{1}{r}}}}}, \quad (8)$$

where $r = 118.1$.

The electric circuit of the controller corresponding to the fraction (8) is shown in Fig. 6. When it was created, well-known methods and rules for performing electrical circuits were used.

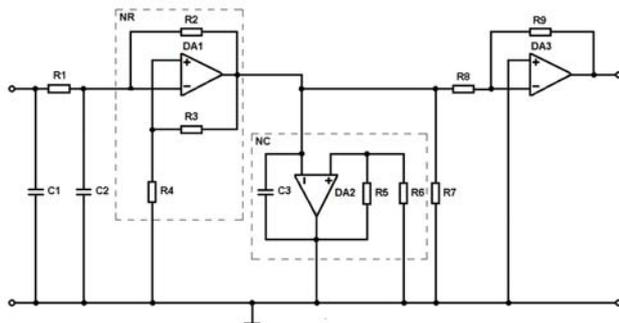


Fig. 6. Electric circuit of a robust controller

The circuit shown in Fig. 6, is made in the form of a four-terminal network and consists of a series-connected the first passive four-terminal network with a capacitor C1 connected in parallel, the second passive four-terminal network with a resistor R1 connected in series and a parallel capacitor C2, the third active four-terminal network with a negatron of the negative resistance NR in series, consisting of an operational amplifier DA1 and resistances R2, R3, R4, the fourth active four-terminal with a parallel connected negatron of the negative capacitance NC, consisting of an operational amplifier DA2, a capacitor C3 and resistors R5, R6, the fifth passive four-terminal with a parallel resistor R7, and an operational amplifier DA3 with resistors R8 and R9 connected to the output of the fifth four-terminal.

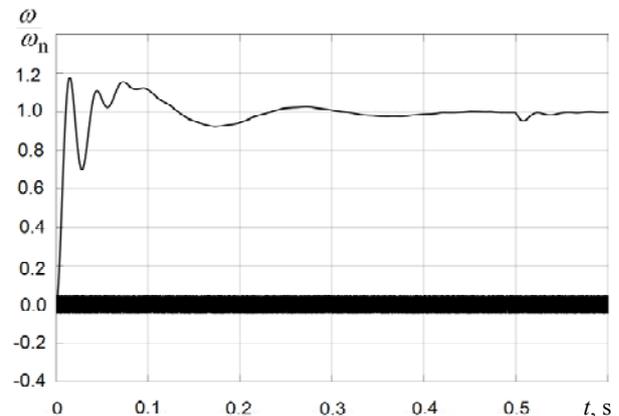
Parameters of its capacitors and resistors $C1 = 28.4 \text{ pF}$; $R1 = 2.32 \text{ k}\Omega$; $C2 = 34 \text{ }\mu\text{F}$; $R2 = 252 \text{ }\Omega$; $C3 = 31.8 \text{ }\mu\text{F}$; $R7 = 118 \text{ k}\Omega$; $R8 = 1 \text{ M}\Omega$; $R9 = 1 \text{ k}\Omega$ and they correspond to the standard values of rounded coefficients, fraction (8) when multiplying its numerator and denominator by a certain constant number, and $R3 = R4$ and $R5 = R6$ are chosen from design considerations.

As calculations performed by the method of [7] show, at such capacitance and resistance values, the values of the coefficients $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ of controller (7) do not go beyond the limits of the range specified above $\pm 15 \%$.

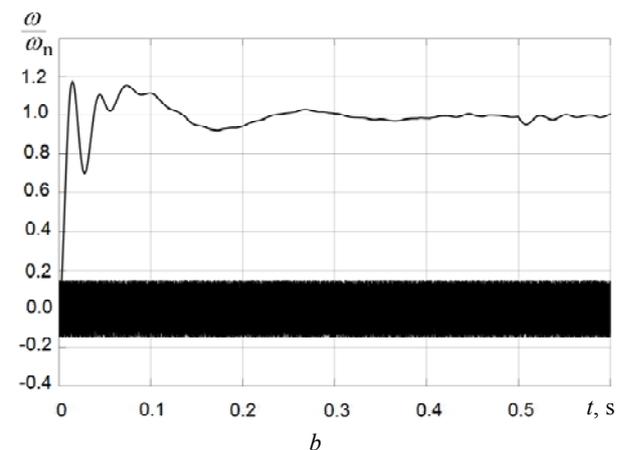
In the robust control system, interferences can occur, caused, for example, by sensor noise, connector pins, electromagnetic fields, interference with the frequency of the supply network and other reasons. A robust regulator, as an element of this system, is known to be able to filter these interferences. Therefore, the calculations of transients of the angular speed of the electric motor were

performed at various values of the noise intensity in the feedback channel of the frequency control system with a robust controller. The calculations were carried out in the Simulink package. A static motor load of $0.75 \cdot M_n$ was applied to the rotor of the machine in steady state.

The results of calculating the curves of transients of the angular speed of the rotor, filtered by the robust control system, for two different values of the generated span of interference with a single spasmodic change in the reference action are shown in Fig. 7.



a



b

Fig.7. Transients of the angular speed ω/ω_n with interferences filtered by a robust system and a rotor load of $0.75M_n$ at time 0.5 s: a – interference span of 10%; b – 30%

An analysis of these curves shows that the level of interference filtering by the robust system largely depends on the intensity of the interference span and in the steady state it is within the tolerance range of $\pm 2.5 \%$, except for the local area of the application of the M_{rs} load at 0.5 s.

Conclusions.

1. A mathematical model and a technique for calculating and designing an electrical circuit of the H_∞ -suboptimal robust speed controller of the frequency control system of an induction electric drive with random variations of the uncertain parameters of the object and the controller within the specified boundaries and the presence of interferences in the feedback channel are developed.

2. The results of modelling of transients of the angular speed of the rotor according to the developed technique confirm the high accuracy of stabilization with random

variations of uncertain parameters at given boundaries and low sensitivity to interferences in the feedback channel.

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