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S.T. Tolmachev, S.L. Bondarevskyi, A.V. Il'chenko

MAGNETIC PROPERTIES OF MULTICOMPONENT HETEROGENEOUS MEDIA WITH A DOUBLY PERIODIC STRUCTURE

Heterogeneous media have a wide range of practical applications. Media with a doubly periodic structure (matrices of high-gradient magnetic separators, etc.) occupy an important place. Their study is usually based on experimental and approximate methods and is limited to simple two-phase systems. The development of universal and accurate methods of mathematical modelling of electrophysical processes in such environments is an urgent task. The aim of the paper is to develop a method for calculating local and effective parameters of a magnetostatic field with minimal restrictions on the number of phases, their geometry, concentration, and magnetic properties. Based on the theory of elliptic functions and secondary sources, an integral equation is formulated with respect to the magnetization vector of the elements of the main parallelogram of the periods. The calculated expressions for the complex potential, field strength, and components of the effective magnetic permeability tensor are obtained. The results of a series of computational experiments confirming the universality and effectiveness of the method are presented. As an example of a practical application, a detailed study of the field of the magnetic forces of the matrix is carried out: the lines of magnetic isodine and potential extraction areas for a complex version of the matrix are constructed. Within the framework of the developed method, the calculation of local and effective field characteristics is carried out by solving the field problem in the field of an arbitrary parallelogram of periods without specifying boundary conditions on its sides with a comprehensive consideration of significant interdependent factors. The practical value of the method is to create new opportunities for improving the technical characteristics of electrophysical devices for which the universality and accuracy of calculating local and effective field characteristics is decisive. An algorithm for optimizing the characteristics of the separator is proposed. References 16, figures 11.

Key words: **doubly periodic heterogeneous medium, integral equation, magnetization vector, strength field, homogenization problem, magnetic permeability tensor, polygradient separation, matrix, magnetic forces.**

Викладено метод розрахунку магнітостатичного поля в двоякоперіодичному гетерогенному середовищі. Сформульовано інтегральне рівняння відносно вектора намагніченості елементів середовища. Розрахунок характеристик поля виконується шляхом вирішення польової задачі в області основного паралелограма періодів без задання граничних умов на його сторонах. Отримано розрахункові вирази для напруженості поля і тензора магнітної проникності. Наведено результати обчислювальних експериментів, що підтверджують універсальність і ефективність методу. Проведено детальне дослідження поля магнітних сил матриці високоградієнтного магнітного сепаратора. Метод відкриває нові можливості підвищення технічних характеристик електрофізичних пристроїв, для яких універсальність і точність розрахунку локальних і ефективних характеристик поля є визначальною. Бібл. 16, рис. 11.

Ключові слова: **двоякоперіодичне гетерогенне середовище, інтегральне рівняння, вектор намагніченості, поле напруженості, тензор магнітної проникності, високоградієнтна сепарація, матриця, магнітні сили.**

Изложен метод расчета магнитостатического поля в двоякопериодической гетерогенной среде. Сформулировано интегральное уравнение относительно вектора намагниченности элементов среды. Расчет характеристик поля осуществляется путем решения полевой задачи в области основного параллелограмма периодов без задания граничных условий на его сторонах. Получены расчетные выражения для напряженности поля и тензора магнитной проницаемости. Приведены результаты вычислительных экспериментов, подтверждающих универсальность и эффективность метода. Проведено детальное исследование поля магнитных сил матрицы высокоградиентного магнитного сепаратора. Метод открывает новые возможности повышения технических характеристик электрофизических устройств, для которых универсальность и точность расчета локальных и эффективных характеристик поля является определяющей. Библ. 16, рис. 11.

Ключевые слова: **двоякопериодическая гетерогенная среда, интегральное уравнение, вектор намагниченности, поле напряженности, тензор магнитной проницаемости, высокоградиентная сепарация, матрица, магнитные силы.**

Introduction. Heterogeneous media (HM) are widely used due to a wide range of their practical application: magnetodielectrics, semiconductors, mixtures, solutions, composite and reinforced materials, electrostatic and magnetic filters, etc.

The theory of HM originates from the classical works by J. Maxwell and J. Rayleigh, which dealt with determining the effective parameters of the HM with canonical inclusions in the shape of cylinders and spheres (the problem of homogenization). Subsequently, these investigations were developed and summarized by many authors: K.M. Polivanov, V.M. Finkelberg, A.V. Netushil, B.M. Fradkin, V.I. Odelevsky, L.D. Stepin, B.Ya. Balagurov, Yu.P. Yemets, V. Buryachenko, M. Kharadly, W. Jackson, K.Z. Markov, S. Nemat-Nasser, M. Hori, W.T. Perrins, D.R. McKenzie, R.C. McFedran, P.D. Qyuivy, S. Torcuato and many others.

Various aspects of the theory and practice of HM were actively developed in Ukraine, in particular, by researchers of the Institute of Electrodynamics of the NAS of Ukraine. Particular attention was paid to the development of methods for analyzing electromagnetic fields in electrically conductive, dielectric, composite and heterogeneous systems at the Department of Electrophysics of Energy Conversion. Yu.P. Yemets developed analytical methods for analyzing electric fields using methods of integral equations and complex variables. 2D two-component systems with a regular distribution structure of inhomogeneities are considered.

The main effective parameters of two-component dielectric and conductive media with canonical inclusions are determined: conductance, magnetoresistance and Hall coefficient. Research results by Yu.P. Emets and his followers in this direction are presented in the monograph [1].

Based on the multipole expansion of high orders, the classical J. Maxwell and J. Rayleigh formulas on spherical and circular cylinders in a rectangular matrix in [2] are generalized and developed for the case of elliptic cylinders and spheroidal elements. There, for the first time, a fairly general field formulation was considered on the doubly periodic problem of magnetostatics for a nonlinear inhomogeneous anisotropic medium with periodic inclusions and with complex geometry of elements.

In recent years, the scope of HM has been steadily expanding: the study of nanocomposite materials [3], ferromagnetic perforated membranes (magnetic sieves) [6], and other devices for micromagnetic separation of ultrafine magnetic particles [5]. More actively in the study of the properties of the HM the capabilities of modern information technology are used.

Significant place in the theory and applications of HM occupy the tasks associated with the use of a magnetic field. In particular, one of such problems is the synthesis of filter matrices of high-gradient magnetic separators (HGMS) for maximum extraction of weakly magnetic minerals. The need for these devices arose in the middle of the last century due to the depletion of reserves of rich raw materials against the background of the rapidly developing technology of homeless metallurgy and the growth of requirements for the quality of steel [6, 7]. Various types of HGMS (Jones, Sala-Carousel, Boxmag Rapid, Krupp-Sol-24/14, 6-ERM-35/135, VGS-100/2, etc.) were created. The operation of these separators showed that at high weight and size parameters (for example, the Jones separator DR 335 with capacity of 180 t/h has a rotor diameter of 3.35 m and mass of 114 t) and specific power consumption they do not always provide the required technological parameters of enrichment. Therefore, the development of new HGMS designs continues, and the optimization of their technical parameters remains an urgent task.

A distinctive feature of the HGMS, which largely determines their effectiveness, is the use of magnetic filters of the matrix structure, the elements of which have complex geometry and high concentration to increase the magnetic field strength and its gradient. The study of various types of matrices is the subject of attention of many authors. A review of the current state of this issue with an extensive bibliography is given in [5]. The interest in this issue is explained by the fact that the matrix significantly affects productivity, separation efficiency, and operating cost. Ideally, it should, with high extraction efficiency, provide the maximum specific capture volume of the useful mineral with the minimum possible pulp resistance.

