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NUMERICAL ANALYSIS OF MATHEMATICAL MODELS OF THE FACTUAL CONTRIBUTION DISTRIBUTION IN ASYMMETRY AND DEVIATION OF VOLTAGE AT THE COMMON COUPLING POINTS OF ENERGY SUPPLY SYSTEMS

Purpose. Perform numerical analysis of the distribution of the factual contributions of line sources of distortion in the voltage distortion at the point of common coupling, based on the principles of superposition and exclusions. Methodology. Numerical analysis was performed on the results of the simulation steady state operation of power supply system of seven electricity consumers. Results. Mathematical model for determining the factual contribution of line sources of distortion in the voltage distortion at the point of common coupling, based on the principles of superposition and exclusions, are equivalent. To assess the degree of participation of each source of distortion in the voltage distortion at the point of common coupling and distribution of financial compensation to the injured party by all sources of distortion developed a one-dimensional criteria based on the scalar product of vectors. Not accounting group sources of distortion, which belong to the subject of the energy market, to determine their total factual contribution as the residual of the factual contribution between all sources of distortion. Originality. Simulation mode power supply system was carried out in the phase components space, taking into account the distributed characteristics of distortion sources. Practical value. The results of research can be used to develop methods and tools for distributed measurement and analytical systems assessment of the power quality. References 8, tables 6, figures 3.
Key words: **power quality, factual contribution, point of common coupling, voltage asymmetry, voltage deviation.**

На основе имитационного моделирования проведен сравнительный анализ математических моделей распределения фактических вкладов линейных источников искажений в искажение напряжений в точке общего присоединения, которые основаны на принципах наложения и исключения. Полученные результаты позволили сделать вывод об эквивалентности двух математических моделей и их произвольном выборе для решения задачи распределения фактических вкладов линейных источников искажений в искажение напряжений в точке общего присоединения. Библ. 8, табл. 6, рис. 3.
Ключевые слова: **качество электрической энергии, фактический вклад, точка общего присоединения, несимметрия напряжений, отклонение напряжения.**

Introduction. Non-compliance of power quality (PQ) to established standards are the reasons the marriage of products, equipment damage, and additional power losses both in consumers and electrical energy (EE) suppliers [1]. According to some estimates [2] annual economical losses in several countries due to the low PQ arise USD 10-20 bln. For certain sectors of production decrease in PQ can cause damage to 3.800.000 EUR per event [3]. Obviously, if this happens, it becomes a question of determining those responsible for lowering the PQ and compensation of economic damages to the injured party. The answer to it is to solve the problem of the distribution of factual contributions (FC) of sources of distortion (SD) in the distortion of the voltage at the point of common coupling (PCC) [4].

Problem definition. One of the new directions of development of the FC SD distribution methods in voltage distortion at PCC involves the use of mathematical models, drawn up in phase coordinates, given the distributed nature of SD in the power supply system (PSS), which are based on the principles of superposition [5] and exclusion [6].

A mathematical model of the distribution of FC of linear SD (undistorting the sinusoidal voltage waveform) in the voltage distortion is based on the principle of superposition, involves the expansion of distorting parts of voltages in each PCC from the activities of all SD according to the following expression:

$$\sum_{i=1}^{n} \boldsymbol{U}_{dis\,i} = \boldsymbol{A}^T \times \boldsymbol{Y}_{undis}^{-1} \times \sum_{i=1}^{n} \boldsymbol{I}_{dis\,i}\ , \qquad (1)$$

where $\boldsymbol{A}$ is the incidence matrix; $\boldsymbol{Y}_{undis}$ is the matrix of undistorted nodal conductivities of the PSS and EE consumers; $\boldsymbol{I}_{dis\,i}$ is column matrix of distorted currents characterizing the *i*-th active or passive element with SD.

A mathematical model of the distribution of FC of linear SD in the voltage distortion is based on the principle of exclusion, involves determining the distorting of the voltage in each PCC, introduced by the *i*-th SD, by the following expression:

$$\boldsymbol{U}_{dis\,i} = \boldsymbol{U}_{dis} - \boldsymbol{U}_{dis}^{ex\,SD\,i}\ , \qquad (2)$$

где $\boldsymbol{U}_{dis}$ the matrix of distorted parts of voltages in the PCC from common action of all SD; $\boldsymbol{U}_{dis}^{ex\,SD\,i}$ is the matrix of distorted parts of voltages in the PCC with excluded distorted part of the *i*-th SD.

