UDC 621.31 **doi: 10.20998/2074-272X.2016.3.10**

Iu.A. Sirotin

ORTHOGONAL COMPONENTS OF THE THREE-PHASE CURRENT AT ASYMMETRICAL ACTIVE-REACTIVE LOAD IN 4-WIRE CIRCUIT

Purpose. For the unbalanced sinusoidal mode with asymmetric voltage in 3-phase 4-wire to receive the orthogonal 4-component decomposition of 3-phase current, are classified symmetry/asymmetry of active and reactive load elements separately. Methodology. The methodology is based on the vector approach, which with one voice allows to analyze the energy characteristics of a 4-wire and 3-wire circuits as balanced and unbalanced modes. At asymmetrical voltage the matrix representation methodology of the equivalent conductivities is used. Results. For 3-phase 4-wire network with a sinusoidal unbalanced mode with asymmetric voltage obtained 4-component orthogonal decomposition of the 3-phase current. The components have a clear electro-energetic sense and are classified irrespective by the load condition. Originality. The resulting decomposition current develops the theory Currents' Physical Components (CPC) for 4-wire circuit with asymmetric voltage. For the first time the unbalanced current is classified by activity and reactivity of asymmetry load elements. Practical value. Practical value of the obtained orthogonal decomposition current and the power equation is a possibility of their utilization for the increase both quality of delivery and quality of consumption of electrical energy. References 8, figures 1.
Key words: **three-phase circuit, active and reactive power, power shift, power equation, unbalanced current and mode**, **active-reactive asymmetrical load, asymmetrical voltage, currents' physical components (CPC)**.

Для 3– фазной схемы электроснабжения рассмотрен синусоидальный несимметричный режим. При несимметричном напряжении и ассиметричной активно–реактивной нагрузке для 4– проводной сети получено ортогональное разложение трехфазного тока. Четыре составляющие разложения классифицированы активностью/реактивностью и симметрией/асимметрией нагрузки и имеют однозначный электроэнергетический смысл. Для 4– проводной цепи с несимметричной нагрузкой при несимметричном напряжении полученное уравнение мощности развивает теорию токовых физических составляющих (Currents' Physical Components – CPC). Библ. 8, рис. 1.
Ключевые слова: **трехфазная цепь, активная и реактивная мощность, мощность сдвига, уравнение мощности, несбалансированный ток и режим, активно-реактивная несимметричная нагрузка, несимметричное напряжение, Currents' Physical Components (CPC).**

Introduction. Active-reactive unbalanced load not only consumes electrical energy (EE) of active power, but also the EE of inactive components of total power (TP) which leads to additional losses. An effective solution to the problem of reducing losses and increasing the accuracy of accounting EE is the combined use of compensating devices and differential accounting means consumption EE. However, even in a sinusoidal mode, taking into account existing means of measuring energy efficiency they measure EE due to only the *symmetry* of the load active and reactive elements (active power and reactive power of shift). In real conditions of asymmetry voltage components of TP due to the asymmetry of the active-reactive load elements lead to additional losses, however, are not measured are not counted and will not be compensated.

Problem definition. Compensation, measuring and accounting for components of TP are related, complementary objectives of effective EE consumption. These tasks are the same positions should be solved within the framework of the general theory of power using orthogonal decomposition of 3-phase current [1-6] The mutual orthogonal component decomposition can uniquely estimate the losses caused by them independently. Widely used power theory *Currents' Physical Components* (CPC) [2, 4-6] uses a methodology of orthogonal decomposition. At the sinusoidal unbalanced mode, 3-phase current comprises two orthogonal components: balanced and unbalanced. Balanced component (due to the symmetry of the active-reactive load elements) comprises orthogonally reactive current and active for three, and for a 4-wire circuit. The asymmetry of the active-reactive load elements, both in symmetric and asymmetric under voltage leads to unbalance current.

Unfortunately, even in a sinusoidal mode CPC theory developed either for 3-wire or 4-wire circuits with a symmetrical voltage [2, 4-6]. Thus, for 3-wire circuit is decomposed into two components with unbalanced voltage in the CPC power theory of unbalance current, using the method of symmetrical components [6], which is not shared by the asymmetry of the load active and reactive elements explicitly.

The goal of the work is for unbalanced mode with asymmetric voltage in 3-phase 4-wire to obtain orthogonal 4-component decomposition of the 3-phase current, are classified by symmetry/asymmetry of separate active and reactive load elements separately.

Periodic power processes. When considering a 3-phase 4-wire circuit we assume that the voltage in the phases are measured relative to the neutral (Fig. 1).

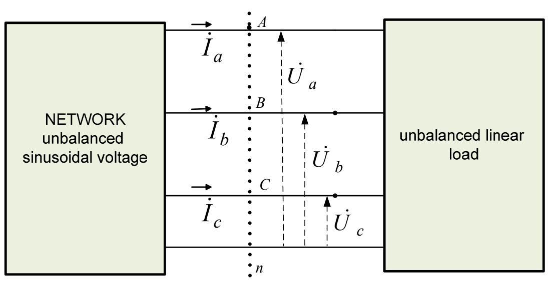

Fig. 1. 3-phase 4-wire power supply with unbalanced load – sinusoidal mode

At any time, instantaneous values (*i.v.*) of voltages (relative to a «neutral» conductor) and *i.v.* of currents in the phases are considered as 3-dimensional vectors (matrix columns) of an arithmetic 3-dimensional space $R^{(3)}$

$$\boldsymbol{u}(t)=\left[u_a(t)\ u_b(t)\ u_c(t)\right]^{\tau},\ \boldsymbol{i}(t)=\left[i_a(t)\ i_b(t)\ i_c(t)\right]^{\tau},\ (1)$$

hereinafter τ is the transposition sign.

