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MATHEMATICAL MODEL OF ELECTROMAGNETIC PROCESSES IN LEHERA LINE AT OPEN-CIRCUIT OPERATION

Purpose. The work proposed for the modeling of transients in Lehera line uses a modified Hamilton-Ostrogradskiy principle. The above approach makes it possible to avoid the decomposition of a single dynamic system that allows you to take into account some subtle hidden movements. This is true for systems with distributed parameters, which in the current work we are considering. Methodology. Based on our developed new interdisciplinary method of mathematical modeling of dynamic systems, based on the principle of modified Hamilton-Ostrogradskiy and expansion of the latter on the non-conservative dissipative systems, build mathematical model Lehera line. The model allows to analyze transient electromagnetic processes in power lines. Results. In this work the model used for the study of transients in the non-working condition Lehera line. Analyzing the results shows that our proposed approach and developed based on a mathematical model is appropriate, certifying physical principles regarding electro-dynamics of wave processes in long power lines. Presented in 3D format, time-space distribution function of current and voltage that gives the most information about wave processes in Lehera line at non-working condition go. Originality. The originality of the paper is that the method of finding the boundary conditions of the third kind (Poincare conditions) taking into account all differential equations of electric power system, i.e. to find the boundary conditions at the end of the line involves all object equation. This approach enables the analysis of any electric systems. Practical value. Practical application is that the wave processes in lines affect the various kinds of electrical devices, proper investigation of wave processes is the theme of the present work.
References 12, figures 12.
Key words: **mathematical modeling, Hamilton-Ostrogradskiy principle, Euler-Lagrange equation, electric power system, power line with distributed parameters.**

В работе, на основе обобщенного междисциплинарного (интердисциплинарного) метода математического моделирования, основанного на модификации интегрального вариационного принципа Гамильтона-Остроградского, предложена математическая модель двухпроводной длинной линии электропередач, которая работает на холостом ходу. Представлены результаты компьютерной симуляции переходных процессов в виде рисунков, которые анализируются.
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Ключевые слова: **математическое моделирование, принцип Гамильтона-Остроградского, уравнение Эйлера-Лагранжа, электроэнергетическая система, линия электропередач с распределенными параметрами.**

Introduction. Mathematical modeling of complex electrical systems today is an important technical problem. With mathematical simulation device can analyze electromagnetic and electromechanical transient processes in electrical facilities and systems, not using for the latest expensive full-scale experiments. No exception here and electricity.

In the current work as an example Lehera system analysis [1] uses a long line with distributed parameters that runs on direct current. We know that these lines have found their proper place in power systems around the world. Transmission of electricity in this way: it reduces losses in the lines (due absence of skin effect phenomenon) makes possible association between local power systems that operate with varying frequency and reduces the cost of construction for large distances due to fewer wires and auxiliary fittings, etc.

Unfortunately, in our country for some reason, however, and economic, in 2014 were brought down the only DC line Volgograd-Donbass, which has been designed for a voltage of 800 kV. But in highly developed foreign countries mentioned lines are not only effective, but commissioned new due to certain advantages mentioned above. Here we can mention the following lines: Line Pacific DC power 1400 MW, ± 400 kV voltage, length of 1362 km for the transmission of electricity from hydropower plants in Oregon grid in Los Angeles; power transmission line HPP «Xiangjiaba» – China's Shanghai ± 800 kV voltage guarantees transmitting 6400 MW over a distance of 2000 km; Canada three transmission line length of about 900 km, built by HPP Nelson River, located in the Arctic Circle, to the city Winnipeg in the South of country. Epps was the third power of 2000 MW at a voltage of ± 500 kV; Brazil put into operation two chains of the line of HPP Itaipu of throughput of 3,150 MW at a voltage of ± 600 kV. The length of each circuit of about 800 km, and others [2].

Analysis of last investigations. Among a number of scientific papers devoted to the analysis of transient processes in power systems look at some of them, next to the theme of this work.

