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MODELLING OF DYNAMIC MODES OF AN INDUCTION ELECTRIC DRIVE AT PERIODIC LOAD

Goal. Development of methods and mathematical models, based on them, for the calculation of transients and steady-state modes of induction electric drives operating in periodic load mode. Methodology. The developed algorithms are based on a mathematical model of an induction motor, which takes into account the saturation of the magnetic core and the displacement of current in the rotor bars. The processes are described by a system of nonlinear differential equations in the orthogonal axes x, y, which enables the results to be obtained with the smallest amount of calculations. The magnetization characteristics by the main magnetic flux and the leakage fluxes are used to calculate the electromagnetic parameters of the motor. To account for the current displacement in the rotor bars, the short-circuited winding is considered as a multilayer structure formed by dividing the bars in height by several elements. Results. Due to the variable load on the motor shaft, electromagnetic processes in both transient and steady state modes of the electric drive in any coordinate system are described by a system of nonlinear differential equations. The result of the calculation of the transients is obtained as a result of their integration time dependencies of coordinates (currents, electromagnetic torque, etc.) at a given law of change of the moment of loading. The proposed method of calculating steady-state mode is based on algebraization of differential equations on the mesh of nodes of the process cyclicity period and allows to obtain periodic dependencies in the time domain. Originality. The problem of calculating a steady-state periodic mode is solved as a boundary problem for a system of first-order differential equations with periodic boundary conditions, which allows to obtain instantaneous dependences during the period of currents, electromagnetic torque, capacities and other coordinates. Practical significance. Using the developed algorithm, it is possible to calculate the static characteristics of periodic processes as dependencies on different parameters of the cycle of periodic load or other coordinates, which is the basis for the choice of the motor for overload, power, heating, etc., as well as to detect the possibility of resonance. References 9, figures 4.
Key words: **induction motor, periodic load, mathematical model, steady-state dynamic mode, transient, static characteristics, saturation of the magnetic core, displacement of current.**

Розроблено математичні моделі і алгоритми, з використанням яких складені програми розрахунку перехідних процесів і усталених режимів асинхронних електроприводів, які працюють в режимі періодичної зміни навантаження. В їх основу покладено математичну модель асинхронного двигуна, розроблену на основі теорії кіл і зображувальних векторів електричних координат, в якій враховується насичення магнітопроводу і витіснення струму в стержнях короткозамкненого ротора. Внаслідок змінного навантаження на валу двигуна електромагнітні процеси як в перехідних, так і усталених режимах в будь-якій системі координат описуються системою нелінійних диференціальних рівнянь. В роботі використано систему ортогональних координатних осей x, y, яка обертається з довільною швидкістю. Для обчислення електромагнітних параметрів двигуна використовуються характеристики намагнічування основним магнітним потоком, а також потоками розсіювання статора і ротора. Для урахування витіснення струму в стержнях ротора короткозамкнена обмотка подається у вигляді багатошарової структури, утвореної розбиттям стержнів по висоті на кілька елементів. Усталений періодичний режим розраховується методом розв'язування крайової задачі, розробленим на основі апроксимації координат кубічними сплайнами, що дає змогу отримати їх періодичні залежності в позачасовій області і розраховувати статичні характеристики як залежності від параметрів циклу періодично-змінного навантаження або інших координат. Бібл. 9, рис. 4.
Ключові слова: **асинхронний двигун, періодичне навантаження, математична модель, усталений динамічний режим, перехідний процес, крайова задача, резонанс, статичні характеристики, насичення магнітопроводу, витіснення струму.**

Разработаны математические модели и алгоритмы, с использованием которых составлены программы расчета переходных процессов и установившихся режимов асинхронных электроприводов, которые работают в режиме периодического изменения нагрузки. В их основу положено математическую модель асинхронного двигателя, разработанную на основе теории цепей и изображающих векторов электрических координат, в которой учитывается насыщение магнитопровода и вытеснение тока в стержнях ротора. Вследствие переменной нагрузки на валу двигателя электромагнитные процессы как в переходных, так установившихся режимах в любой системе координат описываются системой нелинейных дифференциальных уравнений. В работе используется система ортогональных осей x, y, которая вращается с произвольной скоростью. Для вычисления электромагнитных параметров двигателя используются характеристики намагничивания основным магнитным потоком, а также потоками рассеивания статора и ротора. Для учета вытеснения тока в стержнях ротора короткозамкнутая обмотка представляется в виде многослойной структуры, образованной разделением стержней по высоте на несколько элементов. Установившийся периодический режим рассчитывается методом решения краевой задачи, разработанным на основе аппроксимации координат кубическими сплайнами, что дает возможность получить периодические зависимости во вневременной области и рассчитать статические характеристики как зависимости от параметров цикла периодически изменяющейся нагрузки или других координат. Библ. 9, рис. 4.
Ключевые слова: **асинхронный двигатель, периодическая нагрузка, математическая модель, установившийся динамический режим, переходный процесс, статические характеристики, резонанс, насыщение магнитопровода, вытеснение тока.**