Steady power mode in a 3-phase section <*A*, *B*, *C*> is determined by 3-D *T*-periodic curves of current and voltage:

$$\boldsymbol{u}(t)=\boldsymbol{u}(t+T),\ \boldsymbol{i}(t)=\boldsymbol{i}(t+T)\cdot$$

An ensemble of 3-D (3-phase) *T*-periodic vector curves

$$\boldsymbol{x}(t)=[x_a(t)\ x_b(t)\ x_c(t)]^{\tau},\quad t\in(v,v+T)\quad (2)$$

with finite root mean square (*rms*) value

$$\|\boldsymbol{x}\|=\sqrt{\frac{1}{T}\int_{v}^{v+T}\boldsymbol{x}(t)^{\tau}\boldsymbol{x}(t)dt}<\infty\quad (3)$$

form a Hilbert space

$$L_2^{(3)}(T)=\{\boldsymbol{x}(t),\quad t\in(v,v+T):\|\boldsymbol{x}\|<\infty\}.\quad (4)$$

For vector curves $\boldsymbol{x}(t)$, $\boldsymbol{y}(t)\in L_2^{(3)}(T)$ a scalar product (SP) is determined

$$<\boldsymbol{x},\boldsymbol{y}>=\frac{1}{T}\int_{v}^{v+T}\boldsymbol{x}(t)^{\tau}\boldsymbol{y}(t)dt=\frac{1}{T}\int_{v}^{v+T}(\boldsymbol{x}(t),\boldsymbol{y}(t))dt\quad (5)$$

as integral averaged of scalar products of *i.v.* in a 3-D space $R^{(3)}$.

In particular, for active power

$$<\boldsymbol{i},\boldsymbol{u}>=\frac{1}{T}\int_{v}^{v+T}\underbrace{\boldsymbol{i}(t)^{\tau}\boldsymbol{u}(t)}_{p(t)}dt=\frac{1}{T}\int_{v}^{v+T}(\boldsymbol{i}(t),\boldsymbol{u}(t))dt=P.\quad (6)$$

Instantaneous power

$$p(t)=\boldsymbol{i}^{\tau}\boldsymbol{u}=i_a(t)u_a(t)+i_b(t)u_b(t)+i_c(t)u_c(t)\quad (7)$$

equals to the electricity rate through the section <*A*, *B*, *C*>. In the space (4) the inequality of Cauchy-Schwarz is correct

$$<\boldsymbol{x},\boldsymbol{y}>\le\|\boldsymbol{x}\|\cdot\|\boldsymbol{y}\|.\quad (8)$$

In particular, active power does not exceed the apparent (total) power

$$P=<\boldsymbol{i},\boldsymbol{u}>\quad\le\quad\|\boldsymbol{u}\|\cdot\|\boldsymbol{i}\|.$$

Sinusoidal mode and 3-complexes. 3-D curves of *i.v.* sinusoidal processes of voltage and current

$$\boldsymbol{u}(t)=\sqrt{2}\Re e[\boldsymbol{U}e^{j\omega t}],\ \boldsymbol{i}(t)=\sqrt{2}\Re e[\boldsymbol{I}e^{j\omega t}].\quad (9)$$

are *T*-periodic ($T\omega$ = 2π) and fully determined by 3-complexes of voltage and current

$$\boldsymbol{U}=\begin{bmatrix}\dot{U}_a\\\dot{U}_b\\\dot{U}_c\end{bmatrix}=\begin{bmatrix}U_ae^{j\psi_a}\\U_be^{j\psi_b}\\U_ce^{j\psi_c}\end{bmatrix},\ \boldsymbol{I}=\begin{bmatrix}\dot{I}_a\\\dot{I}_b\\\dot{I}_c\end{bmatrix}=\begin{bmatrix}I_ae^{j\varphi_a}\\I_be^{j\varphi_b}\\I_ce^{j\varphi_c}\end{bmatrix},\quad (10)$$

– vectors of complex *rms* of voltage and current.

3-complexes (10) are calculated by 3-D curves of *i.v.* of sinusoidal processes of voltage and current

$$\boldsymbol{U}=\frac{\sqrt{2}}{T}\int_{v}^{v+T}\boldsymbol{u}(t)e^{-j\omega t}dt,\ \boldsymbol{I}=\frac{\sqrt{2}}{T}\int_{v}^{v+T}\boldsymbol{i}(t)e^{-j\omega t}dt.\quad (11)$$

An ensemble of 3-form a 3-D complex space $C^{(3)}$ with a complex SP

$$(\boldsymbol{X},\boldsymbol{Z})=\boldsymbol{X}^{\tau}\boldsymbol{Z}^{*}=\dot{X}_a\dot{Z}_a^{*}+\dot{X}_b\dot{Z}_b^{*}+\dot{X}_c\dot{Z}_c^{*}.\quad (12)$$

Hereinafter $*$ is a sign of complex conjugation. Thus, for rms

$$\|\boldsymbol{x}\|^2=\boldsymbol{X}^{\tau}\boldsymbol{X}^{*}=\sum_m\dot{X}_m\dot{X}_m^{*}=\sum_m X_m^2=|\boldsymbol{X}|^2=X^2.$$

In particular,

$$\|\boldsymbol{u}\|=|\boldsymbol{U}|=U,\ \|\boldsymbol{i}\|=|\boldsymbol{I}|=I.\quad (13)$$

For a couple of sinusoidal processes $\boldsymbol{x}(t)$, $\boldsymbol{z}(t)\in L_2^{(3)}(T)$ the equality is correct

$$<\boldsymbol{x},\boldsymbol{z}>=\mathrm{Re}[\boldsymbol{X}^{\tau}\boldsymbol{Z}^{*}]=\mathrm{Re}[\boldsymbol{Z}^{\tau}\boldsymbol{X}^{*}].\quad (14)$$

So, if 3-complexes are orthogonal then corresponding 3-D curves are orthogonal, too. The converse is not true.