In [3] developed a mathematical model of two- and three wire power line AC to study transient processes and phenomena of overvoltage in a 500 kV line. Based on the software code ATP-EMTP transients were calculated and investigated the phenomenon over during the emergency state line.

A practical approach in the study of transient electromagnetic processes in [4] is represented. After describing the many cases of simulation modeling for selected items grid requirements are presented. Also, a comparative analysis of studies of transient electromagnetic processes in the correct and incorrect model grid is made.

In [5] the mathematical model of electromagnetic transients in electrical systems that is based on discrete nodal equations in phase coordinates and implicit numerical integration methods, which enables modeling transients with symmetric and asymmetric switching and injuries in electrical networks of any configuration.

The work [6] covers a wide range of analysis and derivatives re-established processes in electricity under the original angle. Materials book is based on the classical approach to the modeling of electric-energy-systems. Unfortunately, apparently because of the limitation of the volume of the book, the latter did not present the results of computer simulation of wave processes in lines.

The goal improvement on the base of utilization of variation approaches, methods of mathematical modeling of transients in Lehera line which operates on a non-working pace, and due to this more correctly simulation of wave processes.

Variation model of the Lehera line. To build mathematical models of objects under consideration with a high level of adequacy to properly use basic fundamental laws applied physics, applied in the relevant fields of science [9]. In our case this is electrodynamics [1, 6, 7].

Mathematical modeling is usually using two approaches. The first – a classic approach based on the law of conservation of energy and the second – variation based on minimizing functional of the system [9]. Each of these approaches has its advantages and drawbacks, but when used properly leads to reliable results [8]. In other words, the roads leading to the final model is differ but obtained the result – the same. Usually, choosing the right approach to modeling is a proper of researcher.

We offer analysis of transients in line Lehera use modified Hamilton-Ostrogradskiy principle (variation technique) [9]. The above approach allows you to avoid decomposition of single dynamic system, and to obtain initial state equation energetic exclusively on a single approach, enhanced by constructing Lagrange function [9]. In other words, the proposed way allows you to build dynamic systems models based on interdisciplinary approaches. This is especially true for systems with distributed parameters, and in that long transmission lines, as in the equations of the facility it is necessary to consider: electrostatic effects (arc phenomenon), thermodynamic effects (conductors heating, especially during melting ice) mechanical impacts on wires, in particular, various oscillations (especially resonant and close to resonance (beat fluctuations) processes) and others. In the current work we do not consider the above mentioned effects, but these effects, we plan to further consider our investigations actually for that we offer this approach.

A key element of the principle of modified Hamilton-Ostrogradskiy is extended non-conservative Lagrangian. We present its analytical form [8, 10]:

$$L^* = \tilde{T}^* - P^* + \Phi^* - D^*, \quad (1)$$

where L^* is the modified Lagrange function, $\tilde{T}^*$ is the kinetic co-energy, P^* is the potential energy, Φ^* is the dissipation energy, D^* is the energy of external on-potential forces.

We have already mentioned that the line Lehera generally seen as a system with distributed parameters [10, 11]. Then the elements of the modified Lagrange function will not feature power, and their respective densities [1]. So, functional of action by Hamilton-Ostrogradskiy will have a form [9]:

$$S = \int_{t_1}^{t_2}\left(L^* + \int_l L_l dl\right)dt, \; I = \int_l L_l dl, \text{ here } L^* = 0, \quad (2)$$

where S is the action by Hamilton-Ostrogradskiy, L_l is the linear density of the modified Lagrange function, I is the energetic functional.