To check the adequacy and the comparison of the proposed new mathematical models of the distribution of FC of linear SD in voltage distortion at PCC it is necessary to perform the numerical analysis.

The goal of the investigation. To perform numerical analysis of mathematical models of distribution of FC of linear SD in voltage distortion in the PCC, based on the principles of superposition and exclusion.

Results of the investigation. We consider the PSS of seven EE power consumers (C) (see Fig. 1) consisting of a energy supply (ES), generalized electrical network (EN), a power transformer (T) and three overhead lines (OL).

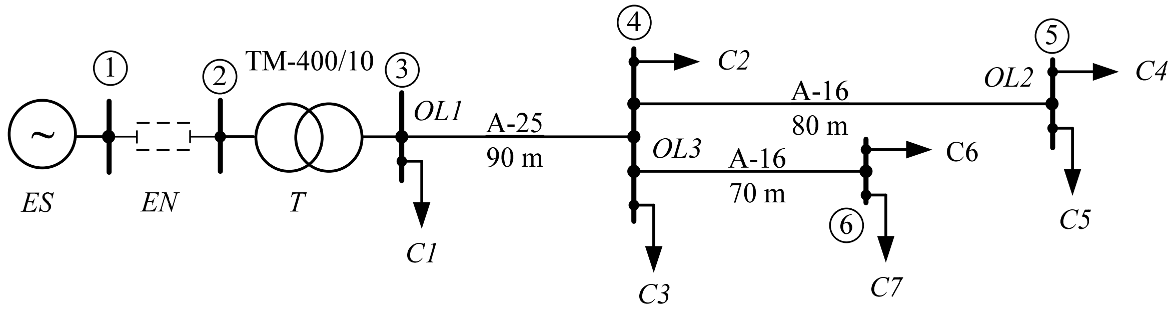

Fig. 1. Power supply system of seven consumers of EE

The parameters of equivalent circuits of the elements of the considered PSS and EE consumers reduced to the voltage of 380 V are the following. The voltage on the buses of the ES: $\underline{U}_A^{ES} = 232\angle 0°$ V; $\underline{U}_B^{ES} = 232\angle 240°$ V; $\underline{U}_C^{ES} = 232\angle 120°$ V. Equivalent resistances of the generalized EN: $\underline{Z}_A^{EN}$ = 0.008 + j0.148 Ω; $\underline{Z}_B^{EN}$ = 0.008 + j0.04 Ω; $\underline{Z}_C^{EN}$ = 0.008 + j0.056 Ω. Equivalent resistances of EE consumers:

i	$\underline{Z}_A^{Ci}$, Ω	$\underline{Z}_B^{Ci}$, Ω	$\underline{Z}_C^{Ci}$, Ω
1	7.2 + j3.7	6.5 + j3.0	6.74+ j3.5
2	6.9 + j5.2	7.7 + j3.7	6.87 + j3.9
3	13.7+ j5.2	15.1 + j4.7	14.2 + j4.6
4	9.7 + j3.2	8.9 + j3.1	10.5 + j3.5
5	6.3 + j1.9	6.8 + j1.4	7.2 + j1.9
6	17.2 + j7.1	19.8 + j8.1	15.6 + j6.5
7	13.9 + j3.9	14.9 + j4.9	15.1 + j4.5

Resistance of the power transformer: $\underline{Z}_{ph\,T}$ = 0.00105 + j0.0072 Ω. Conductivity of the power transformer: $\underline{Y}_{pT}$ = 0.001375 + j0.0021 S. Specific resistance of the OL: $\underline{Z}_s^{A25}$ = 1.26 + j0.34 Ω/km; $\underline{Z}_s^{A16}$ = = 1.97 + j0.345 Ω.

According to calculations carried out, the steady state operation of the considered PSS is characterized by parameters given in the Table 1. It follows from them that in the PCC No. 5, to which EE consumers C4 and C5 are connected, the coefficient of asymmetry of voltage by the zero sequence K_{0U} and steady state deviation of the voltage δU_y exceed the normal allowable values [7]. On this basis, for the PCC No. 5 we determine FC of all SD in the distortion of its voltage.