From (14) it follows that at the sinusoidal mode active power is adequately represented in terms of 3-complexes of voltage and current

$$P=<\boldsymbol{i},\boldsymbol{u}>=\mathrm{Re}[\boldsymbol{I}^{\tau}\boldsymbol{U}^{*}]=\mathrm{Re}[\boldsymbol{U}^{\tau}\boldsymbol{I}^{*}].\quad (15)$$

The temporal shift of 3-D curve of *i.v.* of sinusoidal voltage $\boldsymbol{u}_{\perp}(t)=\boldsymbol{u}(t-T/4)$ is equivalent to a rotation of the 3-complex of voltage in the space $C^{(3)}$ to 90°

$$\boldsymbol{u}_{\perp}(t)=\sqrt{2}\Re e[\boldsymbol{U}_{\perp}e^{j\omega t}]=\sqrt{2}\Re e[-j\boldsymbol{U}e^{j\omega t}].\ (16)$$

Here $\|\boldsymbol{u}_{\perp}\|=\|\boldsymbol{u}\|$. Because of

$$<\boldsymbol{u}_{\perp},\boldsymbol{u}>=\Re e[-j\boldsymbol{U}^{\tau}\boldsymbol{U}^{*}]=\Re e[-j|\boldsymbol{U}|^2]=0,$$

3-D curves of voltages are orthogonal ($\boldsymbol{u}\perp\boldsymbol{u}_{\perp}$).

Integral determination of reactive power (known as power of shift) is represented in terms of 3-complexes of voltage and current

$$Q=<i,\boldsymbol{u}_{\perp}>=\Re e[-j\boldsymbol{U}^{\tau}\boldsymbol{I}^{*}]=\mathrm{Jm}[\boldsymbol{U}^{\tau}\boldsymbol{I}^{*}].\quad (17)$$

Powers (15) and (17) are connected by complex power – SP of 3-complexes of voltage and current

$$\dot{S}=\boldsymbol{U}^{\tau}\boldsymbol{I}^{*}=\mathrm{Re}[\boldsymbol{U}^{\tau}\boldsymbol{I}^{*}]+j\,\mathrm{Jm}[\boldsymbol{U}^{\tau}\boldsymbol{I}^{*}]=P+jQ.\ (18)$$

At the sinusoidal mode at *symmetrical* load the equation of powers is correct

$$P^2+Q^2=\|\boldsymbol{i}\|\cdot\|\boldsymbol{u}\|\cdot\quad (19)$$

Equivalent conductivities of load current. At the sinusoidal mode 3-complexes of current and voltage permit to determine equivalent conductivities of current in the section <*A*, *B*, *C*>

$$\dot{Y}_m=G_m-jB_m=\frac{\dot{I}_m}{\dot{U}_m},\ m\in\{a,b,c\}\quad (20)$$

and represent a 3-complex of 3-phase current in the matrix form

$$\boldsymbol{I}=\begin{bmatrix}\dot{U}_a\dot{Y}_a\\\dot{U}_b\dot{Y}_b\\\dot{U}_c\dot{Y}_c\end{bmatrix}=\begin{bmatrix}\dot{Y}_a&0&0\\0&\dot{Y}_b&0\\0&0&\dot{Y}_a\end{bmatrix}\begin{bmatrix}\dot{U}_a\\\dot{U}_b\\\dot{U}_c\end{bmatrix}=\hat{Y}\boldsymbol{U}\quad (21)$$

by using a diagonal matrix

$$\hat{Y}=diag\{\dot{Y}_a,\dot{Y}_b,\dot{Y}_c\}.\quad (22)$$

For a 4-wire circuit with a star-type load equivalent conductivities (20) in the section <*A*, *B*, *C*> equal to conductivities of phase loads.

Active power and power of shift are adequately represented by quadratic forms of the 3-complex of voltage

$$P = \mathrm{Re}[\boldsymbol{U}^{\tau}\hat{Y}^{*}\boldsymbol{U}^{*}],\ Q = \mathrm{Jm}[\boldsymbol{U}^{\tau}\hat{Y}^{*}\boldsymbol{U}^{*}]. \quad (23)$$

Active power (power of shift) depends only on conductivities of active (reactive) load elements

$$P = \sum_{m} G_m |\dot{U}_m|^2,\ Q = \sum_{m} B_m |\dot{U}_m|^2. \quad (24)$$

Losses of total 3-phase current per one Ω

$$\|\boldsymbol{i}\|^2 = \mathrm{Re}[\boldsymbol{I}^{\tau}\boldsymbol{I}^{*}] = \sum_{m}(G_m^2 + B_m^2)|\dot{U}_m|^2. \quad (25)$$

Active and reactive current. For 3-D curve of sinusoidal current (9) it is correct

$$\boldsymbol{i}(t) = \sqrt{2}\Re e[\hat{Y}\boldsymbol{U}e^{j\omega t}],\ \boldsymbol{I} = \hat{Y}\boldsymbol{U}. \quad (26)$$