Introduction. In modern conditions of development of science and technology, the problem of the development of induction electric drives requires new approaches to their practical implementation, which can be realized only on the basis of the development of adequate mathematical models of electric drive systems that are adapted to their operating conditions. Their use allows not only to correctly select the necessary induction motor (IM), but also to develop a control system under which the motor, operating under these conditions, would ensure the maximum possible efficiency of the electric drive system as a whole.

Modern factory methods make it possible to design an IM that is highly likely to meet the technical conditions of operation in a steady nominal mode with constant load. Such calculations are usually performed using classical substitution circuits [1, 2], but classical substitution circuits are not suitable for the calculation of dynamic modes, and their various adaptations need to be checked on a case-by-case basis.

In the practice of operation, IMs are used not only for actuating mechanisms that operate with unchanged mechanical torque of loading, but also for drives with periodic repeated short-term loading [3, 4]. The duration of the cycle of periodic re-alternating load *T* consists of two parts: the duration of the action of the load pulse and the pause. In particular, for repeated short-term operation (*S*3), the duration of the load pulse is expressed as a percentage to the duration of the full cycle. Switch-on time (ST) = 15; 25; 40; 60 % (e.g. *S*3 – 25 %; *S*3 – 40 %) is considered standard, with a cycle time of 10 minutes [5]. The industry produces IMs to operate in different standard-defined *S*3 modes. The selection of the motor power for the repeated short-term operation mode *S*3 can be made for equivalent power or torque for a given load schedule. Knowing the power of the catalog IM for motors designed to operate in a particular *S*3 mode, it is possible to select a motor to check for starting torque, overload capacity and heating [4].

Both standard long-run motors and motors specially designed for repeated short-term mode can operate in the repeated short-term mode. After all, often the values of the duration of switching on the IM are not standard. There is a need for a comprehensive study of the operation of the motor under the conditions specified by the working mechanism of the periodic load torque, which can be accomplished through mathematical modelling.

The goal of the work is to develop mathematical models for the analysis of the dynamic modes of induction motors operating under conditions of periodic-variable loading.

A mathematical model for the calculation of transients. For the analysis of the operation of electric drives operating in dynamic modes, mathematical models of IM, built on the basis of substituting circuits or linear differential equations (DEs) can be used only for approximate calculations. Because the electromagnetic torque is determined by the flux linkages and currents of the circuits of the motor, the inaccuracy of their determination leads to the inaccuracy of the calculation of the mechanical characteristic [1, 2]. In particular, the value of the inductive resistances of the windings is significantly influenced by the saturation of the magnetic core, the change in the active resistances of the rotor winding due to displacement of current. Taking them into account in dynamic modes with the help of corresponding coefficients [2] does not guarantee the accuracy of the calculation results, especially for deep-slot motors.

The object of study is an IM with short-circuited rotor winding, which is powered by a three-phase network with a symmetric voltage system. For the analysis of electromagnetic processes in IM, we use a mathematical model, created using orthogonal coordinate axes, which allows to consider processes by computer simulation taking into account both saturation and displacement of current in bars of the short-circuit rotor windings with minimal computation. The magnetization characteristics of the main magnetic flux and the scattering fluxes are used to take into account saturation, and to take into account displacement of current the bars are separated by a height into n layers ($2 \le n \le 5$) which results in the n windings being covered by different magnetic scattering fluxes. The calculation algorithms are based on a mathematical model of the IM in the x, y axes, developed on the basis of the theory of imaging vectors [7], which allows to consider processes in the IM based on the theory of circuits.

