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WAYS TO IMPROVE THE EFFICIENCY OF COMPUTER SIMULATION OF ELECTRICAL SYSTEMS MODES BASED ON EQUATIONS IN PHASE COORDINATES

The development of electrical systems must be accompanied by the development of tools and their modeling. However, the possibility of development models, traditionally developed on the basis of the transition from the real to the single-phase three-phase circuits equivalents, represented exhausted. Therefore, along with the use of single-phase three-phase equivalents need to develop models in phase coordinates. Showing the need to move to the development of models based on equations in the phase coordinates and the possibility of increasing the effectiveness of development through the use of implicit methods of integration, the transition to a higher level of decomposition and unification models developed for the implementation of the structural approach to modeling complex systems. References 3.
Key words: **stationary modes, electric networks, mathematical models, phase coordinates, transients.**

Показаны необходимость перехода к разработке моделей на основе уравнений в фазных координатах и возможности повышения эффективности таких разработок за счет использования неявных методов интегрирования, перехода на более высокий уровень декомпозиции и унификации разрабатываемых моделей. Библ. 3.
Ключевые слова: **стационарные режимы работы, электрические сети, математические модели, фазные координаты, переходные процессы.**

Introduction. The current stage of development of electrical systems characterized by the fact that all the more significant becomes the influence of a number of factors affecting the quality of electrical energy. These factors are linked, firstly, with the advent of new processes and new equipment, and, secondly, with aging and wear of the main equipment of electrical systems. The introduction of new technological processes tend to be associated with an increased harmonic sources, distorting the shape of the voltage curve in electrical networks and equipment wear increases the asymmetry sources as individual network elements are forced to work part number of phases for a long time required for maintenance and repair works on the damaged phase.

To solve the problems of analysis of electrical systems in these new conditions and the need to develop new, more complete and accurate mathematical models and corresponding software to allow playback modes systems in the presence of harmonics and unbalance sources. In the traditional approach to modeling, based on the transition from the real three-phase circuit to the single-phase equivalents (in symmetrical components, d-q-0, α-β-0 coordinates, etc.) such models in principle can not be implemented, as the transition itself strictly justified and is possible only when there is symmetry and sinusoidal [1]. For this reason, attempts to develop models based on the transition to single-phase equivalents, in a direction to account unbalance and harmonics, are meaningless.

The goal of the paper is the rationale for the transition to the development of models in the phase coordinates and identification of ways to enhance the effectiveness of such developments.

Statement of the base material. The need for a transition from single-phase to three-phase equivalent model (in phase coordinates), in an environment where development opportunities based on the traditional approach models are exhausted, becomes apparent. However, the three-phase model, reproducing three-phase mode with all the major influencing factors, especially in transitional processes is much more difficult phase and, accordingly, their development and program implementation require greater time and cost [2]. Therefore, the amount of work in this area is still small.

At the Department «Electric Power Transmission» of the NTU «KhPI» the development of models of electrical systems based on the equations in the phase coordinates, in stationary and transient, symmetric and asymmetric modes is conducted for a sufficiently long time, and the existing experience shows that these difficulties are surmountable if:

Firstly, for the solution of systems of differential equations to use implicit numerical integration methods. When using implicit methods of numerical integration eliminates the need to bring the systems of differential equations to the normal form of Cauchy, which significantly reduces the complexity of this stage modeling and its software implementation, especially in the simulation of complex systems. Furthermore, implicit methods provide higher processing stability. And another factor in favor of the choice of implicit numerical integration methods for solving the problems is that while it is possible to complete structural modeling – i.e. we first need to develop a finite-difference (discrete) model of individual elements of a complex system, and then perform the formation of a system model as a whole.