Optimization of matrix parameters is associated with a compromise between a large number of factors affecting its effectiveness. The magnetic force acting on a particle with volumetric magnetic susceptibility χ and volume V, $\boldsymbol{F}=\mu_0\chi V|\boldsymbol{H}|\mathrm{grad}(|\boldsymbol{H}|)$. In this expression, the last two factors are related to the magnetic system of the separator and its matrix, and the rest are related to the extracted magnetic material. If justification of the holding force $|\boldsymbol{F}|_{\min}$ for specific parameters χ and V is the task of technologists, then ensuring the necessary value of the value of $\boldsymbol{F}^*=|\boldsymbol{H}|\mathrm{grad}(|\boldsymbol{H}|)$, at which $|\boldsymbol{F}| \geq |\boldsymbol{F}|_{\min}$, is a rather complicated task requiring special research. Obviously, the $\boldsymbol{F}^*$ value is important for extraction, not the $|\boldsymbol{H}|$ and $\mathrm{grad}(|\boldsymbol{H}|)$ values separately. Moreover, the «weight» of each of the factors is far from obvious. The increase in the field strength $\boldsymbol{H}$ is associated with an increase in the power and, ultimately, the mass and size parameters of the separator. Since here the magnetic field gradient does not change significantly, an increase in the magnetic field strength «blindly» does not necessarily lead to an improvement in the separation efficiency in practical use [7]. As for the field gradient, the possibility of increasing it is potentially much greater, since it substantially depends on the size of the matrix elements and their shape. But here, a compromise solution should be sought, since for selective separation it is necessary to coordinate the dimensions of the matrix element with the particle size distribution. In addition, the large heterogeneity of the matrix field and especially of its gradient greatly complicates the task of ensuring the maximum capture zone while eliminating the possible blocking of the matrix. This explains a large number of studies on precisely the geometric parameters of matrix elements. For example, in [8], the expected decrease in the magnetic force with an increase in the number of sides of regular polygons has been confirmed by calculation. The studies of many authors (see, for example, [7, 9, 10]) recommended the optimal parameters of triangular gear plates, although due to a more uniform force field, replacing triangular elements with elements with a lower surface steepness can increase the ability to collect small particles. On the contrary, the patent [11] proposed strengthening the forces for the extraction of a fine fraction by replacing rods of circular cross section with rods with a diamond-shaped cross section (with a reduction in the size of the capture zone). The publication [12] recommended as promising rod matrices with an elliptical cross section. A number of works (see, for example, [13]) discuss the feasibility of using a combination of rods with different diameters or different cross-sectional shapes, as well as changing the order of their grouping.

A feature of the listed works is their particular and sometimes contradictory nature, as well as the predominant orientation to simple forms of matrix elements. Unfortunately, they do not give an idea of the local distribution of the field of magnetic forces in the working space of the matrix formed by elements of complex geometry and arbitrary concentration, especially when there are difficulties with the formation of boundary conditions in order to localize the calculation domain.

The wide variety of matrix elements used (balls, corrugated plates, rods, nets, spirals, wire wool, etc.) significantly complicate the development of a universal mathematical model for calculating the force field of

HGMS matrices. At the same time, the most common type of rod matrices, which are characterized by periodicity along the plane coordinates, should be selected. As shown in [2, 14], to study such media, the natural mathematical apparatus is the theory of doubly periodic (elliptic) functions, the use of which allows one to efficiently solve doubly periodic problems for HM in a fairly general formulation.

The goal of the paper is the development of a universal method for calculating the magnetic and force fields of a heterogeneous medium with a doubly periodic structure without significant restrictions on the number of phases, their geometry, concentration, and magnetic properties.

Basic definitions and properties of doubly periodic systems. The first and indispensable condition for the investigation of multicomponent HM is the determination of the main periods ω_1 and ω_2, which are the constituents of the main parallelogram of the periods Ω (if it exists). The unequivocal answer to this question is not always obvious, since, as will be shown below, even the doubly periodicity of all phases of a multicomponent HM does not guarantee its doubly periodicity as a whole.

Consider a pair of complex numbers ω_1 and ω_2, with $\mathrm{Im}(\tau=\omega_2/\omega_1)>0$. Points u and v of the complex plane are called congruent if they are connected by the relation $u\equiv v \cdot \mathrm{mod}(\omega_1,\ \omega_2)$ [15] or

$$u = v + m_1\cdot\omega_1 + m_2\cdot\omega_2, \text{ at } m_1, m_2 = 0, \pm1, \pm2, \ldots \; . \quad (1)$$

A parallelogram with vertices u_0, $u_0+\omega_1$, $u_0+\omega_2$, $u_0+\omega_1+\omega_2$ will be called a parallelogram of periods constructed on periods ω_1 and ω_2. Obviously, the set of congruent points corresponds to an infinite number of parallelograms of periods covering the entire complex plane without overlapping.

The concept of a doubly periodic (elliptic) function occupies an important place in the subsequent analysis. Denote $\boldsymbol{\omega}=m_1\cdot\omega_1 + m_2\cdot\omega_2$. The function $f(u)$ with periods $\boldsymbol{\omega}$ will be called doubly periodic, and ω_1 and ω_2 – its main periods.

From the theory of elliptic functions, it is known [13] that a pair of main periods (ω_1, ω_2) is not unique. If for arbitrary integers m_1, m_2 and m_1', m_2' the sets of points $\boldsymbol{\omega}=m_1\cdot\omega_1 + m_2\cdot\omega_2$ and $\boldsymbol{\omega}'=m_1'\cdot\omega_1'+m_2'\cdot\omega_2'$ coincide, then the pairs of periods $\boldsymbol{\omega}$ and $\boldsymbol{\omega}'$ are equivalent. Here, a pair of periods (ω_1, ω_2) is equivalent if and only if to a pair of periods (ω_1',ω_2') ,when the relation $\omega_2'=\alpha\cdot\omega_2+\beta\cdot\omega_1$, $\omega_1'=\gamma\cdot\omega_2+\delta\cdot\omega_1$ is valid, where α, β, γ, δ are the integers that satisfy the condition $\alpha\cdot\delta-\beta\cdot\gamma=1$ at $\mathrm{Im}(\omega_2/\omega_1)>0$. Examples of equivalent periods for two sets of congruent points are shown in Fig. 1.

We note some more obvious statements. The areas of equivalent periods are the same, and the area of the main parallelogram Ω with periods (ω_1, ω_2) is minimal. We will also call two parallelograms with periods (ω_1, ω_2) and (ω_1', ω_2') similar if the directions of the periods ω_1 and ω_1', ω_2 and ω_2' and coincide.

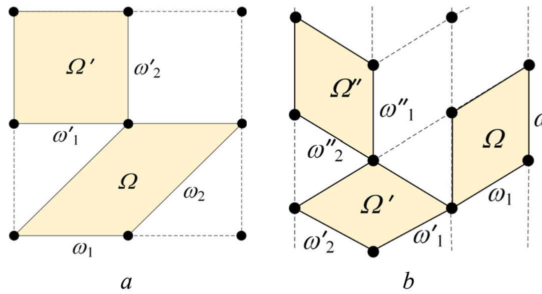

Fig. 1. Sets of congruent points and their corresponding equivalent parallelograms of periods

The concept of a doubly periodic HM is more complicated than the concept of a doubly periodic lattice, since in addition to geometrical properties it is also necessary to take into account the physical and other properties of individual phases, their arrangement in the parallelogram of periods, etc. Moreover, the set of HM can correspond to the same period lattice. For example, we establish a correspondence between doubly periodic HM shown in Fig. 2 and lattices of periods of Fig. 1.

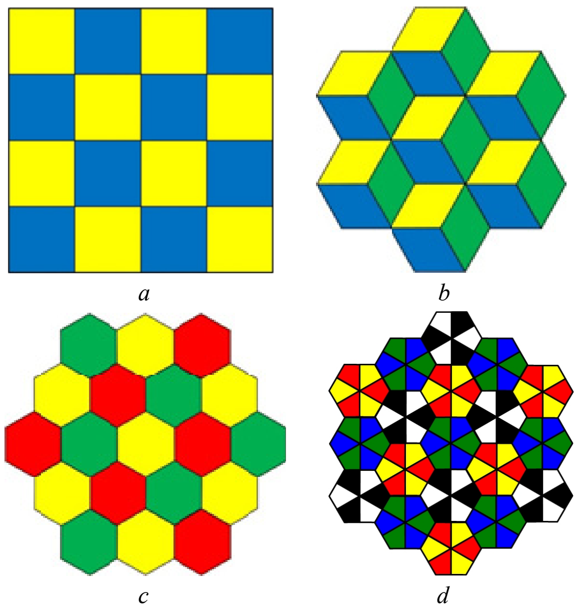

Fig. 2. Examples of doubly periodic multicomponent HM corresponding to the lattices of Fig. 1: two-component – *a*; three-component – *b* and *c*; six-component – *d*

Note an important point on the example of Fig. 2,*a*. The main parallelogram for the set of congruent points is the small square. At the same time, it cannot be the main parallelogram of the HM, since, for example, the entire complex plane cannot be covered by the yellow phase. Therefore, for this HM, the main parallelograms of the periods correspond to Fig. 1,*a* (each of them includes two elements of the yellow and blue phases). HMs, shown in Fig. 2,*b*,*c*,*d* correspond to Fig. 1,*b*. Indeed, considering in Fig. 2,*b*,*c* the system of congruent points of the yellow phase (for example, the upper points of the elements) we see that they coincide with the period lattice of Fig. 1,*b*. The same can be said about other congruent points of the yellow and two other phases. More complicated HM of Fig 2,*d* also corresponds to Fig. 1,*b*. After turning Fig. 2,*d* (or coordinate systems) at 30° it can be seen that topologically Fig. 2,*c*,*d* are the same. The only difference is that each element of the phases of Fig. 1,*c* corresponds to three elements of the two phases of Fig. 2,*d*. Here, as it is easy to see, the sets of the corresponding congruent points of all six phases coincide with the lattice of periods in Fig. 1,*b*.