Dynamics of motion of the rotor of the IM, operating in the mode of periodically variable loading, is described by the system of the DEs of electromechanical equilibrium which in the system of orthogonal axes x, y taking into account the division of each bar in height into n elementary ones, as well as subjecting the image vector of the supply voltage along x axis that is commonly practiced looks like

$$\begin{aligned}
&\frac{d\psi_{sx}}{dt} = -\omega_0\psi_{sy} - R_s i_{sx} + U_m\,;\\
&\frac{d\psi_{sy}}{dt} = -\omega_0\psi_{sx} - R_s i_{sy}\,;\\
&\frac{d\psi_{1x}}{dt} = (\omega_0 - \omega)\psi_{1y} - R_1 i_{1x}\,;\\
&\frac{d\psi_{1y}}{dt} = -(\omega_0 - \omega)\psi_{1x} - R_1 i_{1y}\,;\\
&\vdots\\
&\frac{d\psi_{nx}}{dt} = (\omega_0 - \omega)\psi_{ny} - R_n i_{nx}\,;\\
&\frac{d\psi_{ny}}{dt} = -(\omega_0 - \omega)\psi_{nx} - R_n i_{ny}\,;\\
&\frac{d\omega}{dt} = \frac{p_0}{J}\left(\frac{3}{2}p_0\left(\psi_{sx}i_{sy} - \psi_{sy}i_{sx}\right) - M_c(t)\right),
\end{aligned} \tag{1}$$

where the indices sx, sy denote that the flux linkages (ψ), currents (i), and active resistances (r) belong to the corresponding stator circuits; and $1x,...,nx$, $1y,...,ny$ to the rotor ones; U_m, ω_0 are the amplitude value and angular frequency of the stator winding phase voltage; ω is the angular velocity of rotation of the rotor; J is the moment of inertia of the moving parts of the electric drive reduced to the shaft of the IM; p_0 is the number of pole pairs.

The loading diagram of the mechanism must be known for modelling. Taking into account that the time dependence of the load torque is periodic, it is necessary to represent it in the form of a law of change, which corresponds to a complete cycle in the form $M_c(t) = M_c(t + T)$, where T is the period.

An algorithm of calculation of characteristics. If the IM operates in one of the standard modes (full cycle is 10 minutes), then the transient is almost complete, and for a complete analysis of the motor operation it is enough to calculate the transient during the period. This can be done by integrating the DE system (1) using the numerical method [6].

DE system (1) includes 2 + 2n electrical equilibrium equations and one rotor dynamics equation. Therefore, when calculating the transient, it is necessary to rotate a matrix of the same order at each step (sub-step). In order to reduce the amount of calculations, we reduce the DE system (1), based on the following considerations.

The flux linkages of each IM circuit according to accepted assumptions consists of the sum

$$\psi_j = \psi_{\delta j} + \psi_{\sigma j}$$

of working flux linkage $\psi_{\delta j}$ which is nonlinearly dependent on the currents of all circuits, and scattering flux linkage $\psi_{\sigma j}$ which has a linear dependence, respectively, only on the stator currents or only on the rotor one. In addition, the flux linkage caused by the main work flow and the flux linkage of the slit scattering for all rotor circuits along the x axis are equal. The same applies to similar circuits along the y axis. The above makes it possible to divide the equilibrium equations of the DE system (1) into two parts by distinguishing a linear part in it. For this purpose it is necessary to replace the 5th equation by the difference of the 5th and the 3rd ones, the 6th equation by the difference of the 6th and 4th ones, etc. The first one is of the fourth order

$$\frac{d\psi_{sx}}{dt} = \omega_0 \psi_{sy} - R_s i_{sx} + U_m ;$$

$$\frac{d\psi_{sy}}{dt} = -\omega_0 \psi_{sx} - R_s i_{sy} ;$$

$$\frac{d\psi_{1x}}{dt} = (\omega_0 - \omega)\psi_{1y} - R_1 i_{1x} ;$$

$$\frac{d\psi_{1y}}{dt} = -(\omega_0 - \omega)\psi_{1x} - R_1 i_{1y}$$

and is nonlinear, and the second of the 2(n–1) order is linear

$$\frac{d(\psi_{1x} - \psi_{2x})}{dt} = (\omega_0 - \omega)(\psi_{1y} - \psi_{2y}) - r_1 i_{1x} + r_2 i_{2x} ;$$

$$\frac{d(\psi_{1y} - \psi_{2y})}{dt} = -(\omega_0 - \omega)(\psi_{1x} - \psi_{2x}) - r_1 i_{1y} + r_2 i_{2y} ;$$

$$\vdots$$

$$\frac{d(\psi_{1x} - \psi_{nx})}{dt} = (\omega_0 - \omega)(\psi_{1y} - \psi_{ny}) - r_1 i_{1x} + r_n i_{nx} ;$$

$$\frac{d(\psi_{1y} - \psi_{ny})}{dt} = -(\omega_0 - \omega)(\psi_{1x} - \psi_{nx}) - r_1 i_{1y} + r_n i_{ny} .$$

