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APPLICATION OF VYSHNEGRADKY'S DIAGRAMS FOR TRANSIENT ANALYSIS IN ELECTRIC DISCHARGE INSTALLATIONS WITH STOCHASTIC LOAD

Purpose. To analyze the transient processes in the discharge circuit of reservoir capacitor of electric discharge installations at a change in the circuit configuration during the discharge as well as to determine the appropriate circuit parameters for which the discharge process described by the third-order differential equation remains a damped oscillatory process. Methodology. We have applied the concepts of theoretical electrical engineering, the principles of theory of electrical circuits, theory of automatic control systems and mathematical simulation in the software package MathCAD 12. Results. We have obtained the analytical expressions and graphical dependencies that allow us to determine a relationship between the value of the element parameters of the discharge circuit of installations with an additional active-inductive chain and the character of the transient discharge process without solving a third-order differential equation. Originality. Using Vyshnegradsky's criteria and their graphical representations in the form of diagrams, we have proposed the procedure for determining the inductance value in additional chain shunting the capacitor of electric discharge installation in order to avoid the undesirable aperiodic discharge transient process in the stochastic load. Practical value. The use of this approach makes it possible to determine the ranges of the expedient change in the additional inductance at different load resistances for the realization of transient process required by the technology – the oscillatory discharge of the reservoir capacitor trough the load. References 10, tables 1, figures 3.

Key words: **electric discharge installation, capacitor discharge, transients, Vyshnegradsky's diagram, stochastic load.**

Цель. Целью статьи является анализ переходных процессов в разрядной цепи накопительного конденсатора электроразрядных установок при изменении ее конфигурации во время разряда и определение целесообразных параметров цепи, при которых процесс разряда, описываемый дифференциальным уравнением третьего порядка, остается затухающим колебательным процессом. Методика. Для проведения исследований использовались научные положения теоретической электротехники, теория электрических цепей, теория систем автоматического регулирования и математическое моделирование в программном пакете MathCAD 12. Результаты. Получены аналитические выражения и графические зависимости, устанавливающие связь между значениями параметров элементов разрядной цепи установок с дополнительной активно-индуктивной цепочкой и характером переходного процесса разряда конденсатора без решения дифференциального уравнения третьего порядка. Научная новизна. Используя критерии Вышнеградского и их графические изображения в виде диаграмм, предложена методика определения величины индуктивности дополнительной цепочки, шунтирующей конденсатор электроразрядной установки, для исключения его нежелательного апериодического разряда на стохастическую нагрузку. Практическое значение. Использование данного подхода позволяет определить диапазоны целесообразного изменения дополнительной индуктивности при различных сопротивлениях нагрузки для реализации необходимого для технологии переходного процесса – колебательного разряда накопительного конденсатора на нагрузку. Библ. 10, табл. 1, рис. 3.

Ключевые слова: **электроразрядная установка, разряд конденсатора, переходные процессы, диаграмма Вышнеградского, стохастическая нагрузка.**

Introduction. In the electric discharge installations (EDI) with reservoir capacitors, in particular in the semiconductor (thyristor) installations for volumetric electro-spark dispersion (VESD) of the metals, the oscillatory discharge of capacitor with a small reverse recharge (less than 30 % in voltage) is the most efficient technologically and energetically mode of discharge through electric spark load [1-6]. In this case, there is a fast natural locking of the discharge semiconductor switch, which makes it possible to quickly carry out the subsequent charge of the capacitor and further its discharge trough the load [1, 4-6]. Thus, we can realize a high frequency of charge-discharge cycles and stability of the duration of discharge currents in the EDI load.

At the same time the resistance of such load as a metal granular layer can stochastically increase several times during discharge. As a result, a so-called idle discharge trough the load, i.e., a long-term discharge with a low current without sparking can occur [1, 4, 6-8]. Since the increase in active resistance of load decreases the Q-factor of the discharge circuit, then the oscillatory capacitor discharge transient can become aperiodic one, and discharge duration can increase many times. Because of such long capacitor discharges, we can not to realize high frequency and stability of charge-discharge cycles, and thus the yield of spark-eroded powders.

To reduce the discharge pulse duration in such EDI, we have proposed to connect an additional shunt chain VT_2-L_2-R_2 in parallel to the capacitor at a certain time t_1 as shown in Fig. 1. The parameters of the additional chain must be chosen from the condition for avoiding of aperiodic capacitor discharge.

