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CALCULATION METHOD OF ELECTRIC POWER LINES MAGNETIC FIELD STRENGTH BASED ON CYLINDRICAL SPATIAL HARMONICS

Purpose. Simplification of accounting ratio to determine the magnetic field strength of electric power lines, and assessment of their environmental safety. Methodology. Description of the transmission lines of the magnetic field by using techniques of spatial harmonic analysis in the cylindrical coordinate system is carried out. Results. For engineering calculations of electric power lines magnetic field with sufficient accuracy describes their first spatial harmonic magnetic field. Originality. Substantial simplification of the definition of the impact of the construction of transmission line poles on the value of its magnetic field and the bands of land alienation sizes. Practical value. The environmentally friendly projection electric power lines on the level of the magnetic field. References 6, tables 1, figures 4.

Key words: **electric power line, magnetic field, environmental safety, cylindrical spatial harmonics.**

На основе пространственного гармонического анализа магнитного поля в цилиндрической системе координат предложен метод расчета индукции магнитного поля линий электропередачи. Показано, что магнитное поле линий электропередачи с достаточной для инженерных расчетов точностью описывается первой цилиндрической пространственной гармоникой. Использование предложенного метода позволяет существенно упростить определение влияния конструкции опор линий электропередачи на величину их магнитного поля и на ширину полос отчуждения земельных участков. Библ. 6, табл. 1, рис. 4.

Ключевые слова: **линия электропередачи, магнитное поле, цилиндрические пространственные гармоники.**

Introduction. One of the problems solved by the designers of overhead transmission lines (TL) in assessing their environmental safety is determination of dimensions $\pm X_s$ of the trackside width, as shown in Fig. 1. Among the factors which determine the width of the strips are installed on their border $\pm X_s$ limits [1, 2] of the value of the module of the magnetic field (MF) strength vector B_l produced by TL at the height h_0 of the earth's surface. Under these restrictions, the value of the module B_l away $-X_s \geq x \geq X_s$ from the TL should be less than the specified value B_s of the magnetic field strength. Borders ($-X_s$; $+X_s$) of the strip of alienation by the parameter B_s are determined by the calculated dependence (magnetograms) of the TL magnetic field strength module B_l (Fig. 1).

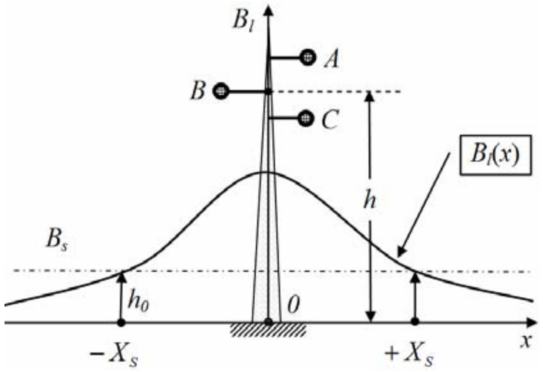

Fig. 1. Magnetograms of the TL

Problem definition. To simplify the calculation of the MF of the TL in the far field (at the border of the exclusion zone of the TL) multidipole transmission line models [3], based on the use of spherical spatial harmonics are utilized. At the same calculation relations are quite complex, and final calculation results are, as a rule, in numerical format, which complicates the practical need to establish cause – effect relationships between design parameters of transmission lines and distribution of their MF strength.

The goal of the work is to simplify the settlement of relations to determine the MF strength of the TL and evaluate their environmental safety.

The goal of the work proposed to be carried through the use of cylindrical space harmonics to calculate the magnetic field strength of the TL.

Presentation of research materials. At the description of the TL magnetic field we assume that:

- Phase conductor lines are parallel current filaments of infinite length and infinitely small diameter.
- Line currents $\dot{I}_A$, $\dot{I}_B$ $\dot{I}_C$ form a symmetrical system:

$$\dot{I}_A = I \,,\ \dot{I}_B = \alpha^2 I \,,\ \dot{I}_C = \alpha I \,, \tag{1}$$

where $a = e^{j4\pi/3}$.