Write these two systems in the form

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} d\vec{i}_I / dt \\ d\vec{i}_{II} / dt \end{bmatrix} = \begin{bmatrix} \vec{B}_1 \\ \vec{B}_2 \end{bmatrix} . \qquad (2)$$

Determine the derivative of equation (2)

$$\frac{d\vec{i}_I}{dt} = \left(A_{11} - A_{12} A_{22}^{-1} A_{21}\right)^{-1} \left(\vec{B}_1 - A_{12} A_{22}^{-1} \vec{B}_2\right),$$

in which only the elements of the matrices A_{11} and A_{12} depend on saturation. This allows to calculate the elements of the matrices A_{22}^{-1} and A_{21} once and use them to determine at each step the integration the derivative

$$\frac{d\vec{i}_{II}}{dt} = A_{22}^{-1}\left(\vec{B}_2 - A_{21} \frac{d\vec{i}_I}{dt}\right).$$

Therefore, it is enough to rotate once the matrix of the 2(n–1) order and rotate the 4th order matrix at each integration step. The obtained formulas make it possible by numerical methods to reduce to Cauchy form the system (2) of the DEs of the electric equilibrium of the circuits.

Flux linkages of circuits are determined based on the use of magnetization curves by the main magnetic flux ψ_μ and the scattering fluxes of the stator $\psi_{\sigma s}$ and rotor $\psi_{\sigma r}$ windings

$$\psi_\mu = \psi_\mu(i_\mu), \; \psi_{\sigma s} = \psi_{\sigma s}(i_s), \; \psi_{\sigma r} = \psi_{\sigma r}(i_r),$$

where

$$i_\mu = \sqrt{(i_{sx} + i_{rx})^2 + (i_{sy} + i_{ry})^2} ;$$

$$i_s = \sqrt{i_{sx}^2 + i_{sy}^2} ; \qquad i_r = \sqrt{i_{rx}^2 + i_{ry}^2} .$$

The currents of the rotor circuits are defined as the sum of the currents of n bar elements

$$i_{rx} = \sum_{j=1}^{n} i_{rjx} ; \qquad i_{ry} = \sum_{j=1}^{n} i_{rjy} .$$

A mathematical model for calculating steady-state dynamic mode. In order to reduce the presentation of the computation of the steady-state dynamic calculation algorithm, we write the DE system (1) as a vector equation of the form

$$\frac{d\vec{x}}{dt} = \left(\frac{\partial \vec{y}}{\partial \vec{x}}\right)^{-1} \vec{z}\left(\vec{y}, \vec{x}, \vec{u}, \vec{f}\right), \qquad (3)$$

where $\frac{d\vec{y}}{dt} = \begin{bmatrix} L_{xy} & 0 \\ 0 & 1 \end{bmatrix}$ is the matrix in which $L_{xy} = \frac{d\vec{\psi}}{d\vec{i}}$ is the complete matrix of differential inductances of the IM in coordinate axes x, y [7];

$$\vec{y} = \begin{bmatrix} \psi_{sx} \\ \psi_{sy} \\ \psi_{1x} \\ \psi_{1y} \\ \vdots \\ \psi_{nx} \\ \psi_{ny} \\ \omega \end{bmatrix} ; \; \vec{u} = \begin{bmatrix} U_m \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix} ; \; \vec{f} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ M_c(t) \end{bmatrix} ; \; \vec{x} = \begin{bmatrix} i_{sx} \\ i_{sy} \\ i_{1x} \\ i_{1y} \\ \vdots \\ i_{nx} \\ i_{ny} \\ \omega \end{bmatrix} .$$

In steady state mode of the electric drive system with periodic change of the load torque $M(t) = M(t + T)$ flux linkages, currents, rotor speed, electromagnetic torque, etc. are varied by periodic laws. The task of calculating a periodic mode is to determine these dependencies. The solution of the system of equations (3) is the periodic dependencies of the components of the vector $\vec{x}(t) = \vec{x}(t+T)$. The calculation by the stable method is inefficient for many reasons. In particular, CPU time is wasted, and if the process is becomes stable too slowly, then the oscillations at time t are little different from those for time $t + T$, so there is a problem of determining the time when the transient ends. Finally, the stable method is practically unsuitable for optimization calculations.