Second, the move to a higher level of decomposition – as elements of the settlement scheme is not considered bipolar *R*, *L*, *C* elements, but three-phase multipoles corresponding to the three-phase network elements. Equivalent circuit complex systems created even on a single phase, have a large number of bipolar *R*, *L*, *C* elements and complex configuration, the transition to three-phase circuits and taking into account along with the longitudinal parameters of transverse capacitive and inductive coupling power line number increases more than threefold which greatly complicates the process of formation of systems of equations in the stationary, and, especially in transient conditions. In the transition to the level of the three-phase switching circuits the number of elements is reduced, simplified circuit topology, all of the features of embodiment, the parameters of the phases and their mutual influence is reflected in the matrices of the third order of the parameters of the corresponding elements, and the process by which systems of equations at the level of the three-phase switching circuits becomes less complicated. It should also be noted that the transition to the level of three-phase multiport matrix coefficients in the general equations in stationary and transient regimes acquire a pronounced block structure, and opportunities to improve the simulation efficiency is provided through the use of new means of modern programming languages (sub-types, object-oriented programming, etc.).

And third, to present three-phase multipolar equation elements in phase coordinates in unified manner. When used for solving systems of differential equations implicit numerical integration methods, as has been said, it is possible the implementation of formalized procedures for the construction of a model system for the pre-formed models of individual elements. At the same time significantly simplify all stages of the simulation can be achieved if the discrete models of all elements of the system at the stage of their formation to present in the unified form.

At modeling elements useful to distinguish two groups of elements:

- static elements (air and cable lines (AL and CL), power transformers and auto-transformers, static elements of the load units, means of reactive power compensation);
- rotating electrical machines (synchronous generators, synchronous compensators, synchronous and asynchronous motors).

The equations of transients of any of the static elements (AL, CL, transformers, etc.) in the phase coordinates in the matrix form are:

$$[L]_{ij}^{F}\frac{d}{dt}[i]_{ij}^{F}+[R]_{ij}^{F}[i]_{ij}^{F}=[\Delta u]_{i}^{F}\,, \qquad (1)$$

and for different elements differ only in the order and structure of the matrices [*L*] and [*R*], where [*L*] is the matrix of self and mutual inductances of the phases (windings of the transformer, wires of AL or CL, and others), [*R*] is the matrix of resistances of the corresponding element phases.

For implicit methods of integration equations of the element (1) in phase coordinates must be presented in the normal form

$$\frac{d}{dt}[i]_{ij}^{F}=[L]_{ij}^{F-1}[\Delta u]_{ij}^{F}-[L]_{ij}^{F-1}[R]_{ij}^{F}[i]_{ij}^{F}\,, \qquad (2)$$

and at using equations discretization to be sampled, such as the Euler-Cauchy method

$$x_{k+1}=x_k+\frac{h}{2}\left(f_{k+1}+f_k\right)$$

to perform an approximation of the original differential equations using formulas:

$$[i]_{L}^{(K+1)}=[i]_{L}^{(K)}+\frac{h}{2}\left(\frac{d}{dt}[i]_{L}^{(K+1)}+\frac{d}{dt}[i]_{L}^{(K)}\right). \qquad (3)$$

Substituting in (3) expressions for the derivatives of (2), we obtain the expression:

$$[i]_{ij}^{(k+1)}=[i]_{ij}^{(k)}+\frac{h}{2}\left([L]_{ij}^{-1}[u]_{ij}^{(k+1)}-[L]_{ij}^{-1}[R]_{ij}[i]_{ij}^{(k+1)}\right)+$$
$$+\frac{h}{2}\left([L]_{ij}^{-1}[u]_{ij}^{(k)}-[L]_{ij}^{-1}[R]_{ij}[i]_{ij}^{(k)}\right),$$

which can be solved for the currents at the current step of integration:

$$[i]_{ij}^{(k+1)}=\frac{h}{2}[K]_{L}^{(-1)}[L]_{ij}^{(-1)}[u]_{ij}^{(k+1)}+\frac{h}{2}[K]_{L}^{(-1)}[L]_{ij}^{(-1)}[u]_{ij}^{(k)}+$$
$$+[K]_{L}^{(-1)}[i]_{ij}^{(k)}-\frac{h}{2}[K]_{L}^{(-1)}[L]_{ij}^{(-1)}[R]_{ij}[i]_{ij}^{(k)},$$

where $[K]_L=[E]+\frac{h}{2}[L]_{ij}^{-1}[R]_{ij}$, [*E*] is the unit matrix.