Under what assumptions spatial harmonic analysis of the magnetic field of the TL can be made in a cylindrical coordinate system (r, φ, Y) which Y-axis passes through the center of a circle of minimum radius $r_{\min}$ where all current filaments fit (Fig. 2).

Relation (1) allows to represent module of the magnetic field strength $B_l(x)$ of three-phase line at an arbitrary point in space P as the modulus of the sum of the magnetic field strengths $\vec{B}_{A-0}(P)$, $\vec{B}_{B-0}(P)$, $\vec{B}_{C-0}(P)$ respectively of three independent closed broaching circuits $A-0$, $B-0$ и $C-0$ (see Fig. 2).

$$B_s(P) = \left| \vec{B}_{A-0}(P) + \vec{B}_{B-0}(P) + \vec{B}_{C-0}(P) \right| . \tag{2}$$

When the selected track (along the Y-axis) of passing of inverse wires with currents, $-\dot{I}_A$, $-\dot{I}_B$ and $-\dot{I}_C$ the position of each of three circuits define respectively filaments coordinates of phases A, B, C.

Spatial harmonic analysis of the MF of a closed current circuit. There is a closed current circuit, for example, $A-0$ (Fig. 2).

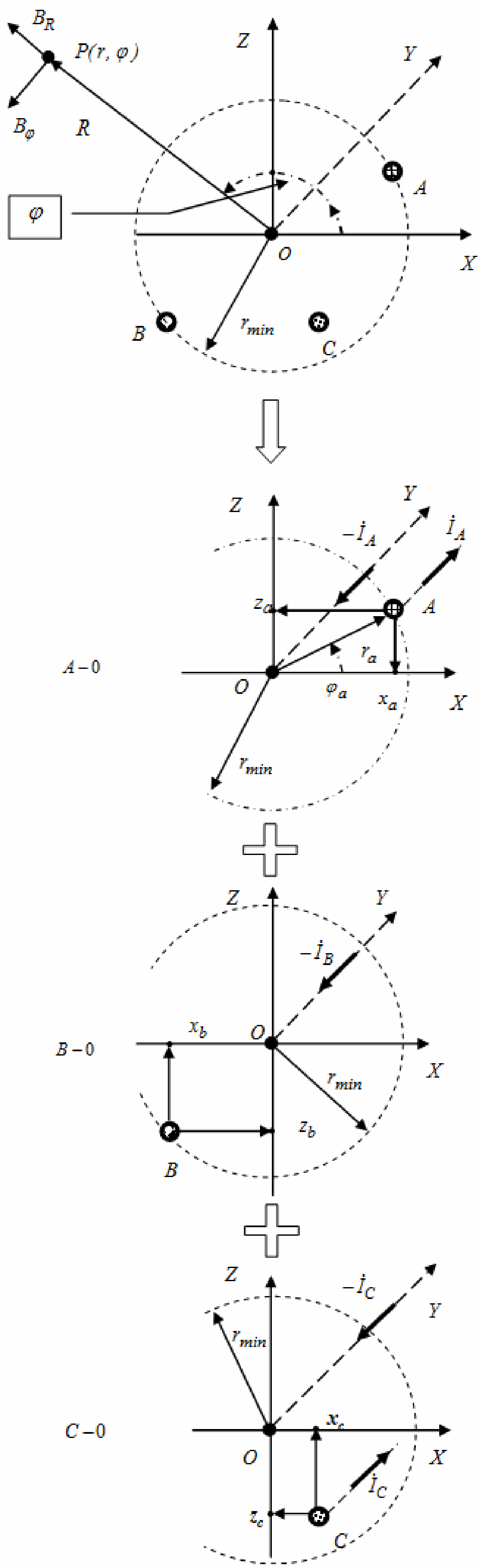

Fig. 2. Representation of a three-phase line as three independent circuits

Vector potential $A_{(A-0)Y}$ of the magnetic field of such a circuit in the arbitrary point of the space $P(r,\varphi,Y)$ is determined as a sum of the corresponding vector potentials A_{ay}, A_{oy} of the current of the phase A and the opposite current and taking into account [4] it can be determined by the relation