The most effective approach to calculating a steady periodic mode is to consider the problem as a boundary one [7], which allows to obtain periodic dependencies of coordinates in the timeless domain, that is, without resorting to the calculation of the transient. To do this, the system of continuous DEs (1) must be reduced to discrete ones, which are a point mapping of the dependencies of the coordinates during the process repetition period. There are many methods of algebraization in the literature that have both positive and negative sides: difference, collocation, including trigonometric, differential transformations, etc. The method, based on spline approximations of coordinates, developed in [8], makes it possible to formalize the algebraization process and is also numerically stable. It allows to obtain continuous dependencies of coordinates on a period on the basis of the obtained by calculation their discrete values in nodes of a mesh on a period. Note that the mesh of nodes can be taken uniformly. In the system of algebraic equations obtained by the approximation of variables, the values of the coordinates in m nodes of the period are unknown. As a result, taking into account periodic boundary conditions $\vec{Y}(t) = \vec{Y}(t+T)$, $\vec{X}(t) = \vec{X}(t+T)$, we obtain the system of $m\times(2n+3)$ nonlinear algebraic equations, which can be represented as a vector equation

$$\vec{Y}(\vec{X}) = H^{-1}\vec{Z}(\vec{Y}, \vec{X}), \tag{4}$$

in which H is the square matrix of size $m(3+2n)$ of transition from continuous change of coordinates to their nodal values, whose elements are determined only by the mesh step [8]; $\vec{Y} = (\vec{y}_1, \ldots, \vec{y}_m)$, $\vec{Z} = (\vec{z}_1, \ldots, \vec{z}_m)$, $\vec{X} = (\vec{x}_1, \ldots, \vec{x}_m)$ are the vectors made up of values of vectors $\vec{y}$, $\vec{x}$, $\vec{z}$ in m nodes of the period.

By defining from vector (4) the vector $\vec{X}$, it is possible to construct periodic dependencies of all coordinates, including electromagnetic of torque, power, etc.

Direct application of the iterative method to the solution of system (4) is practically impossible due to the divergence of the iterative process. A reliable method of solving the problem is the method of continuation by parameter [9]. However, in the system of nonlinear algebraic equations there are two disturbing actions: applied voltage – vector $\vec{U} = (\vec{u}_1, \ldots, \vec{u}_m)$ and vector of nodal values of load torque – $\vec{F} = (\vec{f}_1, \ldots, \vec{f}_m)$. It is impossible to increase them at the same time, so the problem is solved in two stages, the essence of which is to increase them alternately in proportion to a certain parameter. First, we increase the applied voltage, and then, taking it unchanged, we increase the nodal values of the applied torque. This makes it possible to determine the time dependencies of the coordinates in the steady-state periodic mode of operation of the IM at a given law of change of the applied torque.

The steady-state mode calculation algorithm is the basis for the calculation of static characteristics, which can be obtained as a sequence of steady-state modes calculated with a set of coordinate values, which is taken as an independent variable, which can be any value: moment of inertia, pulse density of the load torque; ratio between pulse duration and pause, pulse rate, maximum and minimum torque, period duration, etc. In addition, under cyclic loading mechanical resonance is possible, which can be detected by mathematical modelling.