According to the mathematical models (1) and (2) in the equivalent circuit of the individual elements of the PSS and EE customers distorted part of SD must be extracted and identified [8]. If SD is a passive longitudinal element, its equivalent circuit will be determined by a series connection of two resistances, one of which describes the undistorted part ($\underline{Z}_{el}^{undis}$), and the other – distorting part ($\underline{Z}_{el}^{dis}$). If SD is a passive cross element, its equivalent circuit will be determined by the parallel connection of two conductivities $\underline{Y}_{el}^{undis}$ and $\underline{Y}_{el}^{dis}$. For SD, which is an active element the equivalent circuit is provided as a serial connection of two EMF ($\underline{E}_{ES}^{undis}$ and $\underline{E}_{ES}^{dis}$).

Determination of distorted part of any SD by voltage asymmetry is based on the deflection of its parameters from some symmetric state, for example for passive SD:

$$\begin{cases} \underline{F}_{A(B,C)el}^{undis} = \dfrac{\underline{F}_A^{el} + \underline{F}_B^{el} + \underline{F}_C^{el}}{3}; \\ \underline{F}_{ph\,el}^{dis} = \underline{F}_{ph} - \underline{F}_{ph\,el}^{undis}. \end{cases} \quad (3)$$

The basis of determination of distorted parts of the SD by the deflection voltage are the principles of compliance with the required voltage levels on the ES buses and in control nodes of the PSS voltage as well as load of the individual elements of EN and EE consumers not exceeding permissible or maximum permissible values for them.

So, in the case of excess power of consumer above the maximum permitted its distorted part will be characterized by the following conductivity:

$$\underline{Y}_{ph\,Ci}^{dis} = \Delta\underline{S}_{ph\,Ci}^{*} \Big/ U_{ph\,Ci}^{2}, \quad (4)$$

where $\Delta\underline{S}_{ph\,Ci}$ is the part of the phase power of the i-th EE consumer, exceeding its maximum allowed value; $\underline{U}_{ph\,Ci}$ is the phase voltage of the i-th EE consumer.

Distortion of the actual voltage on the ES buses ($\underline{E}_{ph\,ES}^{fact}$) from the value required by the PSS operation mode ($\underline{E}_{ph\,ES}^{undis}$) will characterize its distorted part:

$$\underline{E}_{ph\,ES}^{dis} = \underline{E}_{ph\,ES}^{undis} - \underline{E}_{ph\,ES}^{fact}. \quad (5)$$

In our case, the maximum permitted power of electrical loads of each EE consumers are listed in Table 2. On operation mode conditions of the PSS the voltage on the ES buses must be maintained as $1{,}065 \cdot U_{nom}$. Voltage regulation by the power transformer is not performed.

On the basis of the above expressions and additional information about the PSS operation, distorted and undistorted parameters of all its SD are determined (see Table 3 and Table 5). According to the mathematical models (1) and (2) the distribution of FC of linear SD in distortion of the voltage in the PCC No. 5 corresponds to the data given in Table 4 and Table 6. For a more visual representation these results are presented in Fig. 3 in graphical form.

Table 1

Parameters of operation modes of the PSS and indicator of PQ in PCC

Parameters of the operation mode of the PSS	PCC PSS					
	1*	2*	3	4	5	6
$\underline{U}_A(\underline{U}_{AB})$, V	401.84∠30	395.33∠28.67	224.09∠–1.17	210.11∠–0.88	202.01∠–0.41	206.53∠–0.85
$\underline{U}_B(\underline{U}_{BC})$, V	401.84∠–90	396.48∠–91.57	228.26∠–122.55	215.2∠–122.42	206.45∠–122.35	212.36∠–122.23
$\underline{U}_C(\underline{U}_{CA})$, V	401.84∠150	394.48∠148.4	231.05∠118.53	217.66∠118.91	210.76∠119.06	213.7∠119.14
δU_y, %	5.75	4.06	3.54	–2.59	–6.19	–4.16
K_{2U}, %	0	0.293	0.29	0.35	0.37	0.39
K_{0U}, %	0	0	1.6	1.86	2.21	1.82