We write components of the expanded Lagrange function (mean linear density) [9]:

$$\frac{\partial T^*}{\partial x} \equiv T_l = \frac{L_0 Q_t^2}{2}, \frac{\partial P^*}{\partial x} \equiv P_l = \frac{1}{2C_0}Q_x^2, Q_t \equiv \frac{\partial Q}{\partial t} = i; \quad (3)$$

$$\frac{\partial \Phi^*}{\partial x} \equiv \Phi_l = \Phi_{l3} - \Phi_{lB} = \int_0^t \left(\frac{R_0}{2}Q_t^2 - \frac{g_0}{2C_0^2}Q_x^2\right)_{|t=\tau} d\tau, \quad (4)$$

where $i(x, t)$ is the current in the line, R_0, g_0, C_0, L_0 are the line parameters, Φ_{l3} is the external energy dissipation, Φ_{lB} is the internal energy dissipation, $Q(x,t)$ is the charge of the line.

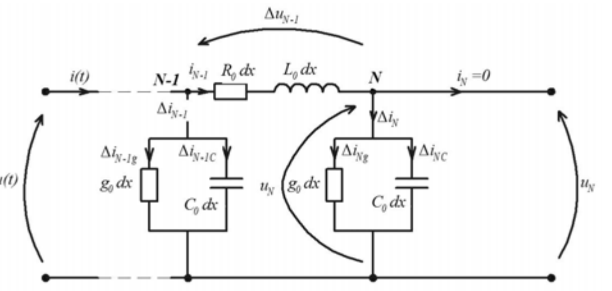

Fig. 1. Electric circuit of the Lehera line at open-circuit operation

It is important to note that in equation (4) minus sign appears! This is due to the fact that the function external dissipation depends on the leakage current that flow between the line wires. Obviously, the electric transmission line during the transfer of energy from the source to the consumer consumes the energy dissipated in in space. In other words, energy is transferred exclusively via the electromagnetic field lines and wires only indicate the direction of electromagnetic wave propagation [1].

Taking into account the equations (3), (4) the energy functional will look like [9]:

$$I = \int_l \left\{\frac{L_0}{2}Q_t^2 - \frac{1}{2C_0}Q_x^2 + \int_0^t \left(\frac{R_0}{2}Q_t^2 - \frac{g_0}{2C_0^2}Q_x^2\right)_{|t=\tau} d\tau\right\} dl. \quad (5)$$

We write the variation of the energy functional (5) and equate it to zero

$$\int_l \left\{\left(L_0 Q_t + R_0 \int_0^t Q_{t|t=\tau} d\tau\right)\delta Q_t - \left(\frac{1}{C_0}Q_x + \frac{g_0}{C_0^2}\int_0^t Q_{x|t=\tau} d\tau\right)\delta Q_x\right\} dl = 0. \quad (6)$$

Next, for each element of integrand expressions we use the rule of integration by parts, also known as Gauss-Ostrogradskiy theorem. Then, for the first bracket will be [9]:

$$-\int_l \frac{\partial}{\partial t}\left(L_0 Q_t + R_0 \int_0^t Q_{t|t=\tau} d\tau \right) \delta Q dl + \Omega_t \qquad (7)$$

and for the second one:

$$-\int_l \frac{\partial}{\partial x}\left(\frac{1}{C_0} Q_x + \frac{g_0}{C_0^2} \int_0^t Q_{x|t=\tau} d\tau \right) \delta Q dl + \Omega_x , \quad (8)$$

where Ω_t, Ω_x are the boundary conditions for the functional (5).

From here we can write

$$\delta I = \int_l \left\{ \frac{1}{C_0}\frac{\partial^2 Q}{\partial x^2} + \frac{g_0}{C_0^2}\int_0^t \frac{\partial^2 Q}{\partial x^2}_{|t=\tau} d\tau - L_0 \frac{\partial^2 Q}{\partial t^2} - \right.$$
$$\left. - R_0 \frac{\partial}{\partial t}\int_0^t \frac{\partial Q}{\partial t}_{|t=\tau} d\tau \right\} \delta Q dl + \Omega = 0, \; \Omega = \Omega_t + \Omega_x. \qquad (9)$$