$$A_{(A-0)y} = A_{oy} + A_{ay} = \mu_0 \frac{I}{2\pi}\ln r - -\mu_0 \frac{I}{2\pi}\ln\sqrt{\left(r^2 + (r_a)^2 - 2rr_a\cos(\varphi-\varphi_a)\right)}. \tag{3}$$

Relation (3) can be represented as Fourier series after that for the external region ($r \geq r_{\min}$) it will have the known form [5]

$$A_{(A-0)Y} = \mu_0 \frac{I}{2\pi}\sum_{n=1}^{N}\left(\frac{1}{r}\right)^n\left(\frac{(a_{an}\cos n\varphi + b_{an}\sin n\varphi)}{n}\right), \tag{4}$$

where a_{an}, b_{an} are the amplitudes of the n-th order of the magnetic field's vector potential of the current circuit A–0

$$a_{an} = (r_a)^n\cos n\varphi_a,\; b_{an} = (r_a)^n\sin n\varphi_a. \tag{5}$$

Magnetic vector potential's harmonics (4) determine also the corresponding harmonics of its magnetic field strength B_{ar} and $B_{a\varphi}$:

$$B_{ar} = \frac{\mu_0}{r}\frac{dA_{(A-0)Y}}{d\varphi} = \frac{\mu_0 I}{2\pi}\sum_{n=1}^{\infty}\frac{[a_{an}\sin n\varphi + b_{an}\cos n\varphi]}{r^{n+1}}, \tag{6}$$

$$B_{a\varphi} = \frac{\mu_0}{2\pi}\frac{dA_{(A-0)y}}{dr} = -\frac{\mu_0 I}{2\pi}\sum_{n=1}^{\infty}\frac{(a_{an}\cos n\varphi + b_{an}\sin n\varphi)}{r^{n+1}}. \tag{7}$$

Magnetic field strength module B_{an} of the n harmonic in the point $P(r,\varphi,Y)$ will be dependent on the r-coordinate

$$B_{an} = \mu_0 \frac{I}{2\pi\cdot r^{n+1}}\sqrt{(a_{an})^2 + (b_{an})^2}. \tag{8}$$

Table 1 represents values of amplitudes a_{an}, b_{an} of two first harmonics for the circuit A–0 in the coordinate system X, Y, Z (Fig. 2).

Table 1

Amplitudes of the magnetic field strength harmonics for the current circuit $A-0$

Amplitude of harmonics	Relations for the circuit with coordinates x_a, z_a
a_{a1}	x_a
b_{a1}	z_a
a_{a2}	$(x_a)^2-(z_a)^2$
b_{a2}	$2x_a\cdot z_a$

This format of the amplitudes a_{an}, b_{an} representation harmonizes well with the design document for TL pylons which regulates coordinates of points of suspension of its wires with respect to earth surface.

By analogy with (5) amplitudes of harmonics a_{bn}, b_{bn} and a_{cn}, b_{cn} of circuits B – 0 and C – 0 are respectively determined:

$$\begin{gathered} a_{bn} = \alpha^2\cdot(r_b)^n\cos n\varphi_b,\; b_{bn} = \alpha^2\cdot(r_b)^n\sin n\varphi_b, \\ a_{cn} = \alpha^2\cdot(r_c)^n\sin n\varphi_c,\; b_{cn} = \alpha^2\cdot(r_c)^n\sin n\varphi_c. \end{gathered} \tag{9}$$

The structure of series (6), (7) is such that as r increases the contribution of high-order harmonic components in the magnetic field strength B_r and B_φ reduces.