Algebraic form of complex equivalent conductivities (20) permits to resolute the diagonal matrix (22)

$$\hat{Y} = \hat{G} - j\hat{B}, \quad (27)$$

$$\hat{G} = diag\{G_a, G_b, G_c\},\ \hat{B} = diag\{B_a, B_b, B_c\} \quad (28)$$

and divide 3-complex of current into two components associated with active and reactive load elements

$$\boldsymbol{I} = \boldsymbol{I}_A + \boldsymbol{I}_R,\ \boldsymbol{I}_A = \hat{G}\boldsymbol{U},\ \boldsymbol{I}_R = -j\hat{B}\boldsymbol{U} = \hat{B}\boldsymbol{U}_{\perp}. \quad (29)$$

Resolution of 3-D curve of current (26)

$$\boldsymbol{i}(t) = \boldsymbol{i}_A(t) + \boldsymbol{i}_R(t) \quad (30)$$

into active and reactive current

$$\boldsymbol{i}_A(t) = \sqrt{2}\Re e[\hat{G}\boldsymbol{U}e^{j\omega t}],\ \boldsymbol{i}_R(t) = \sqrt{2}\Re e[\hat{B}\boldsymbol{U}_{\perp}e^{j\omega t}] \quad (31)$$

is orthogonal in the space of 3-D curves (4).

Because of the quantity

$$\boldsymbol{I}_A^{\tau}\boldsymbol{I}_R^{*} = (\hat{G}\boldsymbol{U})^{\tau}(-jB\boldsymbol{U})^{*} = j\sum_{m} U_m^2 G_m B_m \quad (32)$$

is pure imaginary then 3-D curves (31) are orthogonal

$$<\boldsymbol{i}_A, \boldsymbol{i}_R> = \mathrm{Re}[\boldsymbol{I}_A^{\tau}\boldsymbol{I}_R^{*}] = 0 \ \Rightarrow\ \boldsymbol{i}_A \perp \boldsymbol{i}_R. \quad (33)$$

Because of orthogonality of the resolution (29) for losses per one Ω the Pythagoras equality is correct

$$\|\boldsymbol{i}\|^2 = \|\boldsymbol{i}_A\|^2 + \|\boldsymbol{i}_R\|^2. \quad (34)$$

Losses of active and reactive current

$$\|\boldsymbol{i}_A,\|^2 = \mathrm{Re}[\boldsymbol{I}_A^{\tau}\boldsymbol{I}_A^{*}] = \sum_{m} G_m^2 |\dot{U}_m|^2, \quad (35)$$

$$\|\boldsymbol{i}_R,\|^2 = \mathrm{Re}[\boldsymbol{I}_R^{\tau}\boldsymbol{I}_R^{*}] = \sum_{m} B_m^2 |\dot{U}_m|^2 \quad (36)$$

determine losses of total current (25). Here

$$\|\boldsymbol{i}_A\|^2 \le \|\boldsymbol{i}\|^2,\ \|\boldsymbol{i}_R\|^2 \le \|\boldsymbol{i}\|^2.$$

Active current guarantees EE supply with active power of total current (24)

$$<\boldsymbol{u}, \boldsymbol{i}_A> = \mathrm{Re}[\boldsymbol{U}^{\tau}\boldsymbol{I}_A^{*}] = \mathrm{Re}[\boldsymbol{U}^{\tau}\hat{G}\boldsymbol{U}^{*}] =$$

$$= \sum_{m} G_m |\dot{U}_m|^2 = <\boldsymbol{i}, \boldsymbol{u}> = P. \quad (37)$$

Reactive current guarantees EE transmission of power of shift of total current (24)

$$<\boldsymbol{i}_R, \boldsymbol{u}_{\perp}> = \mathrm{Re}[-j\boldsymbol{U}^{\tau}\boldsymbol{I}_R^{*}] = \mathrm{Jm}[\boldsymbol{U}^{\tau}\hat{B}\boldsymbol{U}^{*}] =$$

$$= \sum_{m} B_m |\dot{U}_m|^2 = <\boldsymbol{i}, \boldsymbol{u}_{\perp}> = Q. \quad (38)$$

In the resolution (30) active (reactive) current is caused by *summarily* symmetry and asymmetry of active (reactive) load elements.

Balanced current component. A sinusoidal mode is *balanced* if 3-complexes of current and voltage (10) are collinear (parallel $\boldsymbol{I} \| \boldsymbol{U}$) [7, 8]

$$\boldsymbol{I} \| \boldsymbol{U} \ \Leftrightarrow\ \boldsymbol{I} = \beta\boldsymbol{U}\ (\dot{\beta} = \beta' + j\beta'',\ \beta \ne 0). \quad (39)$$

A mode is *really* balanced [7, 8] if $Jm[\beta] = \beta'' = 0$. If the load is symmetrical then the mode is balanced at any unbalanced voltage.