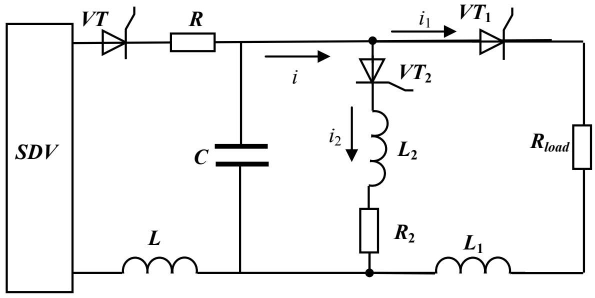

Fig. 1. Electric schematic diagram of EDI with additional RL-chain shunting the capacitor

The **purpose** of this paper is to analyze the transient processes in the discharge circuit of reservoir capacitor of electric discharge installations at a change in the circuit configuration during the discharge as well as to determine the appropriate circuit parameters for which the discharge process described by third-order differential equation remains a damped oscillatory process.

Transient analysis of capacitor discharge through the load when the capacitor is shunted by the *RL*-chain. As an example, we have performed the transient analysis of the capacitor discharge trough the load in the thyristor installation for VESD with an additional parallel active-inductive chain. In the installation for VESD, whose electrical circuit is shown in Fig. 1, the capacitor C is charged to a voltage U_0 from a shaper of direct voltage (*SDV*). Then, after switching on the discharge thyristor VT_1, the capacitor C is discharged through the load with the resistance R_{load} and discharge circuit inductance L_1, which is usually 1–5 µH.

We have assumed that the resistance R_{load} (that take into account not only the resistance of the electric spark load, but the active resistance of the circuit wires) remains unchanged during the discharge, but could change discontinuously between the discharges. It has been also assumed that the thyristor VT_2 was locked until the time t_1, and the discharge process was aperiodic, that is, the Q-factor of the C-VT_1-R_{load}-L_1-C discharge circuit $Q_1 < 0.5$.

During the discharge transient analysis, we have believed that the thyristors VT_1 and VT_2 were ideal switches, that is, the commutation occurred instantaneously and without power loss.

Expressions for the voltage of the capacitor $u_C(t)$ and the current $i(t)$ in the discharge circuit are [10]:

$$u_C(t) = U_0\left(p_1 e^{p_2 t} - p_2 e^{p_1 t}\right)\Big/\left(p_1 - p_2\right), \qquad (1)$$

$$i(t) = U_0\left(e^{p_1 t} - e^{p_2 t}\right)\Big/L_1\left(p_1 - p_2\right), \qquad (2)$$

where U_0 is the initial capacitor voltage; p_1 and p_2 are the roots of the characteristic equation of this circuit:

$$p_1 = -R_{load}\big/2L_1 + \sqrt{R_{load}^2\big/4L_1^2 - 1/L_1 C},$$

$$p_2 = -R_{load}\big/2L_1 - \sqrt{R_{load}^2\big/4L_1^2 - 1/L_1 C}.$$

At point in time $t = t_1$, when the current in the circuit is equal to a certain value $i(t_1) = I_1$, and the capacitor voltage has a certain value $u_C(t_1) = U_1$, the thyristor VT_2 unlocks and an additional L_2R_2-chain is connected to the circuit, that is, the circuit changes its configuration.

In new transient process, which started at $t \geq t_1$ in the circuit with the changed configuration, the following system of equations is valid according to the second Kirchhoff's law:

$$\begin{cases} u_C + u_{L_2} + u_{R_2} = 0; \\ u_C + u_{L_1} + u_{R_{load}} = 0. \end{cases} \qquad (3)$$

As $u_{L_1} = L_1\, di_1/dt,\ u_{L_2} = L_2\, di_2/dt,\ u_{R_{load}} = R_{load} i_1$, $u_{R_2} = R_2 i_2$, then system (3) can be written as:

$$\begin{cases} u_C + L_2\, di_2/dt + R_2 i_2 = 0; \\ u_C + L_1\, di_1/dt + R_{load} i_1 = 0. \end{cases} \qquad (4)$$