Obtained equations can be written as:

$$[i]_{ij}^{(k+1)}=[Y]_{ij}[u]_{ij}^{(k+1)}+[Y]_{ij}[u]_{ij}^{(k)}+[A]_{ij}[i]_{ij}^{(k)}, \qquad (4)$$

where $[Y]_{ij}$, $[A]_{ij}$ are the matrices defined respectively by longitudinal and transverse element parameters.

Equations couple voltage and phase currents at the current step of integration with the phase voltages and currents in the previous step.

They are useful for modeling the transients in the corresponding element and to be included in the system model. And the fact that the equations solved for the currents, allows the formation of a system of differential equations in general, to use the most effective method of node.

Using any other implicit methods of numerical integration, differential equations (1) any static element can be represented in the integration step in the form of (4).

System of differential equations of electrical machines (synchronous, induction) in phase coordinates in the matrix form comprises two groups of equations:

1) flux linkage equations

$$[\Psi]_S = [L]_S[i]_S + [L_{SR}][i]_R;$$
$$[\Psi]_R = [L]_{RS}[i]_S + [L]_R[i]_R, \quad (5)$$

2) voltage equilibrium equations of all electrical circuits in the stator and rotor

$$[U]_S = -\frac{d}{dt}[\Psi]_S - [R]_S[i]_S;$$
$$[U]_R = \frac{d}{dt}[\Psi]_R - [R]_R[i]_R, \quad (6)$$

where indexes S and R taken to refer to quantities relating to the windings of the stator and rotor, respectively.

Winding fluxes are functions of the rotor angle γ. Therefore derivatives of flux linkage with respect to time taking into account this dependence has the form

$$\frac{d}{dt}\begin{bmatrix}\psi_S\\ \psi_R\end{bmatrix} = \begin{bmatrix}\frac{dL(\gamma L)}{d\gamma}\frac{d\gamma}{dt}\end{bmatrix}\begin{bmatrix}i_S\\ i_R\end{bmatrix} + [L(\gamma)]\frac{d}{dt}\begin{bmatrix}i_S\\ i_R\end{bmatrix}.$$

Substituting derivatives of flux linkage in the equation (6), we obtain

$$\begin{bmatrix}L_S & L_{SR}\\ L_{RS} & L_R\end{bmatrix}\frac{d}{dt}\begin{bmatrix}i_S\\ i_R\end{bmatrix} + \left(\omega\begin{bmatrix}\frac{dL(\gamma L)}{d\gamma}\end{bmatrix} + \begin{bmatrix}r_S & \\ & r_R\end{bmatrix}\right)\begin{bmatrix}i_S\\ i_R\end{bmatrix} = \begin{bmatrix}U_S\\ U_R\end{bmatrix}. \quad (7)$$

Since when the rotor rotates the inductance of windings depends on the angular position γ of the rotor, voltage balance equation (7) comprise transformer EMF $L_{ij}\frac{di}{dt}$, due to changes in current in the j-th circuit, and also rotation EMF $\frac{dL_{ij}}{d\gamma}\omega_{ij}$ of a change in inductance when the rotor rotates.

However, considering that the rotational EMF, as the voltage drop dependent on the current in the windings, in the equations (7) for the expression in brackets may be taken designation

$$\left(\omega\begin{bmatrix}\frac{dL}{d\gamma}\end{bmatrix} + [R]\right) = [R_1],$$

and write them as

$$[L]\frac{d}{dt}[i] + [R_1][i] = \begin{bmatrix}u_S\\ u_R\end{bmatrix},$$

we can say that they are similar to the equations of static elements and are characterized in that the matrix elements of the phase inductances are periodic functions of time.