It is easy to see that variation of energy functional can be zero only when equality to zero of integrand or variations function of the charge of the line. As δQ never can be equal to zero [9, 12] the energy functional (9) obtains a stationary value only in the case when integrand equals zero, i.e. at presence of the Euler-Poisson equation [9, 12]

$$\frac{1}{C_0}\frac{\partial^2 Q}{\partial x^2} + \frac{g_0}{C_0^2}\int_0^t \frac{\partial^2 Q}{\partial x^2} d\tau - L_0 \frac{\partial^2 Q}{\partial t^2} - R_0 \frac{\partial Q}{\partial t} = 0 . \quad (10)$$

We write for the equation (10) an expression of steady-state connections [1, 10]

$$-\frac{1}{C_0}\frac{\partial^2 Q}{\partial x^2} = L_0 \frac{\partial^2 Q}{\partial t^2} + R_0 \frac{\partial Q}{\partial t} . \qquad (11)$$

Taking into account the expression [1]

$$\int_0^t \frac{\partial^2 Q}{\partial t^2}_{|t=\tau} d\tau = \frac{\partial Q}{\partial t}, \; \int_0^t \frac{\partial Q}{\partial t}_{|t=\tau} d\tau = Q \qquad (12)$$

we obtain finally the commonly known telegraph equation [1, 10]

$$\frac{\partial^2 Q}{\partial x^2} = L_0 C_0 \frac{\partial^2 Q}{\partial t^2} + \left(R_0 C_0 + g_0 L_0 \right) \frac{\partial Q}{\partial t} + g_0 R_0 Q . \quad (13)$$

The equation of the Lehera line (telegraph equation) is written for the function of the charge of the line. However, it is easily transformed to the common telegraph equation

$$\frac{\partial^2 \lambda}{\partial x^2} = L_0 C_0 \frac{\partial^2 \lambda}{\partial t^2} + \left(R_0 C_0 + g_0 L_0 \right) \frac{\partial \lambda}{\partial t} + g_0 R_0 \lambda, \; \lambda = (Q,u,i) . \; (14)$$

Experience shows that for more optimal description of physical processes in the line it is useful to use as a general function a function of voltage, i.e. $\lambda = u(x,t)$ [1, 10].

We rewrite (14) in such a way:

$$\frac{\partial v}{\partial t} = \left(C_0 L_0 \right)^{-1} \left(\frac{\partial^2 u}{\partial x^2} - \left(g_0 L_0 + C_0 R_0 \right) v - g_0 R_0 u \right), \frac{\partial u}{\partial t} = v . \; (15)$$

The most important problem solving equations (15) is to determine the initial ($v(x,t)|_{t=to}$) and boundary ($u(x,t)|_{x=0}$ and $u(x,t)|_{x=l}$) conditions. As to the first, then the problem is solved in the accustomed way (they calculate their from previous research or take zero). The main problem is to find the boundary conditions. In general, the voltage at the beginning $u(x,t)|_{x=0}$ and at the end $u(x,t)|_{x=l}$. Of the line are unknown. In the particular case (on the current work) voltage is known at the beginning of the line, while at the end of the line – no. Actually finding this voltage we loan.

We write equations (11) in such a way (taking into account $Q_x(x,t) = C_0 u(x,t)$):

$$-\frac{\partial u(x,t)}{\partial x} = R_0 i(x,t) + L_0 \frac{\partial i(x,t)}{\partial t} . \qquad (16)$$