Since according to first Kirchhoff's law $i = i_1 + i_2$, and the current i flowing through the capacitor is $i = C\, du_C/dt$, we can write the following expression:

$$du_C/dt = i_1/C + i_2/C. \qquad (5)$$

Let us perform the differentiation of the system (4):

$$\begin{cases} du_C/dt + L_2\, d^2 i_2\big/dt^2 + R_2\, di_2/dt = 0; \\ du_C/dt + L_1\, d^2 i_1\big/dt_2 + R_{load}\, di_1/dt = 0. \end{cases} \qquad (6)$$

After substituting (5) in (6) and performing the transformations, we obtain the system:

$$\begin{cases} i_1 = -CL_2\, d^2 i_2\big/dt^2 - CR_2\, di_2/dt - i_2; \\ i_2 = -CL_1\, d^2 i_1\big/dt^2 - CR_{load}\, di_1/dt - i_1. \end{cases} \qquad (7)$$

Let us perform the differentiation of second equation of system (7) once, and then twice:

$$di_2/dt = -CL_1\, d^3 i_1\big/dt^3 - CR_{load}\, d^2 i_1\big/dt^2 - di_1/dt, \qquad (8)$$

$$d^2 i_2\big/dt^2 = -CL_1\, d^4 i_1\big/dt^4 - CR_{load}\, d^3 i_1\big/dt^3 - d^2 i_1\big/dt^2. \qquad (9)$$

Substituting (8), (9) and the second equation of system (7) into the first equation of this system and performing the transformations, we get

$$CL_1L_2\, d^4 i_1\big/dt^4 + C(L_1R_2 + L_2R_{load})\, d^3 i_1\big/dt^3 +$$

$$+ (L_1 + L_2 + CR_{load}R_2)\, d^2 i_1\big/dt^2 + (R_{load} + R_2)\, di_1/dt = 0.$$

After integrating this expression, we have

$$\begin{aligned} &CL_1L_2\, d^3 i_1\big/dt^3 + C(L_1R_2 + L_2R_{load})\, d^2 i_1\big/dt^2 + \\ &+ (L_1 + L_2 + CR_{load}R_2)\, di_1/dt + (R_{load} + R_2) i_1 + A_i = 0, \end{aligned} \qquad (10)$$

where A_i is a constant of integration, which we define from the final conditions.

Since at $t = \infty$ the capacitor is discharged to zero and all currents in the circuit (as well as their derivatives) are 0, then $A_i = 0$, and equation (10) takes the form

$$\begin{aligned} &CL_1L_2\, d^3 i_1\big/dt^3 + C(L_1R_2 + L_2R_{load})\, d^2 i_1\big/dt^2 + \\ &+ (L_1 + L_2 + CR_{load}R_2)\, di_1/dt + (R_{load} + R_2) i_1 = 0. \end{aligned} \qquad (11)$$

Thus, we have obtained a third-order differential equation whose characteristic equation can be written as

$$a_0 p^3 + a_1 p^2 + a_2 p + a_3 = 0, \qquad (12)$$

where $a_0 = CL_1L_2,\quad a_1 = C(L_1R_2 + L_2R_{load}),$

$$a_2 = L_1 + L_2 + CR_{load}R_2,\quad a_3 = R_{load} + R_2.$$

For delimitation of areas with different types of transients, which are described by the third-order differential equations, in many cases it is expedient to use Vyshnegradsky's diagrams [10]. Vyshnegradsky's criterion and its graphic representation in the form of diagrams allow us to judge the influence of parameters of third-order system on its stability without solving the differential equation.

Bringing the equation (12) to a normalized form and introducing a new variable

$$q = p \cdot \sqrt[3]{a_0/a_3}, \qquad (13)$$

we obtain, as a result, the normalized equation

$$q^3 + Aq^2 + Bq + 1 = 0, \qquad (14)$$

where $A = a_1\Big/\sqrt[3]{a_3 a_0^2}$ and $B = a_2\Big/\sqrt[3]{a_0 a_3^2}$ coefficients are called the Vyshnegradsky's parameters.

On the plane of A and B parameters we can plot a Vyshnegradsky's diagram that display the regions of stable and unstable operation of the system described by a third-order differential equation whose characteristic equation has the form (12).

The stability conditions for the third-order system, formulated by Vyshnegradsky, are

$$A > 0,\ \ B > 0,\ \text{ and }\ AB > 1. \qquad (15)$$

The equation for oscillatory stability threshold is

$$AB = 1 \ \text{ at } \ A > 0 \ \text{ and } \ B > 0.$$

This is an equilateral hyperbola, for which the coordinate axes are the asymptotes (Fig. 2). The region of system stability according to conditions (15) lies above this curve.