For an unbalanced mode the 3-complex of components of current balanced with 3-phase voltage equals to projection of 3-complex of voltage in the space $C^{(3)}$

$$\boldsymbol{I}_S = (\boldsymbol{I}^{\tau}\boldsymbol{\upsilon}^{*})\boldsymbol{\upsilon} = \frac{(\boldsymbol{I}^{\tau}\boldsymbol{U}^{*})\boldsymbol{U}}{|\boldsymbol{U}|^2}. \quad (40)$$

Hereinafter:

$$\boldsymbol{\upsilon} = [\dot{\upsilon}_a\ \ \dot{\upsilon}_b\ \ \dot{\upsilon}_c]^{\tau},\ |\boldsymbol{\upsilon}|^2 = \upsilon_a^2 + \upsilon_b^2 + \upsilon_c^2 = 1 \quad (41)$$

is the ort of the 3-complex of voltage

$$\boldsymbol{U} = |\boldsymbol{U}|\boldsymbol{\upsilon},\ \dot{U}_m = U\dot{\upsilon}_m\ (m \in \{a, b, c\}). \quad (42)$$

In terms of conductivities of the 3-complex of balance current (40)

$$\boldsymbol{I}_S = \frac{(\boldsymbol{I}^{\tau}\boldsymbol{U}^{*})\boldsymbol{U}}{|\boldsymbol{U}|^2} = \underbrace{(S^{*}/U^2)}_{\dot{\mathrm{y}}_{\mathrm{s}}} \cdot \boldsymbol{U} = \dot{\mathrm{y}}_{\mathrm{s}}\boldsymbol{U}. \quad (43)$$

Hereinafter:

$$\dot{\mathrm{y}}_{\mathrm{s}} = S^{*}/U^2 = \dot{Y}_a\upsilon_a^2 + \dot{Y}_b\upsilon_b^2 + \dot{Y}_c\upsilon_c^2 \quad (44)$$

is the equivalent complex conductivity of the balanced current component;

$$S^{*} = (\boldsymbol{U}^{\tau}\boldsymbol{I}^{*})^{*} = \boldsymbol{I}^{\tau}\boldsymbol{U}^{*} = \boldsymbol{U}^{\tau}\hat{Y}\boldsymbol{U}^{*}$$

is the complex conjugate power.

In terms of the ort of the 3-complex of voltage (42) the active and reactive power have equivalent form of representation:

$$P = \mathrm{Re}[\dot{S}] = \boldsymbol{U}^{\tau}\hat{G}\boldsymbol{U}^{*} = U^2\sum_{m} G_m\upsilon_m^2; \quad (45)$$

$$Q = \mathrm{Jm}[\dot{S}] = \boldsymbol{U}^{\tau}\hat{B}\boldsymbol{U}^{*} = U^2\sum_{m} B_m\upsilon_m^2. \quad (46)$$

Equivalent complex conductivity (44) of 3-D curve of balance current

$$\boldsymbol{i}_s(t) = \sqrt{2}\Re e[\dot{\mathrm{y}}_{\mathrm{s}}\boldsymbol{U}e^{j\omega t}],\ \boldsymbol{I}_s = \dot{\mathrm{y}}_{\mathrm{s}}\boldsymbol{U} \quad (47)$$

in all phases is the same and equal to weighted average sum of equivalent complex phase conductivities (20). Weighting factors are determined by the ort of the 3-complex of voltage (42).

If voltage is symmetric with direct sequence (DS) then

$$\boldsymbol{\upsilon} = (1/\sqrt{3})[1\ \ \alpha^{*}\ \ \alpha]^{\tau},\ \upsilon_a^2 = \upsilon_b^2 = \upsilon_c^2 = 1/3, \quad (48)$$

where $\alpha = e^{j120^0} = -1/2 + j\sqrt{3}/2$.

If the load is unbalanced

$$\dot{\mathrm{y}}_{\mathrm{s}} \ne \dot{Y}_a,\ \dot{\mathrm{y}}_{\mathrm{s}} \ne \dot{Y}_b,\ \dot{\mathrm{y}}_{\mathrm{s}} \ne \dot{Y}_b, \quad (49)$$

then the mode is unbalanced at any voltage

The complex conductivity of the balance current (47)

$$\dot{\mathrm{y}}_{\mathrm{s}} = \mathrm{g}_{\mathrm{s}} - j\mathrm{b}_{\mathrm{s}}, \quad (50)$$

determines conductivities associated with symmetry of active and reactive load elements

$$\mathrm{g}_{\mathrm{s}} = G_a\upsilon_a^2 + G_b\upsilon_b^2 + G_c\upsilon_c^2; \quad (51)$$

$$\mathrm{b}_{\mathrm{s}} = B_a\upsilon_a^2 + B_b\upsilon_b^2 + B_c\upsilon_c^2. \quad (52)$$

These conductivities equal to weighted average sums of phase conductivities. If the load is unbalanced then

$$\text{g}_\text{s} \neq G_a\,,\ \text{g}_\text{s} \neq G_b\,,\ \text{g}_\text{s} \neq G_c\,; \qquad (53)$$

$$\text{b}_\text{s} \neq B_a\,,\ \text{b}_\text{s} \neq B_b\,,\ \text{b}_\text{s} \neq B_c\,. \qquad (54)$$

Conductivities (51, 52) characterize the symmetry of active and reactive load elements by phases for the 3-phase voltage.

3-complex (43) of the balanced component has two components: active and reactive

$$\boldsymbol{I}_{sA} = \text{g}_s \boldsymbol{U}\,,\ \boldsymbol{I}_{sR} = -j\text{b}_\text{s}\boldsymbol{U} = \text{b}_\text{s}\boldsymbol{U}_\perp \qquad (56)$$

and guarantees resolution of the balanced current

$$\boldsymbol{i}_s(t) = \boldsymbol{i}_{sA}(t) + \boldsymbol{i}_{sR}(t) \qquad (57)$$

into components associated with active and reactive load elements:

$$\boldsymbol{i}_{sA}(t) = \sqrt{2}\Re e[\boldsymbol{I}_{sA}e^{j\omega t}] = \sqrt{2}\Re e[\text{g}_\text{s}\boldsymbol{U}e^{j\omega t}], \qquad (58)$$

$$\boldsymbol{i}_{sR}(t) = \sqrt{2}\Re e[\boldsymbol{I}_{sR}e^{j\omega t}] = \sqrt{2}\Re e[\text{b}_\text{s}\boldsymbol{U}_\perp e^{j\omega t}]. \qquad (59)$$