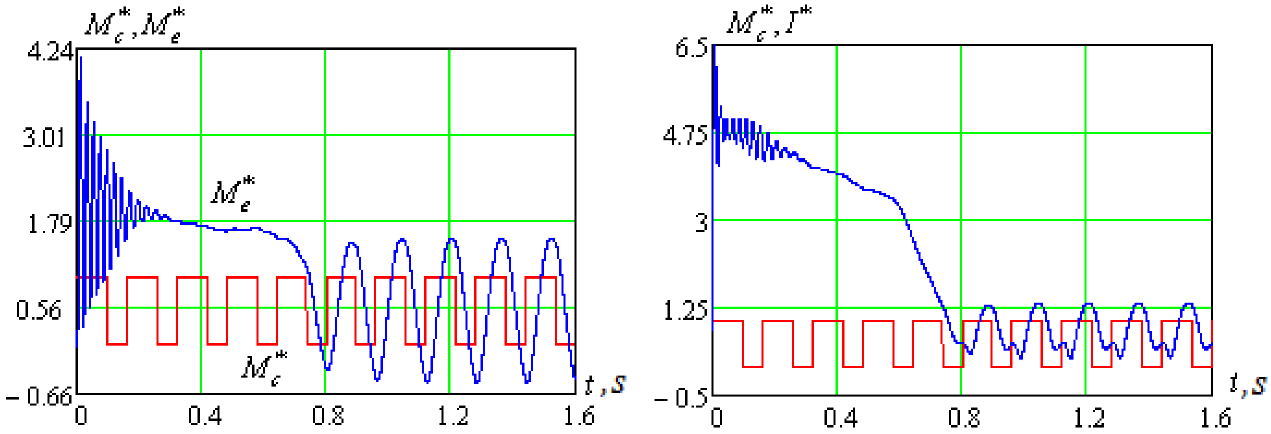
The problem of calculating static characteristics can be solved by a differential method, the essence of which is the differentiation of algebraic equation (4) on an independent variable, for example ε, as a parameter. As a result of differentiation, we obtain a nonlinear system of DE in the form

$$A\frac{d\vec{X}}{d\varepsilon} = \frac{\partial\vec{Z}}{\partial\varepsilon}. \tag{5}$$

The static multidimensional characteristic as a dependence of periodic curves on the independent variable ε is obtained by integrating system (5) with parameter ε. The initial conditions should be those obtained as a result of the first stage of the calculation at a given supply voltage. At each integration step, the result can be refined by the Newton method. During integration as well as iterative refinement, it is necessary to determine the differential inductance of circuits as nonlinear functions of currents.

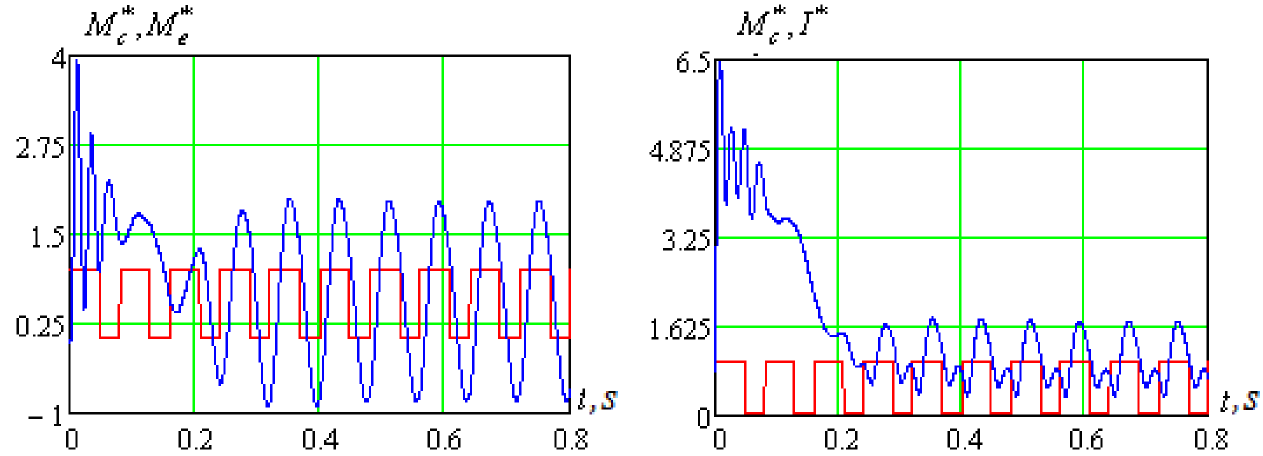
Results of investigations. Below are examples of calculation results performed using the above algorithms on the example of the IM with short-circuited rotor 4AP160S4Y3 (P = 15 kW, U = 220 V, I = 29.9 A, p_0 = 2).