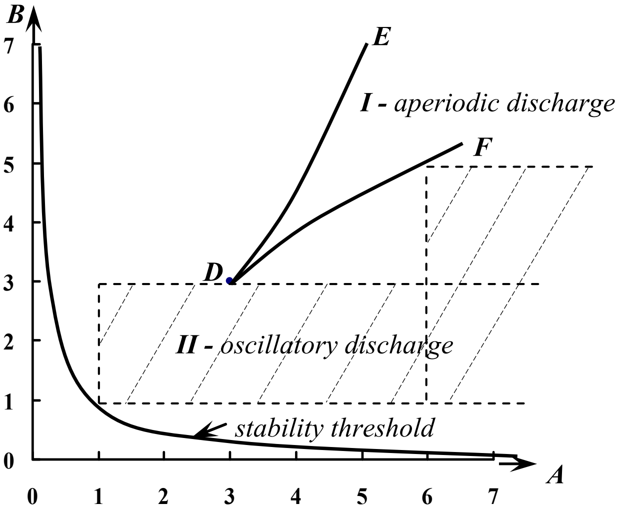

Fig. 2. Vyshnegradsky's diagram for the system, described by third-order differential equation

The stability region can be divided into separate parts corresponding to different combinations of the roots of the characteristic equation. It should be noted that at the point D, where A = 3 and B = 3, the characteristic equation (14) takes the form $(q + 1)^3 = 0$. Consequently, at this point all three roots are equal $q_1 = q_2 = q_3 = -1$. In this case, for the initial equation (13), we obtain $p_1 = p_2 = p_3 = -\sqrt[3]{a_3/a_0}$.

In the general case, two options are possible: 1) all three roots are real; 2) one root is real and two are complex. The boundary between these two cases is determined by the vanishing discriminant of the third-degree equation (14), which can be received, for example, from the Cardano's formula for solving the cubic equation:

$$A^2B^2 - 4\left(A^3 + B^3\right) + 18AB - 27 = 0.$$

This equation gives two curves in the plane of the A and B parameters: DE-curve and DF-one (Fig. 2). Inside of EDF region, the discriminant is positive. Consequently, in this region there are three real roots (region ***I***). In the remaining part of the plane, the discriminant is negative, which corresponds to the presence of a pair of complex roots (region ***II***).

In region ***I***, where all roots are real, an aperiodic transient process takes place, and in region ***II***, where there are one real and two complex roots, the transient process is oscillatory.

Calculating the value of Vyshnegradsky's parameters at changing the parameters of the discharge circuit (parameters of R_2L_2 -chain connected to the capacitor), we can immediately conclude whether they are in the stability region of the system and if this is the case, then in which part of the region they are located (aperiodic discharge region ***I*** or oscillatory one ***II***).

Hence, when the load resistance increases stochastically during the discharge of the capacitor we can easy choose the necessary parameters R_2L_2-chain for connecting to the capacitor in order to prevent a long-term discharge with a low current without sparking in the load.

The investigations carried out in the installation for the volumetric electro-spark dispersion of aluminum in water with the following parameters: L_1 = 5 μH, C = 100 μF, showed that resistance of the load, which is a layer of aluminum granules located between the electrodes, can vary within R_{load} = 0.2 – 5 Ohm. Therefore, the Q-factor of the discharge circuit C-VT_1-R_{load}-L_1-C: Q_1 can be in the range of 1.118 – 0.045. That is, the discharge of the capacitor with certain changes in the load resistance can be aperiodic (Q_1 <0.5). That's why, it is necessary to connect an additional active-inductive chain in order to change the nature of the discharge process. The resistance R_2 of such a chain takes into account the active resistances of both the wires of the inductive coil L_2, and the wires connecting this coil to the discharge circuit. This value is about 0.001 Ohm.

Fig. 3 shows the values of the Vyshnegradsky's parameters calculated using the software package Mathcad 12 for the discharge circuit C-VT_1-R_{load}-L_1-C with the parameters $C = 10^{-4}$ F, $L_1 = 5\cdot10^{-6}$ H, R_{load} = 5 Ohm, 2.5 Ohm, and 1 Ohm. The initial Q-factors are, respectively, Q_1 = 0.045, 0.089, and 0.224, i.e. the capacitor discharge is aperiodic and for changing the discharge character it is necessary to connect an additional active-inductive chain. The resistance of the additional chain was assumed to be R_2 = 0,001 Ohm, and the inductance value varied in the range $L_2 = 10^{-7} \div 14\cdot10^{-4}$ H.