3-D curves (58) и (59) are orthogonal because

$$<\boldsymbol{i}_{sA}, \boldsymbol{i}_{sR}> = \text{Re}[j\sum\nolimits_m U_m^2 \text{g}_\text{s}\text{b}_\text{s}] = 0\,. \qquad (60)$$

Because of orthogonality of the resolution (57) for current component the Pythagoras equality is correct

$$\boldsymbol{i}_{sA}(t) \perp \boldsymbol{i}_{sR}(t) \quad \Rightarrow \quad I_{sA}^2 + I_{sR}^2 = I_s^2\,; \qquad (61)$$

$$I_{sA}^2 = \text{g}_\text{s}^2 U^2\,,\quad I_{sR}^2 = \text{b}_\text{s}^2 U^2\,.$$

From (58) it follows

$$<\boldsymbol{u}, \boldsymbol{i}_{sA}> = \text{Re}[\boldsymbol{U}^\tau \boldsymbol{I}_{sA}^*] = \text{Re}[\boldsymbol{U}^\tau \text{g}_\text{s}\boldsymbol{U}^*] = P\,. \qquad (62)$$

Because of balanced active current is *really* parallel with voltage ($\boldsymbol{I}_{sA} \parallel \boldsymbol{U} \ \Rightarrow \ \boldsymbol{I}_{sA} = \text{g}_\text{s}\boldsymbol{U}$) it guarantees EE supply of active power (62) with minimal losses [7]

$$\|\boldsymbol{i}_{sA}\| \leq \|\boldsymbol{i}_A\| \leq \|\boldsymbol{i}\|\,. \qquad (63)$$

Here $P = <\boldsymbol{u}, \boldsymbol{i}_{sA}> = \|\boldsymbol{u}\| \cdot \|\boldsymbol{i}_{sA}\|$.

Balanced reactive current guarantees EE supply of power of shift

$$<\boldsymbol{u}_\perp, \boldsymbol{i}_{sR}> = \text{Re}[-j\boldsymbol{U}^\tau \boldsymbol{I}_{sR}^*] = <\boldsymbol{u}_\perp, \boldsymbol{i}> = Q\,. \qquad (64)$$

Because of balanced reactive current is *really* parallel with voltage $\boldsymbol{I}_{sR} \parallel \boldsymbol{U}_\perp \ \Rightarrow$ it guarantees EE supply of reactive power of shift with minimal losses $\|\boldsymbol{i}_{sR}\| \leq \|\boldsymbol{i}_R\| \leq \|\boldsymbol{i}\|$. Here

$$|Q| = |<\boldsymbol{u}_\perp, \boldsymbol{i}_{sR}>| = \|\boldsymbol{u}\| \cdot \|\boldsymbol{i}_{sR}\|\,. \qquad (65)$$

Unbalanced current and asymmetry of load conductivities. At the unbalanced mode the unbalanced component of the 3-complex of current (unbalance current) is determined as an orthogonal complement to the balanced component (40)

$$\boldsymbol{I}_D = \boldsymbol{I} - \boldsymbol{I}_S\,,\ (\boldsymbol{I}_D \perp \boldsymbol{I}_S). \qquad (66)$$

Unbalanced component (66) can be represented by using vector product in the space of 3-complexes $C^{(3)}$ [7, 8].

From (21) and (47) it follows

$$\boldsymbol{I}_D = \boldsymbol{I} - \boldsymbol{I}_S = \hat{Y}\boldsymbol{U} - \dot{\text{y}}_\text{s}\boldsymbol{U} = \underbrace{(\hat{Y} - \dot{\text{y}}_\text{s})}_{\hat{\text{y}}_\text{D}}\boldsymbol{U} = \hat{\text{y}}_\text{D}\boldsymbol{U}\,. \qquad (67)$$

The matrix form of the 3-complex of unbalance current

$$\boldsymbol{I}_D = \hat{\text{y}}_\text{D}\boldsymbol{U} = \begin{bmatrix} \dot{\text{y}}_{\text{D}a} & 0 & 0 \\ 0 & \dot{\text{y}}_{\text{D}b} & 0 \\ 0 & 0 & \dot{\text{y}}_{\text{D}c} \end{bmatrix} \begin{bmatrix} \dot{U}_a \\ \dot{U}_b \\ \dot{U}_c \end{bmatrix}. \qquad (68)$$

Uses a complex diagonal matrix

$$\hat{\text{y}}_\text{D} = diag\{\dot{\text{y}}_{\text{D}a}, \dot{\text{y}}_{\text{D}b}, \dot{\text{y}}_{\text{D}c}\} \qquad (69)$$

of equivalent conductivities of the unbalance current

$$\dot{\text{y}}_{\text{D}m} = \dot{Y}_m - \dot{\text{y}}_\text{s}\,,\ m \in \{a,b,c\}\,. \qquad (70)$$

If the voltage is symmetric to DS then (48) and

$$\dot{\text{y}}_{\text{D}m} = (2\dot{Y}_m - \sum\nolimits_{k \neq m} \dot{Y}_k)\big/3\,,\ m \in \{a,b,c\}\,. \qquad (71)$$