* *Note. For the PCC No. 1, 2 the values of the line voltages are indicated.*

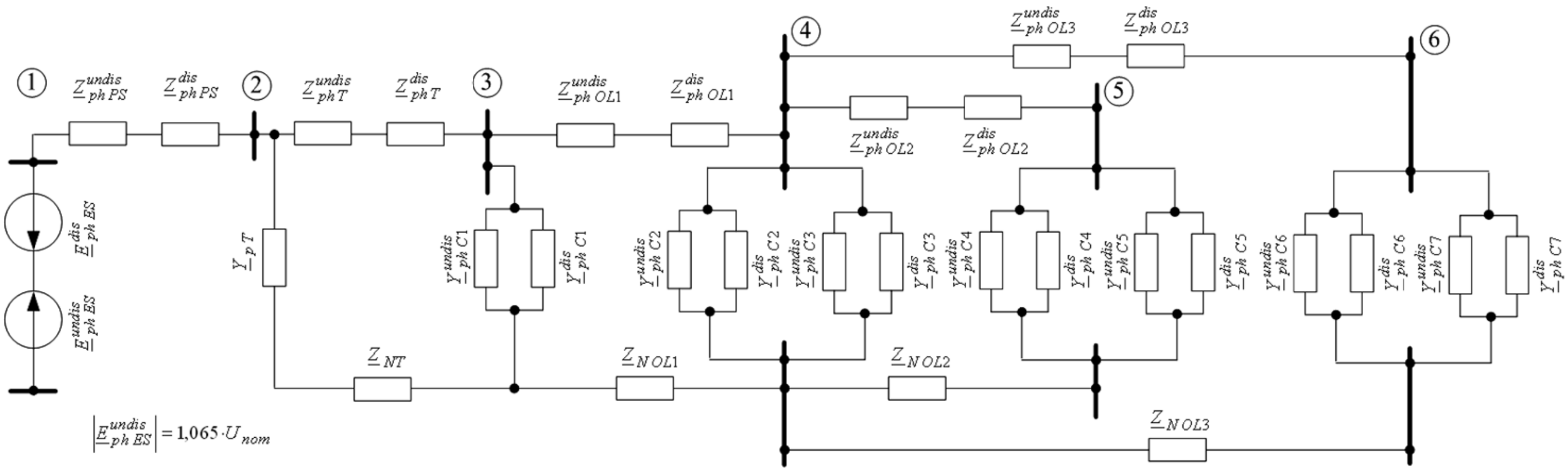

Fig. 2. Equivalent circuit of the PSS and EE consumers in the single-phase version

Table 2

Maximal permitted power of the electrical load of the EE consumers

Power	C1	C2	C3	C4	C5	C6	C7
S_{max}, kVA	8+*j*2.5	5+*j*2	3.5+*j*1.5	4+*j*1.5	7+*j*2.2	3+*j*1.3	3.5+*j*1.5

Table 3

Equivalent circuit parameters of the PSS elements and EE consumers for the FC distribution by the voltage asymmetry

Equivalent circuit	PSS element and power consumers												
	EN	T* (winding)	T* (magnetic)	OL1*	OL2*	OL3*	C1	C2	C3	C4	C5	C6	C7
$\underline{Z}_A^{undis}$, Ω ($\underline{Y}_A^{undis}$, S)	0.008 +j0.048	0.00105 +j0.0072	(0.001375 −j0.0021)	0.1134 +j0.031	0.1576 +j0.0276	0.1379 +j0.0242	(0.118 −j0.059)	(0.109 −j0.058)	(0.063 −j0.021)	(0.093 −j0.031)	(0.139 −j0.036)	(0.049 −j0.02)	(0.063 −j0.019)
$\underline{Z}_A^{dis}$, Ω ($\underline{Y}_A^{dis}$, S)	0	0	0	0	0	0	(−0.00798 +j0.0021)	(0.00227 −j0.00309)	(0.001164 −j0.00299)	(0.000011 −j0.00071)	(0.00669 −j0.00815)	(0.000488 −j0.00018)	(0.00399 −j0.00021)
$\underline{Z}_B^{undis}$, Ω ($\underline{Y}_B^{undis}$, S)	0.008 +j0.048	0.00105 +j0.0072	(0.001375 −j0.0021)	0.1134 +j0.031	0.1576 +j0.0276	0.1379 +j0.0242	(0.118 −j0.059)	(0.109 −j0.058)	(0.063 −j0.021)	(0.093 −j0.031)	(0.139 −j0.036)	(0.049 −j0.02)	(0.063 −j0.019)
$\underline{Z}_B^{dis}$, Ω ($\underline{Y}_B^{dis}$, S)	−j0.008	0	0	0	0	0	(0.008973 −j0.00029)	(−0.00342 −j0.00745)	(−0.00261 +j0.0024)	(0.007239 −j0.00352)	(0.002271 +j0.0067)	(−0.00592 +j0.0026)	(−0.00213 −j0.00099)
$\underline{Z}_C^{undis}$, Ω ($\underline{Y}_C^{undis}$, S)	0.008 +j0.048	0.00105 +j0.0072	(0.001375 −j0.0021)	0.1134 +j0.031	0.1576 +j0.0276	0.1379 +j0.0242	(0.118 −j0.059)	(0.109 −j0.058)	(0.063 −j0.021)	(0.093 −j0.031)	(0.139 −j0.036)	(0.049 −j0.02)	(0.063 −j0.019)
$\underline{Z}_C^{dis}$, Ω ($\underline{Y}_C^{dis}$, S)	j0.008	0	0	0	0	0	(−0.00099 −j0.0021)	(0.00153 −j0.00435)	(0.00109 +j0.0006)	(−0.00725 +j0.0028)	(−0.00896 +j0.0015)	(0.00543 −j0.00243)	(−0.00187 +j0.0008)
$\underline{Z}_N$, Ω	–	≈0	–	0.1134 +j0.031	0.1576 +j0.0276	0.1379 +j0.0242	≈0	≈0	≈0	≈0	≈0	≈0	≈0