Shown in Fig. 2 multicomponent HMs have obvious doubly periodicity with the same lattice parameters for the periods of each phase within the HM. As will be shown below, in this case, the main periods of the HM generally coincide with the corresponding periods of the phases.

A number of important conclusions can be drawn from the analysis performed. In particular, it is legitimate to introduce the concept of congruent areas, whose geometry is fully reproduced in each parallelogram of periods. Moreover, these areas can be multiply connected and multicomponent. This follows from the statement that each period parallelogram of system (1) owns only one point of this system [15]. Considering the set of arbitrary points ν with the sets of congruent points (1) generating them, we naturally come to the concept of congruent doubly periodic domains.

We will illustrate some additional features of a doubly periodic HM using the example of a complex HM shown in Fig. 3.

The discrete phase of this HM is represented by three fractions – red, blue and green. The main parallelograms of the periods of these fractions are highlighted in the corresponding colors. They are similar (i.e., the corresponding sides of the parallelograms are parallel), but have different basic periods and concentration of inclusions. For example, if for the green fraction introduce the designation $\Omega^1=(\omega_1,\ \omega_2)$, then for the red fraction $\Omega^2=(\omega_1,\ 2\cdot\omega_2)$, and for the blue one $\Omega^3=(3\cdot\omega_1,\ \omega_2)$. Note that each of the highlighted main parallelograms of periods Ω^i has a set of equivalent ones, however, for the HM under consideration, all of them are reduced to similar ones. This procedure is necessary to answer the important question: is this HM a doubly periodic, and if so, what are the main periods of this medium. This question was posed in [16]. therefore, we confine ourselves here to some refinements and additions.

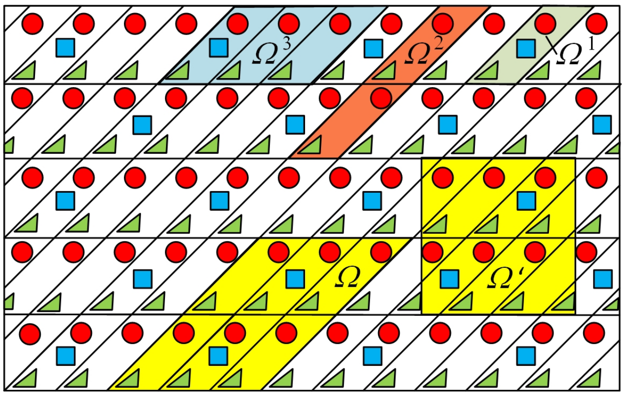

Fig. 3. Basic parallelograms of periods Ω^1, Ω^2, Ω^3 of separate phases and equivalent parallelograms of periods Ω and Ω' of a three-component HM

The condition of doubly periodicity of a multicomponent HM. Let some multicomponent HM is composed of a number of doubly periodic HMs of a lower level. Denote by $\{\Omega^i\}$ the set of primitive lattices $\boldsymbol{\omega}^i=(\omega_1^i,\omega_2^i)$, i=1, 2, 3, … P. In the simplest case, when $\omega_1^i=n^i\cdot\tilde{\omega}_1$, $\omega_2^i=m^i\cdot\tilde{\omega}_2$, where $\tilde{\omega}_1$ and $\tilde{\omega}_2$ are some complex numbers, $\mathrm{Im}(\tilde{\omega}_2/\tilde{\omega}_1)>0$, and n^i, m^i are the arbitrary natural numbers , the HM under consideration is doubly periodic and its main periods ω_1, ω_2 are defined as follows. Denote by N and M the least common multiples for the sets $\{n^i\}$, $\{m^i\}$: $N=\mathrm{lcm}(n^1,\ n^2,\ \ldots,\ n^P)$, $M=\mathrm{lcm}(m^1,\ m^2,\ \ldots,\ m^P)$. Then $\omega_1=N\cdot\tilde{\omega}_1$, $\omega_2=M\cdot\tilde{\omega}_2$. For example, for the HM considered in Fig. 3, $N=\mathrm{lcm}(1,\ 1,\ 3)=3$, $M=\mathrm{lcm}(1,\ 2,\ 1)=2$. Thus, $\omega_1=3\cdot\tilde{\omega}_1$, $\omega_2=2\cdot\tilde{\omega}_2$. In Fig. 3 two equivalent periods of HM Ω and Ω' are highlighted in yellow.

Note several important consequences of the analysis carried out.

1. A necessary condition for doubly periodicity of the HM is the existence in the set of main periods of the phases $\{\Omega^i\}$ of a subset of similar periods $\{\Omega''^i\}$.

2. The doubly periodicity and similarity of periods $\{\Omega''^i\}$ does not guarantee the doubly periodicity of the HM as a whole. For example, let $\omega_1^1=a,\ \omega_1^2=b,\ \omega_2^1=\omega_2^2$, and $a\neq b$ are any irrational numbers, for example, $a=e$, $b=\pi$. Obviously, it is impossible to select integer multiplicities for the specified periods.

3. At a linear displacement of the main parallelogram of periods Ω or when moving to the equivalent parallelogram, all congruent components (or their parts), the concentration of individual phases and physical parameters that are doubly periodic, such as the magnetization vectors in the corresponding congruent points, are conserved. This is important in the practical solution of problems on the definition of local and effective parameters of HM.

It should be noted that the establishment of the fact that the HM is doubly periodic and the determination of its main periods significantly expand the possibilities of investigation the HM, since it provides the possibility of applying the theory of elliptic functions and limits the scope of analysis by the main parallelogram of periods.

Basic calculation relations. In the complex plane E, we consider a medium with a regular structure formed by a set of congruent groups of magnetics, each of which corresponds to a bounded (in the general case multiply connected) domain $D_{mn}\equiv\cup D_{mn}^j$ with a sufficiently smooth boundary $S_{mn}\equiv\cup S_{mn}^j$ ($j=1, 2, \ldots, k$; $m, n=0, \pm1, \pm2, \ldots$). For convenience, we denote the domain D_{00} corresponding to the main parallelogram Ω with periods ω_1 and ω_2, by D. Accordingly, $S_{00}\equiv S$ and $\bar{D}\equiv D\cup S$. The region external to the magnetics is denoted by $D_e\equiv E\backslash D_{mn}$.

Let $B^j=B^j(H,\ z)$ be a known function that, in the general case, defines the inhomogeneous, nonlinear and anisotropic properties of the set of congruent elements $j, z \in D_{mn}$. If $z \in D_e$, then $B=\mu_0 H$.

Consider a system of dipoles with identical moments M located at the points $\xi \equiv \xi \mathrm{mod}(\omega_1, \omega_2)$. Their complex potential and field strength [2]

$$W_M(z) = \frac{M}{2\pi}\cdot\zeta(z-\xi) + C(z-\xi), \qquad (2)$$

$$H_M(z) = -\overline{W}'_M(z) = -\frac{\overline{M}}{2\pi}\cdot\overline{\zeta'(z-\xi)} - \overline{C} = \frac{\overline{M}}{2\pi}\cdot\overline{\wp(z-\xi)} - \overline{C}, \qquad (3)$$

where $\zeta(z-\xi) = \zeta(u)$, $\wp(z-\xi) = \wp(u)$ are the Weierstrass functions, and a bar over a complex number means a conjugation operation.

Without loss of generality, we combine the period ω_1 with the x axis and take $\mathrm{Im}\omega_1=0$. Taking into account the Legendre relation [13]

$$\eta_1\cdot\omega_2 - \eta_2\cdot\omega_1 = 2\pi \mathrm{j},$$

for the constant C we obtain the expression [2]

$$C = -\frac{1}{2\pi}\cdot\left(M_1\frac{\eta_1}{\omega_1} + M_2\frac{\eta_2}{\omega_2}\right) = \frac{\mathrm{j}\cdot M_2}{\omega_1\cdot\omega_2} - \frac{M}{2\pi}\cdot\frac{\eta_1}{\omega_1} = \frac{\mathrm{j}\cdot\mathrm{Im}M}{F_\Omega} - \frac{M}{2\pi}\cdot\frac{\eta_1}{\omega_1}, \qquad (4)$$

where M_1, M_2 is the decomposition of the vector M in the directions of the periods ω_1 and ω_2, $F_\Omega=\omega_1\cdot\mathrm{Im}\omega_2$ is the area of the main parallelogram of periods Ω.