Complex conductivities (70) characterize dissipation by phases of load conductivities regarding balance conductivity. Unbalance (asymmetry) determines the unbalanced current

$$\boldsymbol{i}_u(t) = \sqrt{2}\Re e[\boldsymbol{I}_D e^{j\omega t}]\,, \qquad (72)$$

which is orthogonal to voltage

$$<\boldsymbol{u}, \boldsymbol{i}_u> = \text{Re}[\boldsymbol{U}^\tau (\hat{\text{y}}_D\boldsymbol{U})^*] = \text{Re}[U^2 \sum\nolimits_m \upsilon_m^2 (\dot{Y}_m - \dot{\text{y}}_\text{s})] =$$

$$= \text{Re}[U^2 (\underbrace{\sum\nolimits_m \upsilon_m^2 \dot{Y}_m}_{\dot{\text{y}}_\text{s}}) - \dot{\text{y}}_\text{s})] = 0\,. \qquad (73)$$

Here from (67) the resolution follows

$$\boldsymbol{I} = \boldsymbol{I}_s + \boldsymbol{I}_D\ ,\ \boldsymbol{i}(t) = \boldsymbol{i}_s(t) + \boldsymbol{i}_u(t)\,. \qquad (74)$$

Dissipation (unbalance) by phases of separately active and reactive load elements

$$\text{g}_{\text{D}m} = G_m - \text{g}_\text{s}\ ,\ \text{b}_{\text{D}m} = B_m - \text{b}_\text{s}\,,\ m \in \{a,b,c\} \qquad (75)$$

is represented by diagonal matrices of conductivities of active and reactive load elements:

$$\hat{\text{g}}_\text{D} = diag\{\text{g}_{\text{D}a}, \text{g}_{\text{D}b}, \text{g}_{\text{D}c}\}\,,$$

$$\hat{\text{b}}_\text{D} = diag\{\text{b}_{\text{D}a}, \text{b}_{\text{D}b}, \text{b}_{\text{D}c}\}\,.$$

Unbalance by phases (asymmetry of phase conductivities) separately of active and reactive load elements determines resolution of 3-complex of unbalance current into two components

$$\boldsymbol{I}_D = \boldsymbol{I}_{DA} + \boldsymbol{I}_{DR}\,, \qquad (76)$$

$$\boldsymbol{I}_{DA} = \hat{\text{g}}_\text{D}\boldsymbol{U} = [\text{g}_{\text{D}a}\dot{U}_a \quad \text{g}_{\text{D}b}\dot{U}_b \quad \text{g}_{\text{D}c}\dot{U}_c]^\tau\,, \qquad (77)$$

$$\boldsymbol{I}_{DR} = -j\hat{\text{b}}_\text{D}\boldsymbol{U} = -j[\text{b}_{\text{D}a}U_a \quad \text{b}_{\text{D}b}U_b \quad \text{b}_{\text{D}c}U_c]^\tau\,. \qquad (78)$$

The resolution of the unbalanced current is correct

$$\boldsymbol{i}_u(t) = \boldsymbol{i}_{uA}(t) + \boldsymbol{i}_{uR}(t)\,, \qquad (79)$$

where $\boldsymbol{i}_{uA}(t) = \sqrt{2}\Re e[\boldsymbol{I}_{DA}e^{j\omega t}] = \sqrt{2}\Re e[\hat{\text{g}}_\text{D}\boldsymbol{U}e^{j\omega t}]$, (80)

$$\boldsymbol{i}_{uR}(t) = \sqrt{2}\Re e[\boldsymbol{I}_{DR}e^{j\omega t}] = \sqrt{2}\Re e[\hat{\text{b}}_\text{D}\boldsymbol{U}_\perp e^{j\omega t}] \qquad (81)$$

are the components determined by asymmetry of active and reactive load elements.

3-D curves (80) and (81) are orthogonal because

$$<\boldsymbol{i}_{uA}, \boldsymbol{i}_{uR}> = \text{Re}[\boldsymbol{I}_{DA}^\tau \boldsymbol{I}_{DR}^*] = \text{Re}[j\sum\nolimits_m U_m^2 \text{g}_{\text{D}m}\text{b}_{\text{D}m}] = 0\,.$$

Because of orthogonality of the resolution (79) the Pythagoras equality is correct

$$\boldsymbol{i}_{uA}(t) \perp \boldsymbol{i}_{uR}(t) \quad \Rightarrow \quad I_{DA}^2 + I_{DR}^2 = I_D^2\,, \qquad (82)$$

$$I_{DA}^2 = U^2(\upsilon_a^2 \text{g}_{\text{D}a}^2 + \upsilon_b^2 \text{g}_{\text{D}b}^2 + \upsilon_c^2 \text{g}_{\text{D}c}^2)\,, \qquad (83)$$

$$I_{DR}^2 = U^2(\upsilon_a^2 \text{b}_{\text{D}a}^2 + \upsilon_b^2 \text{b}_{\text{D}b}^2 + \upsilon_c^2 \text{b}_{\text{D}c}^2)\,. \qquad (84)$$

So, for the resolution of current two dichotomous factors are used:

- the first factor is determined by activity and reactivity of load elements;
- the second factor is determined by symmetry and asymmetry of load elements by phases.