According to Fig. 2 in zones defined by conditions, for example $\begin{cases} A > 1 \\ 1 < B < 3 \end{cases}$ or $\begin{cases} A > 6 \\ 1 < B < 5 \end{cases}$, the discharge of the capacitor when the additional active-inductive circuit is connected becomes an oscillatory. Further, according to the dependences shown in Fig. 3, the inductance values L_2 satisfying the above conditions are determined.

The results of the analysis of the value ranges of the additional inductance L_2, which are required for the realization of the oscillatory discharge of the capacitor in the circuit with different load resistance, are given in Table. 1.

According to the proposed procedure, Vyshnegradsky's diagrams can be used to estimate the transient processes in the circuits of electrical discharge installations with different parameters and configuration.

Since the oscillatory discharge duration is proportional to the circuit inductance value, then in order to ensure short-term discharges, the appropriate values of L_2 should be minimum values from corresponding ranges: 103 μH, 48 μH, 15 μH.

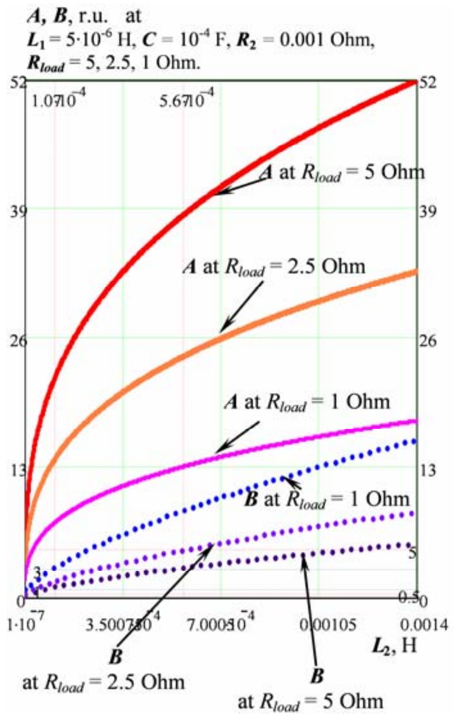

Fig. 3. The calculated values of the Vyshnegradsky's parameters for a discharge circuit with $L_1 = 5\cdot10^{-6}$ H, $C = 10^{-4}$ F, R_{load} = 5 Ohm, 2.5 Ohm, 1 Ohm

Table 1

Value ranges of additional inductance L_2 for different load resistance R_{load}

R_{load}, Ohm	L_2, μH at $\begin{cases} A > 1 \\ 1 < B < 3 \end{cases}$	L_2, μH at $\begin{cases} A > 6 \\ 1 < B < 5 \end{cases}$
5	103 – 567	103 – 1240
2.5	48 – 281	48 – 617
1	15 – 109	55 – 242

Conclusions.

1. In the discharge circuit of the capacitor of electric discharge installations whose load resistance can increase randomly, a low-current discharge (so-called idle discharge) through the load may occur. In order to transfer such a discharge into the required high-current and quickly damped discharge, we have connected the additional inductance to the discharge circuit.

2. To determine the value of the additional inductance, it is advisable to apply Vyshnegradsky's criteria and their graphic representations in the form of diagrams. This approach allows us to determine the range of values of such inductance for various load resistances in order to transfer aperiodic long-term capacitor discharge through the load in a short-term oscillatory discharge without solving a third-order differential equation.

3. As an example, we have performed a transient analysis in the discharge circuit of installation for volumetric electro-spark dispersion of the metals in a liquid with parameters $C = 10^{-4}$ F, $L_1 = 5\cdot10^{-6}$ H, $R_2 = 0.001$ Ohm. We have calculated the Vyshnegradsky's parameters for load resistance of the installation R_{load} = 5 Ohm, 2.5 Ohm, and 1 Ohm. Using the Vyshnegradsky's diagram, we have determined the ranges of the values of the additional inductance L_2 for the realization of the oscillatory discharge of capacitor of the installation with a change in its stochastic load resistance.

Appropriate values of L_2 are the minimum values from the corresponding ranges: 103 μH, 48 μH, 15 μH.

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