Let $J(z)$, $z \in D$ be the distribution of magnetization arising under the action of the field of primary sources $H_0(z)$. A joint consideration in the domain D of the action of primary and secondary sources (the magnetization of all magnetics in E) leads to expressions for the complex potential and intensity [12]:

$$W(z) = W_0(z) + W_J(z) = W_0(z) + \frac{1}{2\pi}\cdot\int_\Omega J(\xi)\cdot[\zeta(z-\xi) - \frac{\eta_1}{\omega_1}\cdot(z-\xi)]\cdot\mathrm{d}\tau_\xi + \frac{\mathrm{j}}{F_\Omega}\cdot\int_\Omega \mathrm{Im}J(\xi)\cdot(z-\xi)\cdot\mathrm{d}\tau_\xi, \qquad (5)$$

$$H(z) = H_0(z) + H_J(z) = H_0(z) + \frac{1}{2\pi}\times \int_\Omega \overline{J}(\xi)\cdot\left[\overline{\wp(z-\xi) + \frac{\eta_1}{\omega_1}}\right]\mathrm{d}\tau_\xi + \frac{\mathrm{j}\cdot\mathrm{Im}P}{F_\Omega}, \qquad (6)$$

where P is the full dipole moment of the main parallelogram Ω. The integral in (6) is singular.

Let us consider in more detail the linear case: $B^j=\mu_a^j\cdot H = \mu_0\,\mu^j\cdot H$, $B_e = \mu_0\cdot H$ for $z \in D$ and $z \in D_e$, respectively. In this case, outside the domain D $J(z)=0$ and the problem of calculation of the field characteristics at an arbitrary point of the HM is reduced to calculation of the distribution of the magnetization vector $J(z)$ in D.

Introduce the integral operator that is important for the analysis below [2]

$$\Pi_\omega J = -\frac{1}{\pi}\int_D J(\xi)\cdot\left[\wp(z-\xi) + \frac{\eta_1}{\omega_1}\right]\mathrm{d}\tau_\xi + \frac{2\mathrm{j}\cdot\mathrm{Im}P}{F_\Omega}. \qquad (7)$$

Denote $\tilde{B} = \mu_0^{-1}B$ and consider a chain of equalities:

$$J = \tilde{B} - H = (\mu-1)\cdot H;\ \tilde{B}_J + H_J = \overline{\Pi_\omega J};$$
$$\tilde{B} + H = (\mu+1)H = 2H_0 + \tilde{B}_J + H_J = 2H_0 - \overline{\Pi_\omega J}. \qquad (8)$$

From (7), (8) it is easy to obtain an integral equation for the medium magnetization vector $J(z)$, $z \in D$:

$$J(z) = \lambda\cdot(2H_0(z) - \overline{\Pi_\omega J}) = \lambda\left\{2H_0(z) + \frac{1}{\pi}\int_D \overline{J(\xi)\cdot\left[\wp(z-\xi) + \frac{\eta_1}{\omega_1}\right]\mathrm{d}\tau_\xi + \frac{2\mathrm{j}\cdot\mathrm{Im}P}{F_\Omega}}\right\}, \qquad (9)$$

where $\lambda=(\mu-1)/(\mu+1)$.

We give one more expression for the singular operator $\Pi_\omega J$. Denoting by σ_ε a circle of small radius ε, and by $D_\varepsilon=D\backslash\sigma_\varepsilon$ a domain D with punctured point $z = \xi$, expression (7) is transformed to

$$\Pi_\omega J = -\frac{1}{\pi}\int_{D_\varepsilon} J(\xi)\cdot\partial_\xi\left[\zeta(z-\xi) - \frac{\eta_1}{\omega_1}(z-\xi)\right]\mathrm{d}\tau_\xi + \frac{2\mathrm{j}\cdot\mathrm{Im}P}{F_\Omega} = -\frac{1}{\pi}\int_{D_\varepsilon}\partial_\xi\left\{J(\xi)\cdot\left[\zeta(z-\xi) - \frac{\eta_1}{\omega_1}(z-\xi)\right]\right\}\mathrm{d}\tau_\xi - \frac{1}{\pi}\cdot\int_{D_\varepsilon}\partial_\xi J\left[\zeta(z-\xi) - \frac{\eta_1}{\omega_1}(z-\xi)\right]\mathrm{d}\tau_\xi + \frac{2\mathrm{j}\cdot\mathrm{Im}P}{F_\Omega}. \qquad (10)$$

It is easy to establish that

$$P = \int_D J(\xi)\cdot\mathrm{d}\tau_\xi = \int_D \{\partial_\xi[\xi\cdot J(\xi)] - \xi\cdot\partial_\xi J\}\cdot\mathrm{d}\tau_\xi. \qquad (11)$$

Applying the Green formula to (10), (11), assuming the differentiability of the function f

$$\int_{D_\varepsilon}\partial_\xi f\cdot\mathrm{d}\tau_\xi = -\frac{1}{2\mathrm{j}}\cdot\int_S f(\xi)\cdot d\overline{\xi} + \frac{1}{2\mathrm{j}}\cdot\int_{|z-\xi|=\varepsilon} f(\xi)\cdot\mathrm{d}\overline{\xi} \qquad (12)$$

and taking into account that the integral (12) along a circle of a sufficiently small radius ε is zero, we find:

$$P = \int_D J(\xi)\cdot\mathrm{d}\tau_\xi = \int_D\{\partial_\xi[\xi\cdot J(\xi)] - \xi\cdot\partial_\xi J\}\cdot\mathrm{d}\tau_\xi = -\frac{1}{2\mathrm{j}}\cdot\int_S \xi\cdot J(\xi)\cdot\mathrm{d}\overline{\xi} - \int_D \xi\cdot\partial_\xi J\cdot\mathrm{d}\tau_\xi.$$

Here S is the boundary of the domain D (in the general case, multiply connected). At $\lambda(z)$=const $\partial_\xi J=0$, therefore, the singular operator $\Pi_\omega J$ is expressed through the surface (boundary) integral

$$\Pi_\omega J = \frac{1}{2\pi\mathrm{j}}\cdot\int_S J(\xi)\cdot\left[\zeta(z-\xi) - \frac{\eta_1}{\omega_1}(z-\xi)\right]\cdot\mathrm{d}\overline{\xi} - \frac{1}{F_\Omega}\cdot\mathrm{Re}\left[\int_S \xi\cdot J(\xi)\cdot\mathrm{d}\overline{\xi}\right]. \qquad (13)$$

The practical implementation of the basic relations. Let us now consider some questions of the practical use of the expressions obtained. We represent the domain D as a set of triangular elements $D \equiv \cup D^k$ with a constant magnetization J^k corresponding to the center of gravity ξ^k of triangle D^k. In this case, the solution $J(z) \equiv \cup J^k(z)$ can be obtained by simple iteration method for the equation

$$J_i^m \equiv J_i(z^m) = 2\lambda\cdot H_0^m + \frac{\lambda}{\pi}\cdot\sum_k\int_{D^k}\overline{J_{i-1}(\xi^k)}\times \overline{\left[\wp(z^m-\xi) + \frac{\eta_1}{\omega_1}\right]}\cdot\mathrm{d}\tau_k + \frac{2\mathrm{j}\cdot\lambda}{F_\Omega}\cdot\mathrm{Im}\sum_k\int_{D^k}\overline{J_{i-1}(\xi^k)}\cdot\mathrm{d}\tau_k. \qquad (14)$$

(m = 1, 2, 3, …, M; i = 1, 2, 3, …).

If we consider the magnetized domains D^k as dipoles with magnetic moments $M^k = J^k\cdot\Delta\tau_k$, located at the points ξ^k, then (14) is greatly simplified:

$$J_i^m = 2\lambda\cdot H_0^m + \frac{\lambda}{\pi}\cdot\sum_k \int_{D^k} \overline{A_m^k\cdot J_{i-1}^k}\cdot\Delta\tau_k + \frac{2\mathrm{j}\cdot\lambda}{F_\Omega}\cdot \mathrm{Im}\sum_k\int_{D^k}\overline{J_{i-1}^k}\cdot\Delta\tau_k, \quad (15)$$

where $A_{mk}=\wp(z^m-\xi^k)+\eta_1/\omega_1$.