Further, for the power transmission line we write equations (15), (16) in the discrete space (we discretize them by the line method)

$$\frac{dv_j}{dt} = \left(C_0 L_0 \right)^{-1} \left(\frac{u_{j-1} - 2u_j + u_{j+1}}{(\Delta x)^2} - \left(g_0 L_0 + \right.\right.$$
$$\left.\left. + C_0 R_0 \right) v_j - g_0 R_0 u_j \right), \; u_1 = u(x,t)\big|_{x=0}, \; u_N = u(x,t)\big|_{x=l} \qquad (17)$$

$$-\frac{u_{j+1} - u_{j-1}}{2\Delta x} = R_0 i_j + L_0 \frac{di_j}{dt} ; \qquad (18)$$

$$\frac{du_j}{dt} = v_j , \quad j = 2,..., N-1 . \qquad (19)$$

We rewrite equations (17), (18) for the N-th node of discretization in the correspondence with Fig. 1 in such a form:

$$\frac{dv_N}{dt} = \frac{1}{C_0 L_0} \left[\frac{1}{(\Delta x)^2} \left(u_{N-1} - 2u_N + u_{N+1} \right) - \right.$$
$$\left. - \left(g_0 L_0 + C_0 R_0 \right) v_N - g_0 R_0 u_N \right] ; \quad (20)$$

$$-\frac{u_{N+1} - u_{N-1}}{2\Delta x} = 0 , \qquad (21)$$

where u_{N+1} is the discretization node voltage function at the fictitious layer [10] which will be found from the equation (21).

Then,

$$u_{N+1} = u_{N-1} . \qquad (22)$$

Taking into account (20) and (22) we write the final equation of the long line for the N-th node

$$\frac{dv_N}{dt} = \frac{2}{C_0 L_0 (\Delta x)^2} u_{N-1} - \left(\frac{2}{C_0 L_0 (\Delta x)^2} + \frac{g_0 R_0}{C_0 L_0} \right) u_N -$$
$$- \frac{g_0 L_0 + C_0 R_0}{C_0 L_0} v_N , \qquad (23)$$

$$\frac{du_N}{dt} = v_N . \qquad (24)$$

An important functional dependence that is interesting for the potential users is current value in elements of the Lehera line. Its calculation is possible in such a way. Discretizing equation (16) by the line method (right derivative) we have:

$$-\frac{u_{j+1} - u_j}{\Delta x} = R_0 i_j + L_0 \frac{di_i}{dt} . \qquad (25)$$

From here we obtain finally

$$\frac{di_j}{dt} = \frac{1}{L_0 \Delta x}\left(u_j - u_{j+1} \right) - \frac{R_0}{L_0} i_j , \; j = 1,..., N-1 . \; (26)$$

Compatible integration is subject to this system of differential equations: (17), (19), (23), (24), (26).

Computer simulation results. Computer simulation is carried out for the Lehera line at DC at open-circuit operation. The line has the following parameters: $R_0 = 0.86 \cdot 10^{-1}$ Ω/km, $L_0 = 0.134 \cdot 10^{-12}$ H/km, $C_0 = 0.85 \cdot 10^{-8}$ F/km, $g_0 = 0.375 \cdot 10^{-7}$ S/km, length of the line l = 600 km. The line is supplied by the DC voltage $u(x,t)|_{x=0}$ = 400 kV.

In Fig. 2-4 spatial distribution of the electromagnetic wave as functional dependences of currents (1) and voltages (2) is presented. From these Figures we can see physical basics of electromagnetic processes in the long Lehera line. Let us analyze these processes.