Resolution of 3-phase current and power equation of unbalanced mode. Combination of values of two factors classifying the load:

- «activity/reactivity» – the first factor)

$$\boldsymbol{i} = \boldsymbol{i}_A + \boldsymbol{i}_R ;$$

- («symmetry/asymmetry» – the second factor)

$$\boldsymbol{i} = \boldsymbol{i}_s + \boldsymbol{i}_u \ ,$$

permitted to obtain four mutually orthogonal components of 3-phase current

$$\boldsymbol{i}_{sA}, \quad \boldsymbol{i}_{uA}, \quad \boldsymbol{i}_{sR}, \quad \boldsymbol{i}_{uR} ,$$

which guarantee resolution into four mutually orthogonal 3-phase current components

$$\boldsymbol{i} = \boldsymbol{i}_A + \boldsymbol{i}_R = \underbrace{(\boldsymbol{i}_{sA} + \boldsymbol{i}_{uA})}_{\boldsymbol{i}_A} + \underbrace{(\boldsymbol{i}_{sR} + \boldsymbol{i}_{uR})}_{\boldsymbol{i}_R} . \quad (85)$$

Because of current resolution (85) is orthogonal then identity (equation of losses per one Ω) is correct

$$\| \boldsymbol{i} \|^2 = \| \boldsymbol{i}_{sA} \|^2 + \| \boldsymbol{i}_{uA} \|^2 + \| \boldsymbol{i}_{sR} \|^2 + \| \boldsymbol{i}_{uR} \|^2 . \quad (86)$$

Multiplication of equation (86) by square of rms of voltage $\| \boldsymbol{u} \|^2$ gives the equation for powers of sinusoidal unbalanced mode

$$S_T^2 = P^2 + Q^2 + D_G^2 + D_B^2 . \quad (87)$$

Here:

$$S_T = \| \boldsymbol{i} \| \cdot \| \boldsymbol{u} \| \quad (88)$$

is the total power;

$$P = \| \boldsymbol{i}_{sA} \| \cdot \| \boldsymbol{u} \| = < \boldsymbol{i}, \boldsymbol{u} > \quad (89)$$

is the active balance power determined by symmetry of active load elements;

$$| Q | = \| \boldsymbol{i}_{sR} \| \cdot \| \boldsymbol{u}_\perp \| = | < \boldsymbol{i}, \boldsymbol{u}_\perp > | \quad (90)$$

is the reactive balance power determined by symmetry of active load elements;

$$D_G = \| \boldsymbol{i}_{uA} \| \cdot \| \boldsymbol{u} \| \quad (91)$$

is the unbalance power determined by asymmetry of active load elements;

$$D_G = \| \boldsymbol{i}_{uR} \| \cdot \| \boldsymbol{u} \| \quad (92)$$

is the unbalance power determined by asymmetry of reactive load elements;

Power equation (87) generalizes the equation for the sinusoidal unbalanced mode [7]

$$I^2 \cdot U^2 = P^2 + Q^2 + D_u^2 , \quad (93)$$

because of $D_u^2 = D_G^2 + D_B^2$.

Practical value of the received orthogonal decomposition of current and power equations is the ability to use them not only for separated measurement and recording of inactive components of TP but also to solve the compensation problem at sinusoidal unbalanced mode.

Conclusions. For a 3-phase 4-wire network with a sinusoidal unbalanced mode at asymmetric voltage a 4-component orthogonal decomposition of the 3-phase current is obtained. Components having a clear power sense independently classify the load condition. The resulting decomposition expands the CPC theory to 4-wire circuits with unbalanced voltage imbalance by resolution of the unbalance current into two components determined by the asymmetry of the active and reactive load elements.

REFERENCES

1. *1459-2010 IEEE Standard definitions for the measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions*. **doi: 10.1109/ieeestd.2010.5439063**.

2. Czarnecki L.S. Orthogonal decomposition of the currents in a 3-phase nonlinear asymmetrical circuit with a nonsinusoidal voltage source. *IEEE Transactions on Instrumentation and Measurement*, 1988, vol.37, no.1, pp. 30-34. **doi: 10.1109/19.2658**.

3. Ferrero A., Superti-Furga G. A new approach to the definition of power components in three-phase systems under nonsinusoidal conditions. *IEEE Transactions on Instrumentation and Measurement*, 1991, vol.40, no.3, pp. 568-577. **doi: 10.1109/19.87021**.

4. Lev-Ari H., Stankovic A.M. A decomposition of apparent power in polyphase unbalanced networks in nonsinusoidal operation. *IEEE Transactions on Power Systems*, 2006, vol.21, no.1, pp. 438-440. **doi: 10.1109/tpwrs.2005.860903**.

5. Czarnecki L.S., Haley P.M. Unbalanced power in four-wire systems and its reactive compensation. *IEEE Transactions on Power Delivery*, 2015, vol.30, no.1, pp. 53-63. **doi: 10.1109/tpwrd.2014.2314599**.

6. Czarnecki L.S., Bhattarai P.D. Currents' physical components (CPC) in three-phase systems with asymmetrical voltage. *Przegląd Elektrotechniczny*, 2015, no.6, pp. 40-47. **doi: 10.15199/48.2015.06.06**.

7. Sirotin Iu.A. Vectorial instantaneous power and energy modes in three-phase circuits. *Tekhnichna elektrodynamika – Technical electrodynamics*, 2013, no.6, pp. 57-65. (Rus).

8. Sirotin Iu.A. Non-pulsed mode of supply in a three-phase system at asymmetrical voltage. *Przeglad Elektrotechniczny*, 2013, no.7, pp. 54-58.

Received 29.02.2016

Iu.A. Sirotin, Doctor of Technical Science, Professor,
National Technical University «Kharkiv Polytechnic Institute»,
21, Frunze Str., Kharkiv, 61002, Ukraine,
e-mail: yuri_sirotin@ukr.net

How to cite this article:
Sirotin Iu.A. Orthogonal components of the three-phase current at asymmetrical active-reactive load in 4-wire circuit. *Electrical engineering & electromechanics*, 2016, no.3, pp. 62-66. doi: 10.20998/2074-272X.2016.3.10.