To move to the difference equations it is necessary to solve the resulting equations for the derivatives

$$\frac{d}{dt}\begin{bmatrix}i_S\\ i_R\end{bmatrix} = -[L(\gamma)]^{-1}\left(\omega\begin{bmatrix}\frac{dL(\gamma)}{d\gamma}\end{bmatrix} + \begin{bmatrix}r_S & \\ & r_R\end{bmatrix}\right)\begin{bmatrix}i_S\\ i_R\end{bmatrix} +$$
$$+[L(\gamma)]^{-1}\begin{bmatrix}U_S\\ U_R\end{bmatrix};$$

and go to the difference approximation in accordance with the Euler-Cauchy formula

$$\begin{bmatrix}i_S\\ i_R\end{bmatrix}^{(k+1)} = \begin{bmatrix}i_S\\ i_R\end{bmatrix}^{(k)} - h\left[L(\gamma)^{k+1}\right]^{-1}\times$$
$$\times\left(\omega\begin{bmatrix}\frac{dL(\gamma L)^{k+1}}{d\gamma}\end{bmatrix} + \begin{bmatrix}r_S & \\ & r_R\end{bmatrix}\right)\begin{bmatrix}i_S\\ i_R\end{bmatrix}^{(k+1)} +$$
$$+h\left[L(\gamma)^{(k+1)}\right]^{-1}\begin{bmatrix}u_S\\ u_R\end{bmatrix}^{(k+1)}.$$

If the obtained equation is solved for the currents in the windings at the (k+1)-th step, equations will have a form:

$$\begin{bmatrix}i_S\\ i_R\end{bmatrix}^{(k+1)} = \left[Y(\gamma)^{(k+1)}\right]\begin{bmatrix}u_S\\ u_R\end{bmatrix}^{(k+1)} +$$
$$+\left[Y(\gamma)^{(k+1)}\right]\begin{bmatrix}u_S\\ u_R\end{bmatrix}^{(k)} + \left[A(\gamma)^{(k+1)}\right]^{-1}\begin{bmatrix}i_S\\ i_R\end{bmatrix}^{(k)}, \quad (8)$$

where $[Y(\gamma)]$, $[A(\gamma)]$ are the matrices whose elements are determined by the self and mutual inductances of the windings and are functions of the rotor angle γ.

In equations (8), as in the discrete equations of static electricity network elements, the currents in the windings of the current step numerical integration of the equations expressed transient voltage on the windings through the current step and the currents in the windings of the previous step integration. In contrast to the static elements of the discrete parameters are variable and must be calculated at each step in the process of computing the angular position of the rotor function. In such form the unified equations may be incorporated into the system of equations to be solved at the numerical integration step.

Positive effect obtained when presenting items of electrical systems in a unified form (4) – (8) lies in the fact that:

- transient simulation algorithms in the individual elements can be regarded as a modification of a single generic algorithm, the main elements of which are: the calculation of the matrix elements of the original R, L, the formation of discrete parameters matrices Y, A, the calculation of the transition process in step parameters;
- unification of models of elements allows you to unify and other stages of the modeling system as a whole – topological analysis of network circuit, the formation of a system of equations, the solution of the resulting system, which, in turn, facilitates the realization of models in the phase coordinates on the basis of a structured approach to the modeling of complex systems.

The realization of the proposed approach, made in the study of operating modes of electrical systems with the asymmetry [3], confirms its effectiveness and feasibility – on a single algorithmic and methodological basis of the performed development of software tools to simulate the stationary and transient modes in the presence of any number of asymmetrical elements and switching.

Conclusions.

Further development of methods and means of electrical systems modes simulation for solving actual problems of control is possible only on the basis of the equations in the phase coordinates.

Improving the efficiency of development of models in the phase coordinates is provided on the basis of implicit numerical integration methods, the transition to a higher level of decomposition, the unification of models of elements and the use of the new features of modern programming languages.

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