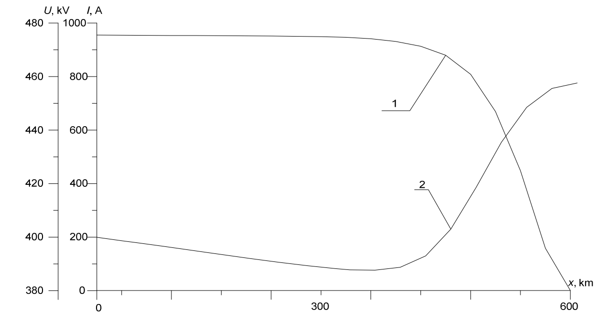

Fig. 2. Distributions of current (1) and voltage (2) in the line at t = 0.002 s

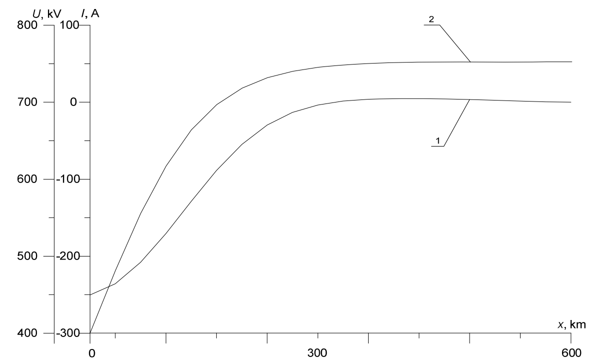

Fig. 3. Distributions of current (1) and voltage (2) in the line at t = 0.004 s

Fig. 2 shows the spatial distribution of functions of current and at time 0.002 s. Analyzing the mentioned Figure it is easy to see that the function of voltage begins to decrease, and the central line rises sharply upward. A stream function in the same place on the contrary – falls.

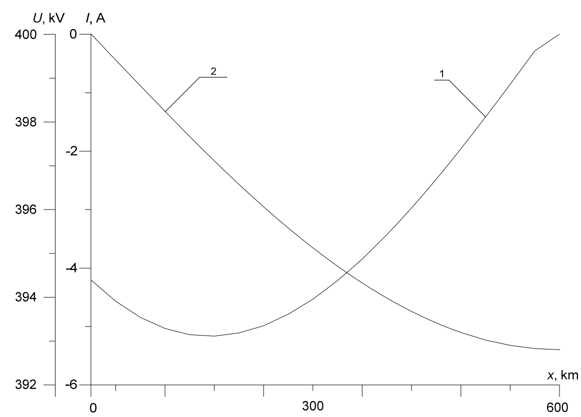

Fig. 4. Distributions of current (1) and voltage (2) in the line at t = 0.1 s

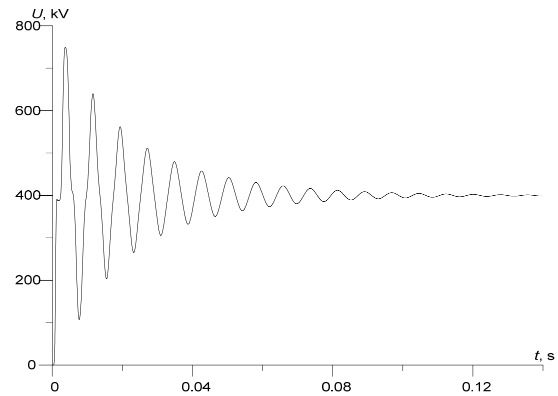

Fig. 5. Representation of the voltage transient function in the central point of the line

Recall that although the line is at open-circuit operation, leakage currents and currents in cell lines will be present. Actually the reason – is capacitive currents between the wires line. Obviously, at the end of the transmission line current will equal zero because the line is unloaded.

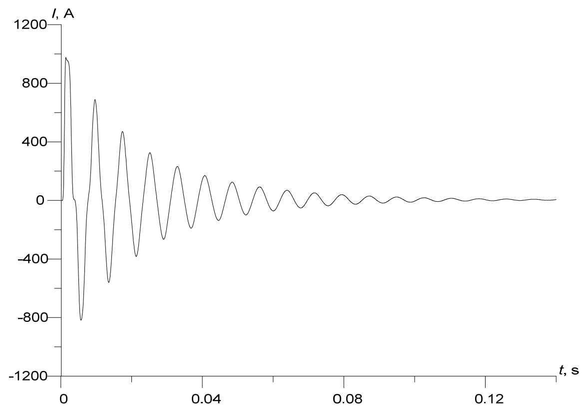

Fig. 6. Representation of the current transient function in the central point of the line

Fig. 3 shows the same, but in time 0.004 s. If at time 0.002 s (see Fig. 2) function of voltage increased to 460 kV but at time 0.004 s this growth was 750 kV. Voltage has almost doubled. As for the current, they fell by almost four times.