* *The power transformer and OL are accepted symmetrical elements.*

Table 4

Distribution of FC of linear SD in voltage distortion by the voltage asymmetry

PCC	Indicator of PQ		Voltages of symmetrical components										
			Voltage of zero sequence, V/grad		FC of the i-th SD in voltage asymmetry								
					EN	C1	C2	C3	C4	C5	C6	C7	∑
Mathematical model based on the principle of SD superposition													
5	K_{0U}, %	2.21	$\lvert\underline{U}_0\rvert$	4.57	$4.9\cdot10^{-7}$	1.125	1.336	0.363	1.522	3.138	0.982	0.402	4.57
			$\arg(\underline{U}_0)$	142.3	2.68	49.42	−162.52	175.32	82.07	141.12	−97.23	−151.89	142.3
Mathematical model based on the principle of SD exclusion													
5	K_{0U}, %	2.21	$\lvert\underline{U}_0\rvert$	4.57	$7\cdot10^{-4}$	1.125	1.337	0.363	1.524	3.138	0.983	0.402	4.569
			$\arg(\underline{U}_0)$	142.3	167.27	49.43	−162.49	175.35	82.02	141.1	−97.21	−151.8	142.3

Table 5

Equivalent circuit parameters of the PSS elements and EE consumers for the FC distribution by the voltage deviation

Equivalent circuit	PSS element and power consumers												
	EN	T* (winding)	T* (magnetic)	OL1*	OL2*	OL3*	C1	C2	C3	C4	C5	C6	C7
$\underline{Z}_A^{undis}$, Ω ($\underline{Y}_A^{undis}$, S)	0.008 +j0.048	0.00105 +j0.0072	(0.001375 −j0.0021)	0.1134 +j0.031	0.1576 +j0.0276	0.1379 +j0.0242	(0.11 −j0.049)	(0.111 −j0.045)	(0.064 −j0.024)	(0.093 −j0.031)	(0.145 −j0.044)	(0.05 −j0.021)	(0.067 −j0.019)
$\underline{Z}_A^{dis}$, Ω ($\underline{Y}_A^{dis}$, S)	0	0	0	0	0	0	(−j0.00697)	(−j0.016)	0	0	0	0	0
$\underline{Z}_B^{undis}$, Ω ($\underline{Y}_B^{undis}$, S)	0.008 +j0.04	0.00105 +j0.0072	(0.001375 −j0.0021)	0.1134 +j0.031	0.1576 +j0.0276	0.1379 +j0.0242	0.127 −j0.048	0.106 −j0.043	0.06 −j0.019	0.094 −j0.035	0.141 −j0.029	0.043 −j0.018	0.061 −j0.02
$\underline{Z}_B^{dis}$, Ω ($\underline{Y}_B^{dis}$, S)	0	0	0	0	0	0	(−j0.01)	(−j0.0073)	0	(0.0059)	0	0	0
$\underline{Z}_C^{undis}$, Ω ($\underline{Y}_C^{undis}$, S)	0.008 +j0.056	0.00105 +j0.0072	(0.001375 −j0.0021)	0.1134 +j0.031	0.1576 +j0.0276	0.1379 +j0.0242	0.117 −j0.047	0.106 −j0.042	0.064 −j0.021	0.086 −j0.029	0.13 −j0.034	0.055 −j0.023	0.061 −j0.018
$\underline{Z}_C^{dis}$, Ω ($\underline{Y}_C^{dis}$, S)	0	0	0	0	0	0	(0.014)	(0.00453 −j0.002)	0	0	0	0	0
$\underline{Z}_N$, Ω	–	≈0	–	0.1134 +j0.031	0.1576 +j0.0276	0.1379 +j0.0242	≈0	≈0	≈0	≈0	≈0	≈0	≈0