The calculation of A_{mk} values can be performed using the formulae [15]

$$\wp(u)+\frac{\eta_1}{\omega_1} = -\frac{4\pi^2}{\omega_1^2}\cdot\left\{\frac{1}{(h-h^{-1})^2}+\sum_{r=1}^{\infty}\left[\frac{q^{2r}\cdot h^{-2}}{(1-q^{2r}\cdot h^{-2})^2}+\frac{q^{2r}\cdot h^{2}}{(1-q^{2r}\cdot h^{2})^2}\right]\right\}, \quad (16)$$

or at $-\mathrm{Im}\,\tau<\mathrm{Im}\,\nu<\mathrm{Im}\,\tau$

$$\wp(u)+\frac{\eta_1}{\omega_1} = \frac{\pi^2}{\omega_1^2\cdot\sin^2(\pi\nu)} - \frac{8\pi^2}{\omega_1^2}\cdot\sum_{n=1}^{\infty}\frac{nq^{2n}}{1-q^{2n}}\cos(2\pi\cdot n\nu), \quad (17)$$

where $u\equiv u^{mk}=z^m-\xi^k$, $q=\exp(\mathrm{j}\pi\tau)$, $\nu=u/\omega_1$.

For uniformly magnetized triangles, the integrals in (14) can be calculated analytically. Applying (13) for the k-th triangle and taking into account that [15]

$$\zeta(u)-\frac{\eta_1}{\omega_1}u = \frac{d}{du}\ln\vartheta_1(\nu,\tau) = -\frac{d}{d\xi}\ln\vartheta_1(\nu,\tau), \quad (18)$$

we obtain

$$\begin{aligned}\Pi_\omega J &= \sum_k \frac{J^k}{2\pi\mathrm{j}}\cdot\int_{S^k}\partial_\xi[\ln\vartheta_1(z^m-\xi)]\cdot\mathrm{d}\overline{\xi} - \frac{1}{F_\Omega}\sum_k \mathrm{Re}\left[J^k\cdot\int_{S^k}\xi\cdot\mathrm{d}\overline{\xi}\right] = \\ &= \sum_k\frac{J^k}{2\pi\mathrm{j}}\left[\begin{array}{l} a_{ijk}\ln\vartheta_1(z^m-z^j)+a_{kij}\ln\vartheta_1(z^m-z^i)+ \\ +a_{jki}\ln\vartheta_1(z^m-z^k)\end{array}\right] - \frac{1}{F_\Omega}\sum_k\mathrm{Re}\left[J^k\cdot\int_{S^k}\xi\cdot\mathrm{d}\overline{\xi}\right].\end{aligned} \quad (19)$$

In this expression S^k is the boundary of the k-th triangle, z^i, z^j, z^k are the complex coordinates of its vertices, $a_{kij} = a_{ki} - a_{ij}$, $a_{ijk} = a_{ij} - a_{jk}$, $a_{jki} = a_{jk} - a_{ki}$, $a_{mn}=\overline{(z^n-z^m)}/(z^n-z^m)$, m, n = i, j k; ϑ_1 is the ϑ-function with high convergence rate:

$$\vartheta_1 = 2q^{1/4}\cdot\left[\sin(\pi\nu)-q^2\sin(3\cdot\pi\nu)+q^6\sin(5\cdot\pi\nu)-...\right].$$

Let us consider in more detail the calculation of the complex potential (5) from the known distribution of the magnetization vector $J(z)$ in D. In the simplest case, the discrete analog of this equation, by analogy with (15), takes the form

$$W(z)=W_0(z)+\frac{1}{2\pi}\cdot\sum_k J^k\cdot[\zeta(z-\xi^k) - \frac{\eta_1}{\omega_1}(z-\xi^k)]\cdot\Delta\tau_k+\frac{\mathrm{j}}{F_\Omega}\sum_k \mathrm{Im}\,J^k\cdot(z\cdot\Delta\tau_k-g_k), \quad (20)$$

where for g_k using the Green formula we obtain

$$g_k=\int_{D^k}\xi\cdot\Delta\tau_k = \frac{1}{2}\int_{D^k}\partial_\xi(\xi^2)\cdot\Delta\tau_k = \frac{\mathrm{j}}{4}\int_{S^k}\xi^2\cdot\mathrm{d}\overline{\xi} = \frac{\mathrm{j}}{12}(z_i^3\cdot a_{kij}+z_j^3\cdot a_{ijk}+z_k^3\cdot a_{jki}),$$

and for $\zeta(u)-(\eta_1/\omega_1)\cdot u$ one can use absolutely and uniformly convergent series

$$\zeta(u)-\frac{\eta_1}{\omega_1}\cdot u = \frac{\pi\mathrm{j}}{\omega_1}\cdot\left(\frac{h+h^{-1}}{h-h^{-1}}+2\cdot\sum_{n=1}^{\infty}\left[\frac{q^{2n}\cdot h^{-2}}{1-q^{2n}\cdot h^{-2}}-\frac{q^{2n}\cdot h^{2}}{1-q^{2n}\cdot h^{2}}\right]\right), \quad (21)$$

or at $-\mathrm{Im}\,\tau<\mathrm{Im}\,\nu<\mathrm{Im}\,\tau$

$$\zeta(u)-\frac{\eta_1}{\omega_1}\cdot u = \frac{\pi}{\omega_1}\cdot\left(\mathrm{ctq}(\pi\nu)+4\cdot\sum_{n=1}^{\infty}\frac{q^{2n}}{1-q^{2n}}\cdot\sin(2\pi n\cdot\nu)\right). \quad (22)$$

A more accurate expression for $W(z)$ can be obtained by passing in (5) to the integral over the boundary S^k. Using (18) and (20), we rewrite (5) as

$$\begin{aligned}W(z) &= W_0(z)-\frac{1}{2\pi}\sum_k J^k\int_{D^k}\partial_\xi[\ln\vartheta_1(\nu,\tau)]\cdot\mathrm{d}\tau_\xi + \frac{\mathrm{j}}{F_\Omega}\sum_k\mathrm{Im}\,J^k(z\Delta\tau_k-g_k) = W_0(z)- \\ &-\frac{\mathrm{j}}{4\pi}\sum_k J^k\int_{S^k}\ln\vartheta_1(\nu,\tau)d\overline{\xi}+\frac{\mathrm{j}}{F_\Omega}\sum_k\mathrm{Im}\,J^k(z\Delta\tau_k-g_k).\end{aligned} \quad (23)$$

To calculate the integrals in (23), we use the well-known expansion for the ϑ-function

$$\ln\left(\pi\cdot\frac{\vartheta_1(\nu,\tau)}{\vartheta_1'(0,\tau)}\right) = \ln\sin(\pi\nu)+2\sum_{n=1}^{\infty}\frac{q^{2\cdot n}}{n\cdot(1-q^{2\cdot n})}[1-\cos(2\pi n\cdot\nu)]. \quad (24)$$

Taking into account that in accordance with (12) $\int_\Gamma d\overline{\xi}\equiv 0$,

$$\int_S \ln\vartheta_1(\nu,\tau)\,\mathrm{d}\overline{\xi} = \int_S\ln\sin(\pi\nu)\cdot\mathrm{d}\overline{\xi} - 2\sum_{n=1}^{\infty}\int_S a_n\cdot\cos(2\pi n\cdot\nu)\cdot\mathrm{d}\overline{\xi}, \quad (25)$$

where $a_n=q^{2n}/[n\cdot(1-q^{2n})]$.

Calculation of the first integral in (25) leads to the expression

$$\begin{aligned}I_1 &= \int_{S^k}\ln\sin(\pi\nu)\cdot d\overline{\xi} = \int_{S^k}\ln\left[\pi\nu\cdot\prod_{n=1}^{\infty}(1-\frac{\nu^2}{n^2})\right]\cdot\mathrm{d}\overline{\xi} = \\ &= \int_{S^k}\ln(z-\xi)\cdot\mathrm{d}\overline{\xi}+\sum_{n=1}^{\infty}\int_{S^k}\ln\left[(z-\xi)^2-(n\cdot\omega_1)^2\right]\cdot\mathrm{d}\overline{\xi} = \\ &= -a_{kij}\cdot\sum_{n=0}^{\infty}b_n\cdot[u_{1i}\cdot(\ln u_{1i}-1)+u_{2i}\cdot(\ln u_{2i}-1)]- \\ &-a_{ijk}\cdot\sum_{n=0}^{\infty}b_n\cdot[u_{1j}\cdot(\ln u_{1j}-1)+u_{2j}\cdot(\ln u_{2j}-1)]- \\ &-a_{jki}\cdot\sum_{n=0}^{\infty}b_n\cdot[u_{1k}\cdot(\ln u_{1k}-1)+u_{2k}\cdot(\ln u_{2k}-1)],\end{aligned} \quad (26)$$

where $u_{1p} = z - n\cdot\omega_1 - z_p$, $u_{2p} = z + n\cdot\omega_1 - z_p$, $p = i, j, k$; b_n=0,5 at $n = 0$ and $b_n = 1$ at $n \neq 0$.