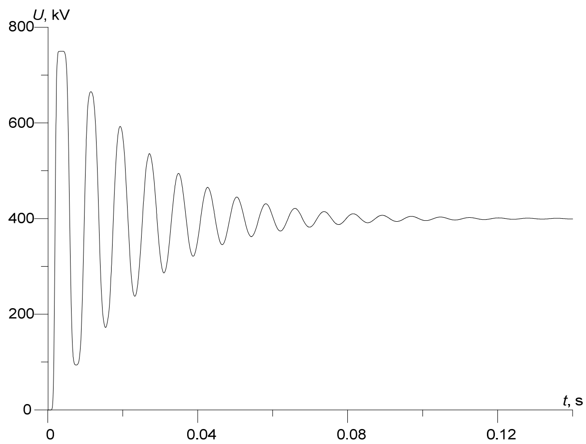

Fig. 7. Representation of the voltage transient function at the end of the line

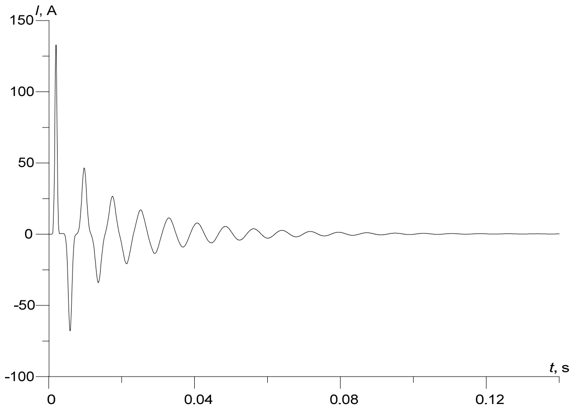

Fig. 8. Representation of the current transient function at the end of the line

Fig. 4 shows again the same as in Fig. 2 and 3 at the time when the transition process is almost completed. Of the Figure shows that deviation of functions of voltage and current almost took a minimum value. In other words, the amplitude of the electromagnetic wave due dissipative process significantly decreased. Oscillation process practically attenuates.

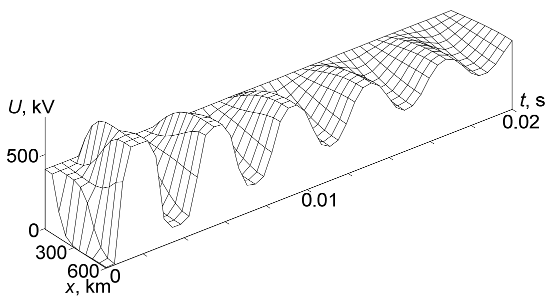

Fig. 9. Temporal-spatial distribution of the voltage function at $t \in [0; 0.02]$ s

Fig. 5-8 show transient functional dependences of voltage and current (temporal distribution). The first two Figures concern central node of the line for voltage and the central segment of the line for line. The second two Figures – the penultimate node of the line and penultimate discrete circuit of the line.

Through a comparative analysis of the above Figures it is easy to see that the function of voltage (see Fig. 5 and 7) changes little. See quite a different picture (see Fig. 6 and 8) regarding currents. The current changes almost 8 times. This is because the line power transmission line is (open-circuit operation).

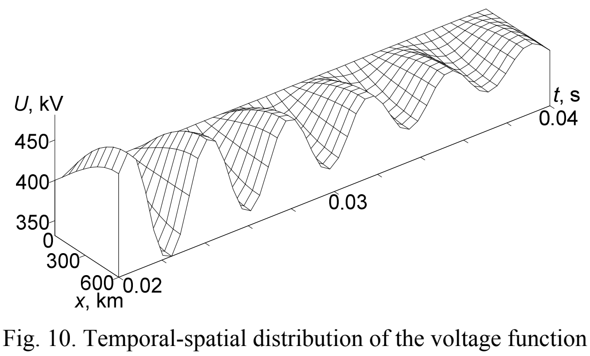

Fig. 10. Temporal-spatial distribution of the voltage function at $t \in [0.02; 0.04]$ s