** The power transformer and OL are accepted symmetrical elements.*

Table 6

Distribution of FC of linear SD in voltage distortion by the voltage deviation

PCC	Indicator of PQ		Voltage deviation by direct sequence										
			Voltage deviation from lower normal permitted (np) bound, V/grad		FC of the i-th excluded SD in the voltage deviation								
					ES	C1	C2	C3	C4	C5	C6	C7	∑
Mathematical model based on the principle of SD superposition													
5	δU_y, %	−6.19	$U_{\min}^{np} - \lvert\underline{U}_1\rvert$	2.615	2.051	0.12	0.423	0	0.122	0	0	0	2.575
			$-\arg(\underline{U}_1)$	178.8	178.68	166.26	129.12	0	−161.52	0	0	0	171.84
Mathematical model based on the principle of SD exclusion													
5	δU_y, %	−6.19	$U_{\min}^{np} - \lvert\underline{U}_1\rvert$	2.615	2.046	0.119	0.423	0	0.121	0	0	0	2.57
			$-\arg(\underline{U}_1)$	178.8	178.76	166.6	129.17	0	−161.68	0	0	0	171.9

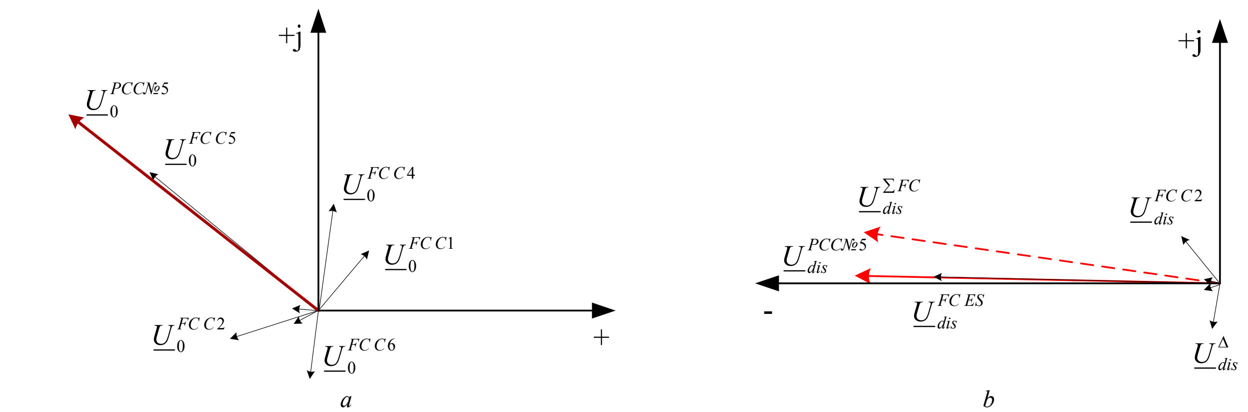

Fig. 3. Graphical representation of the distribution of FC of linear SD in voltage distortion in the PCC No. 5 on the base of the mathematical model: (1): *a*) by the voltage asymmetry; *b*) by the voltage deviation

We estimate the divergence of results for the distribution of FC of linear SD in distortion of voltages in the PCC No. 5, obtained on the basis of (1) and (2) mathematical models, by the relative root-mean-square deviation:

$$\delta = \frac{\sum_{i=1}^{n}\left\{\begin{array}{l}\left[\mathrm{Re}\left(\underline{U}_{dis(1)}^{FCi}\right)-\mathrm{Re}\left(\underline{U}_{dis(2)}^{FCi}\right)\right]^2+\\ +\left[\mathrm{Im}\left(\underline{U}_{dis(1)}^{FCi}\right)-\mathrm{Im}\left(\underline{U}_{dis(2)}^{FCi}\right)\right]^2\end{array}\right\}}{\sum_{i=1}^{n}\left[\mathrm{Re}\left(\underline{U}_{dis(1)}^{FCi}\right)\right]^2+\left[\mathrm{Im}\left(\underline{U}_{dis(1)}^{FCi}\right)\right]^2}\cdot 100\%, \qquad (6)$$

where n is the total number of SD; symbols 1 and 2 correspond the mathematical model (1) and (2), respectively.