Calculation of the second integral in (25) gives:

$$I_2 = -2\int_{S^k}a_n\cdot\cos(2\pi\cdot n\nu)\cdot\mathrm{d}\overline{\xi} = \sum_{n=1}^{\infty}\frac{a_n\cdot\omega_1}{\pi n}\times\left[a_{ijk}\sin(2\pi n\nu^j)+a_{kij}\sin(2\pi n\nu^i)+a_{jki}\sin(2\pi n\nu^k)\right], \quad (27)$$

where

$$\nu^j = (z - z^j)/\omega_1, \nu^i = (z - z^i)/\omega_1, \nu^k = (z - z^k)/\omega_1.$$

It should be borne in mind that the logarithm is a multivalued function, therefore, when integrating along the boundary S^k, it is necessary to choose its continuous branches.

Examples of numerical implementation. Below are examples of numerical simulations illustrating the capabilities of the method described. Figures 4-8 show field patterns (field lines in blue, equipotentials in red) for a three-component HM with basic periods ω_1=8, ω_2=6j. The external field H_0 is uniform and directed at different angles about the horizontal axis.

In Fig. 5 with the same parameters as in Fig. 4, the field is calculated in the equivalent parallelogram of periods Ω' with $\omega_1' = 8$, $\omega_2' = -8 + 6\mathrm{j}$. Comparison of the field distribution in Fig. 4 and Fig. 5 confirms the conclusion about the conservation of field characteristics at congruent points of equivalent periods. The freedom to choose from equivalent periods of more convenient for calculating and visualizing the results in this case clearly speaks in favor of Fig. 4.

The choice of indicated in Fig. 6 angle 9.2535° is due to the direction of the external field H_0 in the direction of the main axis of anisotropy of the homogenized HM (calculation – see below).

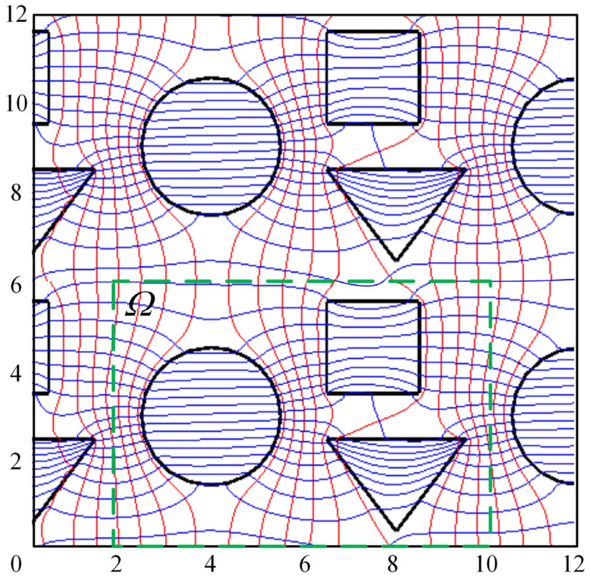

Fig. 4. The field pattern in the three-component HM at relative magnetic permeabilities of discrete elements μ=1000 and the external medium μ_e=1. Green dotted line illustrates the main parallelogram of the periods. External field H_0=1

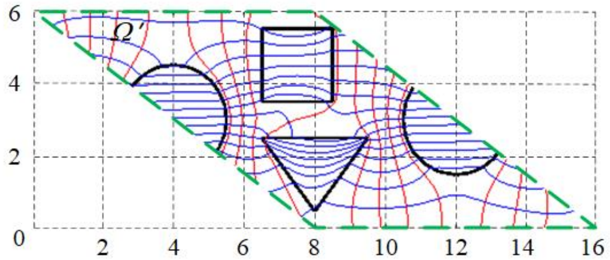

Fig. 5. The field pattern in the equivalent basic period (highlighted in green dotted line) with the HM parameters of Fig. 4

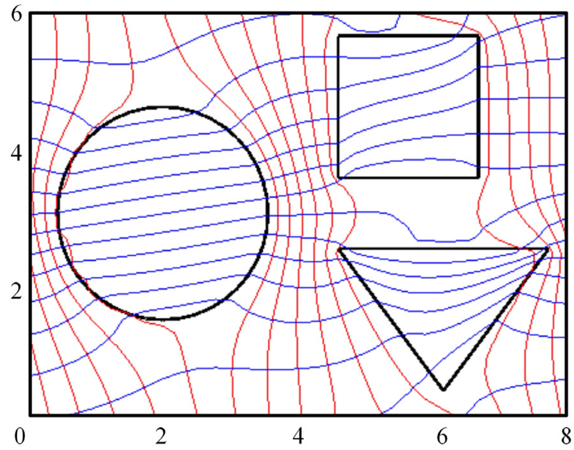

Fig. 6. The same as in Fig. 4, but the external field H_0 is directed at an angle of 9.2535° to x-axis

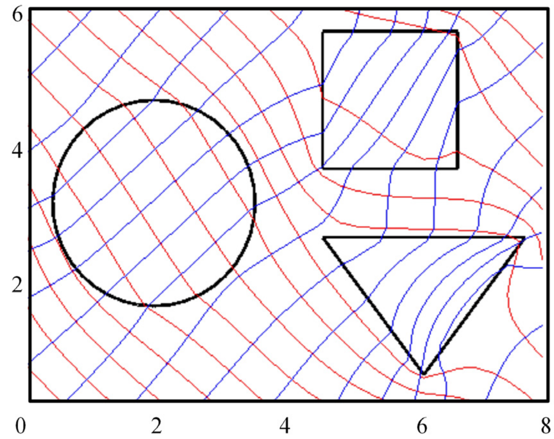

Fig. 7. Relative magnetic permeabilities of triangular, square and round rods, respectively, equal to 1000, 10 and 2, μ_e=1. The external field H_0 is directed at an angle of 45° to x-axis

Figures 8, 9 show the results of solution of the flow problem: Fig. 8 shows the flow lines, and Fig. 9 presents the distribution of the magnetization vector in a magnetic sheet with discrete air voids (see Fig. 8). In this case, instead of (9) the following equation is used:

$$J(z) = 2H_0 \cdot \frac{\mu_i(\mu_e - 1)}{\mu_i + \mu_e} - \lambda \cdot \overline{\Pi_\omega J}, \; \lambda = \frac{\mu_i - \mu_e}{\mu_i + \mu_e}.$$

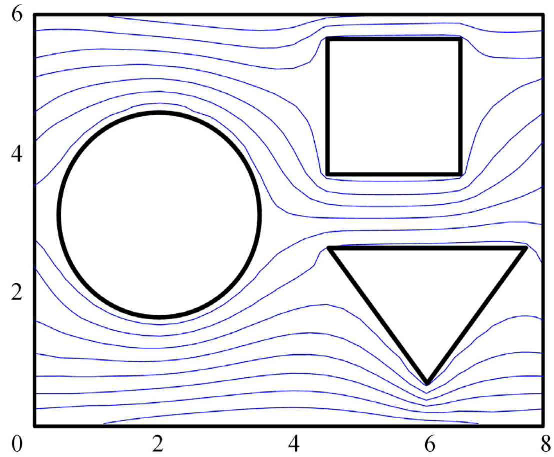

Fig. 8. The flow problem. The relative magnetic permeabilities of discrete elements μ =1, of the external medium μ_e=1000

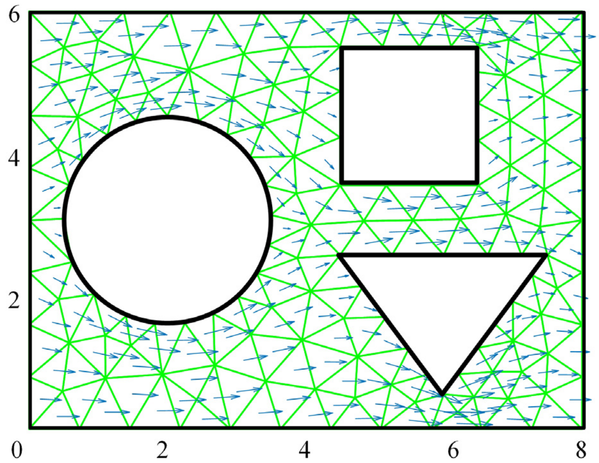

Fig. 9. Discretization of the computational domain and distribution of the magnetization vector in the flow problem (see Fig. 8)

Attention should be paid to an important detail: despite the simple shape of the main parallelogram of periods, the boundary conditions on its sides are not known a priori and cannot be reduced to the conditions commonly used in FEM.