Figure 1 shows the time dependences of the relative values of the electromagnetic torque (Fig. 1,*a*) and the current value (Fig. 1,*b*) in the transient during start up of the IM with cyclic loading, at which the torque of load varies with the period T = 0.16 s in the range from idling to nominal value, the moment of inertia J = 0.5 kg·m^2, and the density is 60 %, and Fig. 2 presents the same dependencies, but with less moment of inertia J = 0.1 kg·m^2.



a *b*

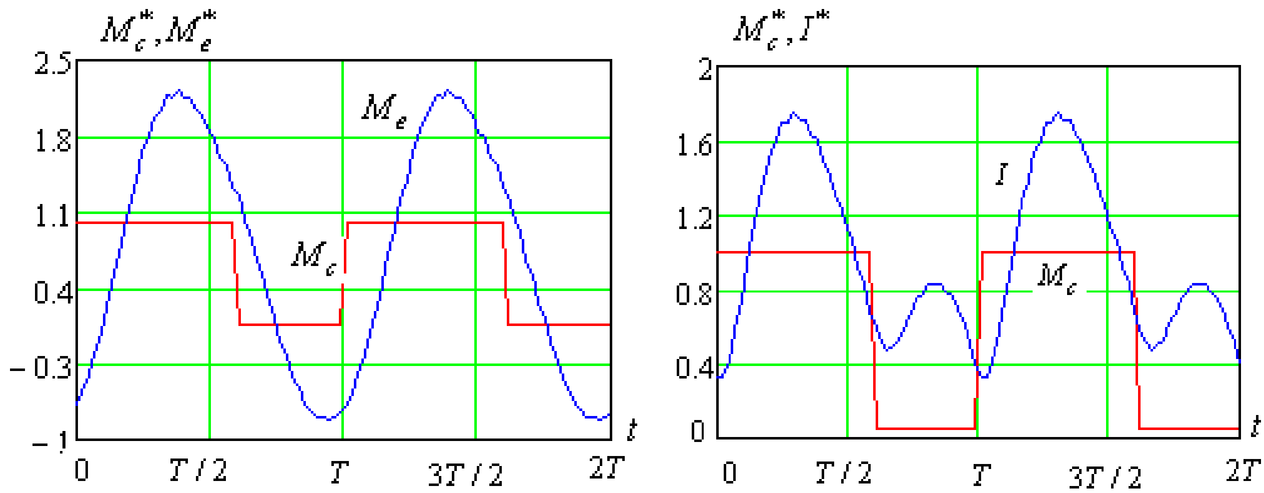
Fig. 1



a *b*

Fig. 2

Figures 3, 4 present an example of calculated by the method of solving the boundary value problem of periodic curves of current, electromagnetic torque and load, presented in this work which correspond dependencies in steady state shown in Fig. 2.



a *b*

Fig. 3. Periodic dependencies (two periods shown) of the relative values of the load torque (M_c^*), electromagnetic torque (M_e^*) and current (I^*) calculated at the moment of inertia $J = 0.1$ kg·m^2 by the method of solving the boundary value problem

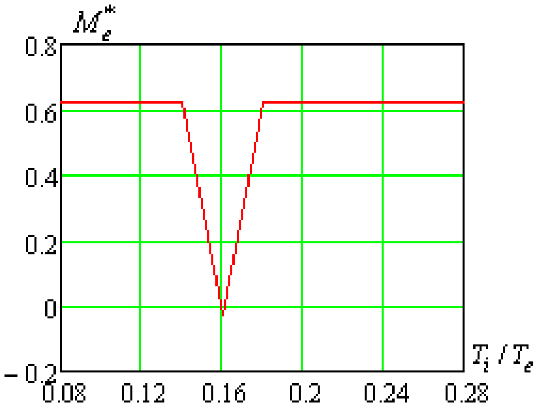


Fig. 4. Dependence of the electromagnetic torque of the motor on the relative value of the duration of the period of change of load (at the point T_i/T_e = 0.16 there is a mechanical resonance)

Conclusions.

1. The developed calculation methods and corresponding algorithms make it possible to use mathematical modelling to analyze the operation of induction motors with a short-circuited rotor taking into account the saturation and displacement of currents in the rotor bars under different laws of change of periodic load.
2. The algorithm of calculation of the steady-state periodic modes at cyclic loading allows to obtain the periodic dependencies of coordinates in the timeless domain, which ensures high speed.
3. The mathematical models developed can be used to design and analyze the operation of electric drives with periodic load.

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