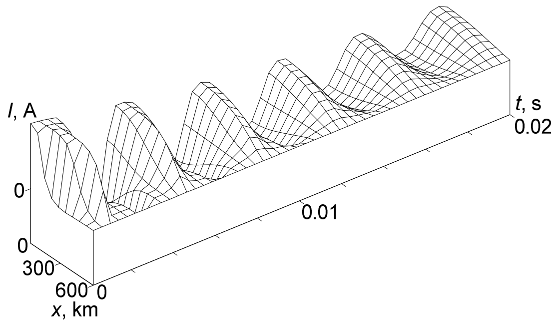

Fig. 11. Temporal-spatial distribution of the current function at $t \in [0; 0.02]$ s

Fig. 9 and 10 represent the line voltage as a function of time and spatial coordinates. These figures are presented in 3D format. Notably relatively high information content of these figures, which is that the spatial and temporal distribution of creating three-dimensional space. It is advisable to analyze the Figures mentioned in comparison with Fig. 2-5 and 7.

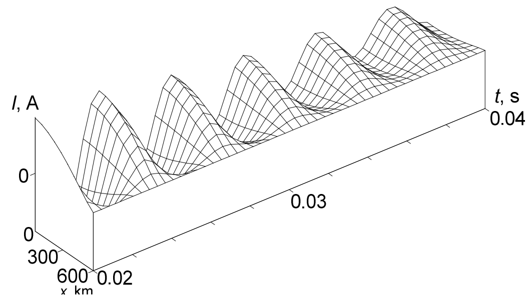

Fig. 12. Temporal-spatial distribution of the current function at $t \in [0.02; 0.04]$ s

Fig. 11 and 12 show the same as the previous two, but the function of current. As we see the function of voltage and current are in opposite phase. Because the nature of the stress associated with the electric field and the magnetic current we can make a conclusion of space perpendicular fields *E* and *B* which maintains the classic electrodynamics [1]. The presented figures it is advisable to analyze in comparison with Fig. 2-4, 6, 8.

Conclusions.

1. Variation approaches to modeling processes in long power lines make it possible to avoid the decomposition of a unified system, while the final form of the equation of state exclusively from single energy approach by building Lagrange expanded function.
2. An important point in the solution of differential equations of state of long line is a search of boundary conditions that often is veiled, incorrectly set, and the use of boundary conditions Neumann and Poincare boundary conditions. Finding these conditions entails full engagement of system of differential equations of studied object including transformers, reactors, compensation devices, etc., which greatly complicates the calculation of transients in a long line.
3. Experience shows that during the analysis of local power systems as the best option long line the telegraph equation it is advisable to write in a function of voltage. In the case of modeling of local energy systems where they use electromagnetic model elements of these systems

(Ψ – type and A – type) have difficulty with utilization of known method of nodal voltages which makes impossible to determine voltage at the beginning and end of the line and therefore it is impossible to correctly solve the equation. All this calls into question the degree of adequacy o eventual results that are obtained by known engineering program Mathematica, MatLab, etc., a especially the use of these programs becomes impossible when considering the circuit-field model elements. In this case, each actual task we must use appropriate apparatus of mathematical modeling.

4. Based on the results of computer simulation we can make a number of conclusions:

- voltage function has the greatest amplitude of oscillations at the end of the line when the current function – at the beginning of the line;
- spatial distribution of functional dependence of the line of sending (Fig. 2, 3 and 4) confirms the physical principles regarding electrodynamics of wave processes in long lines of power supply;
- presented in 3D format temporal-spatial distribution of functions of current and voltage provides most information on wave processes in Lehera line at open-circuit operation.

Materials of this work will be used in further studies that will cover long three-phase power lines with various kinds and types of loads.

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