Calculation of effective parameters of a multicomponent HM. Since the above method is based on the determination of the magnetization vector J in the main parallelogram of the periods, solution of the homogenization problem poses no significant difficulties. To do this, it is necessary to calculate $J(z)$, $z\in\Omega$ for two mutually perpendicular external fields H_0, for example, for $H_0 = 1$ and $H_0 = j$. Let us denote the total magnetization of all elements in Ω by $\wp_x=\wp_{xx}+\mathrm{j}\,\wp_{xy}$ and $\wp_y=\wp_{yx}+\mathrm{j}\,\wp_{yy}$, respectively. Then the relative magnetic permittivity tensor $\boldsymbol{\kappa}$ is easily determined through the effective magnetization of the medium: in vector notation $\boldsymbol{J}=\boldsymbol{\wp}/F_\Omega=\boldsymbol{\kappa}\cdot\boldsymbol{H}_0$. Obviously, $\kappa_{xx}=\wp_{xx}/F_\Omega$, $\kappa_{xy}=\wp_{xy}/F_\Omega$, $\kappa_{yx}=\wp_{yx}/F_\Omega$, $\kappa_{yy}=\wp_{yy}/F_\Omega$.

In the general case, for the chosen coordinate system, the tensor κ should be symmetric, but not necessarily diagonal. To bring it to a diagonal tensor $\tilde{\boldsymbol{\kappa}}$ with principal $\tilde{\kappa}_{xx}, \tilde{\kappa}_{yy}$ $(\tilde{\kappa}_{xy} = \tilde{\kappa}_{yx} = 0)$ values, we introduce a new coordinate system (x', y') by rotating the old one by the angle α. This angle can be determined from the expression

$$\alpha = \frac{1}{2}\operatorname{arctg}\left(\frac{2\kappa_{xy}}{\kappa_{xx}-\kappa_{yy}}\right), \tag{28}$$

and the principal values of the tensor $\tilde{\boldsymbol{\kappa}}$ can be determined from the relations

$$\tilde{\kappa}_{xx} = \frac{(\kappa_{xx}+\kappa_{yy})+\sqrt{(\kappa_{xx}-\kappa_{yy})^2+4\kappa_{xy}{}^2}}{2},$$
$$\tilde{\kappa}_{yy} = \frac{(\kappa_{xx}+\kappa_{yy})-\sqrt{(\kappa_{xx}-\kappa_{yy})^2+4\kappa_{xy}{}^2}}{2}. \tag{29}$$

In accordance with the above, for a medium with parameters corresponding to Fig. 4, the following results are obtained:

$$\boldsymbol{\kappa} = \begin{vmatrix} 1,0054 & 0,0210 \\ 0,0211 & 0,8801 \end{vmatrix}, \ \tilde{\boldsymbol{\kappa}} = \begin{vmatrix} 1,0088 & 0 \\ 0 & 0,8766 \end{vmatrix}, \ \alpha = 9,2535^\circ.$$

For the components of the effective relative magnetic permeability tensor, we obtain the obvious values: $\tilde{\mu}_{xx}$ =2,0088, $\tilde{\mu}_{yy} = 1{,}8801$.

To confirm the correctness of the calculations, Fig. 6 shows a picture of the field obtained with an external field strength H_0=1, directed at an angle α = 9.2535° to the x-axis (i.e. along the main axis of anisotropy). For effective magnetization of the medium, a sufficiently accurate result is obtained: $\wp$=1.0089·exp(j·9.2561·π/180).

For the parameters of the HM corresponding to Fig. 7, the corresponding results are equal to:

$$\boldsymbol{\kappa} = \begin{vmatrix} 0,4462 & -0,0029 \\ -0,0025 & 0,5571 \end{vmatrix}, \ \tilde{\boldsymbol{\kappa}} = \begin{vmatrix} 0,4461 & 0 \\ 0 & 0,5572 \end{vmatrix}, \ \alpha = 1,4169^\circ.$$

The components of the tensor of effective relative magnetic permeability: $\tilde{\mu}_{xx}$ =1.4461, $\tilde{\mu}_{yy} = 1.5572$. Their decrease in comparison with the above values is explained by a decrease in the effective magnetization of the HM due to smaller values of the magnetic permeabilities of the discrete phases. The insignificant asymmetry of the tensor $\boldsymbol{\kappa}$ is explained by its almost zero non-diagonal components.

Calculation of the field of magnetic forces. To further illustrate the capabilities of the developed method, we present the results of calculating the distribution of the force field $|\boldsymbol{H}|\mathrm{grad}|\boldsymbol{H}|$ = 0,5 grad$(|\boldsymbol{H}|^2)$. As can be seen from the last expression, the force field of the HGMS matrix is completely determined by the distribution of the modulus of the magnetic field vector $\boldsymbol{H}$ in the working space of the matrix. Within the framework of the developed method, this distribution is easily obtained on the basis of expression (6) using its discrete analogue or the relation $\boldsymbol{H} = -\mathrm{grad}(\mathrm{Re}\,W(z))$.

To determine the force field $\boldsymbol{F}^*$, it is necessary to specify the vector of the external (background) field strength $\boldsymbol{H}_0$ and the dimensions of the matrix elements. For example, for the HM corresponding to Fig. 4, with the value of the main period ω_1=8 mm (the dimensions of the matrix elements are determined by proportional conversion and are visible from Fig. 4-9) and the external field H_0 =5 kA/m direction along this period, in Fig. 10 lines $|\boldsymbol{H}|^2$=const and the magnetic force vectors $\boldsymbol{F}^*$ perpendicular to them are shown. Since the force field of the matrix is highly heterogeneous, Fig. 10 shows a fragment of the domain with the most intense force field. The areas of magnetic particle capture zones are determined by the known value of the minimum extraction force $\left|\boldsymbol{F}^*\right|_{\min}$, the determination of which is beyond the scope of this paper. As noted above, this force depends on the magnetic susceptibility of the initial product, the size of the extracted fraction, and other technological parameters. For example, at $\left|\boldsymbol{F}^*\right|_{\min} = \left|\boldsymbol{F}\right|_{\min} / (\mu_0 \chi V) = 5.5\cdot 10^9$ A^2/m^3, which roughly corresponds to the real values, the isodines $\left|\boldsymbol{F}^*\right|$ = const and the particle extraction zones are shown in Fig. 11.

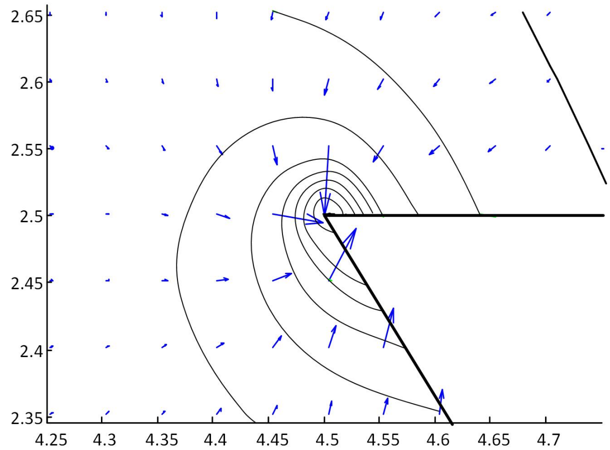

Fig. 10. Characteristics of the force field of the matrix of Fig. 4 in the corner zone of a triangular element

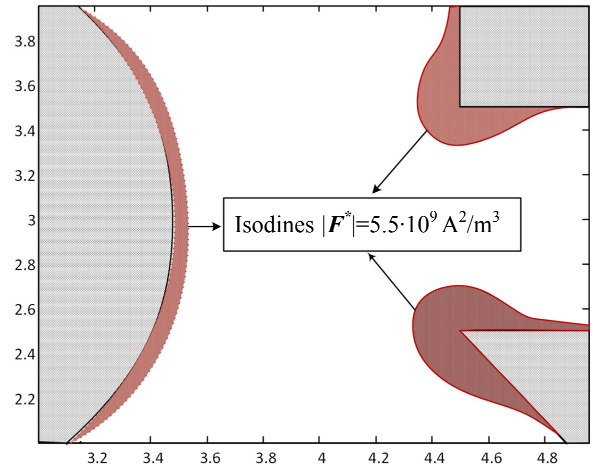

Fig. 11. Isodines $\boldsymbol{F}^*|$ = const and their corresponding extraction domains of the force field at $|\boldsymbol{F}^*|_{\min}$= 5.5·$10^9$ A^2/m^3 for a fragment of the matrix working space

The above analysis shows that the high heterogeneity of the force field (even in the extraction zone, the forces can differ by 2-3 orders of magnitude) is a negative factor. It is more preferable to have a field sufficient for extraction with a minimum spread of magnetic forces (ideally, isodynamic). We also note the high sensitivity of the force field to the strength value H_0 and the size of the filter elements. This casts doubt on the universality of the recommendations for determining the optimal geometric shapes of matrix elements without reference to the magnetic system of a particular HGMS and its comprehensive study.