In our case, δ by the voltage asymmetry is $6.4\cdot 10^{-5}$ %, and by the voltage deviation – $8.1\cdot 10^{-4}$ %. These values lead to the conclusion of equivalence of (1) and (2) mathematical models and, consequently, their arbitrary choice for the solution of the problem of the distribution of FC of linear SD in the voltage distortion in the PCC.

We analyze the obtained FC distributions. Firstly, FC distribution of linear SD in the voltage distortion in the PCC is a vector (two-dimensional) quantity. It is obvious that in such a form the FC can not be used for the distribution of financial compensations for the reduction of PQ and it is necessary to provide a corresponding one-dimensional criterion. We set as the basis of the one-dimensional criterion of the FC distribution the FC scalar product in vector form:

$$\alpha_i = \left\langle \underline{U}_{dis}^{FC\,DS\,i}, \underline{U}_{dis}^{PCC} \right\rangle;\; FC_{DS\,i} = \left[|\alpha_i| \Big/ \sum_{i=1}^{n}|\alpha_i|\right]\cdot 100\%. \qquad (7)$$

Such an approach means that this criterion assesses the FC by projections of vector FC $\underline{U}_{dis}^{FC\,DS\,i}$ on the total vector of the voltage distortion in the PCC $\underline{U}_{dis}^{PCC}$. Omitting the module in the expression (7) it is possible to additionally take into account the effect of the voltage distortion compensation introduced by separate SD. In our case, this effect is most clearly demonstrated by the vectors $\underline{U}_{0}^{FC\,C4}$ and $\underline{U}_{0}^{FC\,C6}$ (Fig. 3,*a*).

Secondly, in the voltage distortion in the PCC No. 5 all SD PSS take part. Here, FC SD outside the PCC No. 5 may be comparable to or greater than FC SD connected directly to the PCC considered.

Third, the discrepancy of the FC ($\underline{U}_{dis}^{\Delta} = \underline{U}_{dis}^{PCC№5} - \underline{U}_{dis}^{\Sigma FC}$) between all SD (Fig. 3,*b*) which is caused by not taking into account or the inaccuracy of the determination of distorted parts of some SD is possible. To eliminate it is enough to group the unknown or ill-defined SD, belonging to the same subject of the energy market, for example, the PSS, and to determine their total FC by excluding from the total distortion level voltages in the PCC:

$$\underline{U}_{dis}^{FC\,PSS} = \underline{U}_{dis}^{PCC№5} - \underline{U}_{dis}^{\Sigma FC\,Ci}. \qquad (8)$$

On the basis of the above, a one-dimensional distribution of the FC by the voltage asymmetry in the PCC No. 5 will be:

$FC_{DS\,i}$, %	EN	C1	C2	C3
	$8.6\cdot 10^{-6}$	1.0	13.43	5.36
	П4	П5	П6	П7
	13.31	55.24	8.77	2.9

Assuming that distorted parts of the SD part from the side of the EE customers are identified accurately and distorted parts of the SD from the side of PSS elements are grouped, the one-dimensional distribution of the FC by the voltage deviation in the considered PCC will be:

$FC_{DS\,i}$, %	PSS	C1	C2	C3
	80.64	4.46	10.56	0
	П4	П5	П6	П7
	4.34	0	0	0

The obtained results show that most part of the payments for compensation of economic losses for the subjects of the energy market in the PCC No. 5 from the voltage asymmetry falls on EE customers C5 (55.24 %) and C2 (13.43 %), and from the voltage deviations – on the PSS (80.64 %) and EE C2 customer (10.56 %).

Conclusions. Mathematical models of determination of FC of linear sources of SD in the voltage distortion in the PCC, based on the principles of superposition and exclusion, are equivalent. To assess the degree of participation of each SD in the voltage distortion in the PCC and the distribution of financial compensation to the injured party between all SD, a one-dimensional criterion of FC distribution based on the scalar product of vectors is developed. Not accounting the group of SD, belonging to one subject of the energy market, permits to determine their total FC as the discrepancy of the distribution of FC between all SD.

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