Field analysis of the force field in the matrix can be continued in the following direction. Obviously, the formed extraction zones reduce the area and geometry of the pulp free flow area. The hydraulic permeability of the matrix can be investigated by solving the flow problem (see Fig. 8) with the geometry of liquid-impermeable regions modified due to particle sticking.

Thus, the information obtained on the basis of the developed method can be used in the development of new and modernization of existing HGMS in the following directions:

- calculation of the magnetic permeability tensor values (homogenization problem) makes it possible to quite accurately determine the magnetic resistance of the matrix as the main element of the magnetic system of the separator, and as a result of calculating the distribution of the magnetic flux in it, to determine the average magnetic flux density in the matrix and the calculated value of the field strength H_0. For the considered example, $\tilde{\mu}_{xx} \approx 2$ and H_0=5 kA/m, the average magnetic flux density is B = 0.126 T;
- for the selected geometrical and magnetic parameters of the matrix elements with a known value of strength $\boldsymbol{H}_0$, it is necessary to calculate the field of magnetic forces $|\boldsymbol{F}^*| \geq |\boldsymbol{F}^*|_{\min}$ (according to the example of Fig. 10), and for a given value of the minimum holding force $|\boldsymbol{F}^*|_{\min}$ – the areas of potential extraction zones and the fill factor of the working space (Fig. 10). It should be borne in mind that the specific magnetic resistance of the matrix does not depend on the absolute dimensions of its elements and the strength of the external field. At the same time, the magnetic forces $|\boldsymbol{F}^*|$ are proportional to $|\boldsymbol{H}_0|^2$ and inversely proportional to the absolute sizes of the elements. From this it follows that the recalculation of the force field in these cases should not be done, since the picture of isodines $\left|\boldsymbol{F}^*\right|$= const remains unchanged, only the values of their values change;
- calculated configuration of liquid-tight areas makes it possible to evaluate the hydraulic permeability of the pulp and decide on a change in the force field $\boldsymbol{F}^*$ in one direction or another;
- varying the geometric sizes and shapes of the matrix elements and conducting a series of corresponding computational experiments, it is possible to optimize the HGMS magnetic system as a whole with given technological limitations.

Thus, the use of the proposed method will create additional opportunities for improving the technical characteristics of electrophysical devices with HM elements, for example, high-gradient magnetic separators, electrostatic filters, and other structures for which the universality and accuracy of calculating effective and especially local field characteristics are decisive.

Conclusions.

1. A universal method has been developed for calculating the local and effective characteristics of the magnetic field of a multicomponent heterogeneous medium with a doubly periodic structure which is based on solving the integral equation with respect to the magnetization vector of the elements of the main parallelogram of the periods.

2. The performed computational experiments confirm the high efficiency and accuracy of the proposed method. Its main advantages are the compactness of the computational domain, the absence of the need to specify unknown boundary conditions on the sides of the parallelogram of the periods and severe restrictions on the geometry and number of components of a heterogeneous medium.

3. One of the effective areas of application of the developed method is the analysis of the force fields of matrices of high-gradient magnetic separators. The ability to comprehensively take into account the factors

determining the effective and local field characteristics opens up additional possibilities for optimizing the matrix parameters and improving the overall dimensions and technological characteristics of the separator as a whole.

4. Without significant changes, the method can be used in the analysis of other potential fields in doubly periodic systems (design of electrostatic filters, problems of flowing around gratings of a complex profile, etc.).

REFERENCES

1. Yemets Y.P. *Elekricheskie harakteristiki kompozicionnih materialov s reguliarnoi ctrukturoi* [Electrical characteristics of regular structure composites]. Kyiv, Naukova Dumka Publ., 1986. 191 p. (Rus).

2. Tolmachev S.T. *Specialnie metody resheniia zadach magnitostatiky* [Special methods for solving magnetostatic problems]. Kyiv, Vyshcha shkola Publ., 1983. 166 p. (Rus).

3. Kowalczyk P., Bielski W., Idzik A. Effective conductivity in two-dimensional two-component structures: macroscopic isotropy. *Photonics Applications in Astronomy, Communications, Industry, and High-Energy Physics Experiments*, 2014, T. 9290, P. 92901A. **doi: 10.1117/12.2075144.**

4. Earhart C.M., Nguyen E.M., Wilson R.J., Wang Y.A., Wang S.X. Designs for a microfabricated magnetic sifter. *IEEE Transactions on Magnetics*, 2009, vol. 45, no. 10, pp. 4884-4887. **doi: 10.1109/tmag.2009.2026486.**

5. Ge W., Encinas A., Araujo E., Song S. Magnetic matrices used in high gradient magnetic separation (HGMS): A review. *Results in Physics*, 2017, vol. 7, pp. 4278-4286. **doi: 10.1016/j.rinp.2017.10.055.**

6. Oder R. High gradient magnetic separation theory and applications. *IEEE Transactions on Magnetics*, 1976, vol. 12, iss. 5, pp. 428-435. **doi: 10.1109/TMAG.1976.1059076.**

7. Svoboda J. *Magnetic Techniques for the Treatment of Materials*. Kluwer Academic Publ., 2004. 642 p. **doi: 10.1007/1-4020-2107-0.**

8. Ren L., Zeng S., Zhang Y. Magnetic field characteristics analysis of a single assembled magnetic medium using ANSYS software. *International Journal of Mining Science and Technology*, 2015, vol. 25, no. 3, pp. 479-487. **doi: 10.1016/j.ijmst.2015.03.024.**

9. Song C.C., Ning G.H., Yuan Z.Y., Jing L.X., Hui C.C., Yao M.S. Investigation of the influence of different matrix rotation angles on the surrounding magnetic field in a uniform magnetic field. *Ming Metall Eng*, 2014, no. 34, pp. 290-294.

10. Gerlici J., Shvedchykova I.O., Romanchenko J.A., Nikitchenko I.V. Determination of the rational geometrical parameters of plate type elements of magnetic matrix of the polygradient separator. *Electrical engineering & electromechanics*, 2018, no.4, pp. 58-62. **doi: 10.20998/2074-272X.2018.4.10.**

11. Hou L.S., Geng L. *A kind of high gradient magnetic matrix for high-intensity magnetic separator*, CN. Patent, 2012.

12. Zheng X., Wang Y., Lu D., Li X. Study on the application of elliptic cross-section matrices for axial high gradient magnetic separation: key considerations for optimization. *Physicochemical Problems of Mineral Processing*, 2019, vol. 55(3), pp. 655-666. **doi: 10.5277/ppmp18178**.

13. Ding L., Chen L.Z., Zeng J.W. Investigation of combination of variable diameter rod elements in rod matrix on high gradient magnetic separation performance. *Advanced Materials Research*, 2014, vol. 1030-1032, pp. 1193-1196. **doi: 10.4028/www.scientific.net/amr.1030-1032.1193.**

14. Tolmachev S.T., Rozhnenko Z.G. Complex solve of a magnetostatic problem in systems with ordered heterogeneous medium. *Bulletin of NTU «KhPI»*, 2008, no. 40, pp. 139-145. (Rus).

15. Hurwitz A., Courant R. *Vorlesungen über allgemeine Funktionentheorie und elliptische Funktionen*. J. Springer, 1922. 399 p. (Ger).

16. Tolmachev S.T., Bondarevskyi S.L. Classification of heterogeneous structures and conditions of their doubly periodicity. *Eastern-European journal of enterprise technologies*, 2013, vol. 5, no. 5(65), pp. 24-29. (Rus).

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S.T. Tolmachev[1], Doctor of Technical Science, Professor,
S.L. Bondarevskyi[1], Candidate of Technical Science, Associate Professor,
A.V. Il'chenko[1], Candidate of Technical Science, Associate Professor,
[1] Kryvyi Rih National University,
11, Vitaly Matusevich Str., Kryvyi Rih, Dnipropetrovsk Region, 50027, Ukraine,
e-mail: kafem.knu@gmail.com

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