So, the magnetic field strength at a distance of two-wire line $x \geq r_{\min}$ is described mainly by its first (n = 1) harmonic constructed as illustrated by equation (8) magnetogram in Fig. 3. It also presents the results of calculations by the Biot-Savart-Laplace low in accordance with [6].

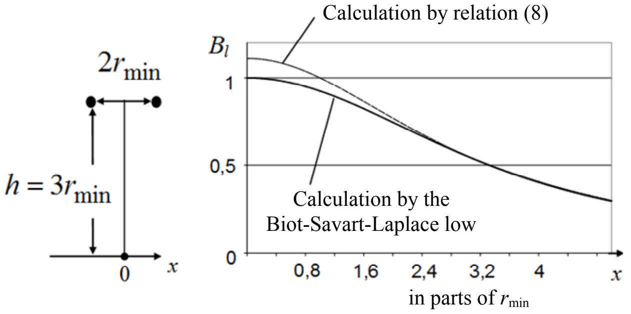

Fig. 3. Magnetograms of B_l of a two-wire line at unit current I

Comparison of the calculation results (Fig. 3) shows that the distance from the transmission line axis at a distance of more than $r_{\min}$ the error of the proposed method in comparison with the exact method [6] does not exceed 10 %, which confirms the possibility of using the first cylindrical space harmonics to calculate the MF of the TL at the boundary of their protected areas.

The magnetic field of single-circuit TL. Single circuit lines have one set of phase conductors. Their relative positions to each other and the Earth's surface determines the design of the (profile) of a TL pylon.

According to that shown in Fig. 2 «magnetic» interpretation of the transmission line, amplitudes a_{ln} and b_{ln} of harmonics of its magnetic field taking into account (1) and (2) are presented in the form of a sum corresponding to the amplitude of its independent circuits A–0, B–0, C–0:

$$a_{ln} = a_{an} + \alpha^2 a_{bn} + \alpha a_{cn}, b_{ln} = b_{an} + \alpha^2 b_{bn} + \alpha b_{cn}. \quad (10)$$

The first significant harmonic of single-circuit TL is the harmonic of the order (n = 1). Its amplitudes a_{ln} and b_{ln} taking into account (5), (10) equal:

$$a_{l1} = x_a + \alpha^2 \cdot x_b + \alpha \cdot x_c, b_{l1} = z_a + \alpha^2 \cdot z_b + \alpha \cdot z_c. \quad (11)$$

It should be note that values of the amplitude a_{l1} and b_{l1} *of the* first harmonic (n = 1) do not depend on the beginning of the selected coordinate system X, Y, Z.

Knowledge of amplitudes of the first harmonic a_{l1} and b_{l1} of the magnetic field of the TL allows by using the relation (7) to build its magnetogram

$$B_l(x) \approx B_{l1}(x) = \mu_0 I \frac{\left|\sqrt{(a_{l1})^2 + (b_{l1})^2}\right|}{2\pi \cdot \left((h-h_0)^2 + x^2\right)}, \quad (12)$$

where h is the distance from the ground level (Fig. 1) to the center of the circle $r_{\min}$ which fit all current lines of the TL.

For ease of calculation the distance h can be set equal to the average height h_a, h_b, h_c of the respectively suspension of phase conductors A, B and C

$$h \approx 1/3\left(h_a + h_b + h_c\right). \quad (13)$$

After simple but cumbersome transformations the relation (12) can be reduced to the form:

$$B_l(x) \approx \mu_0 I \frac{d_{rms}}{2\sqrt{2}\pi \cdot \left((h-h_0)^2 + x^2\right)}, \quad (14)$$

where d_{rms} is the mean square distance between the wires of the TL

$$d_{rms} = \sqrt{(d_{AB})^2 + (d_{BC})^2 + (d_{CA})^2},$$

where d_{AB}, d_{BC}, d_{CA} is the distance between the suspension points on a support phase wires A and B, B and C, C and A, respectively.

Analytical representation of magnetograms (14) permits to determine the size of the band $\pm X_s$ of the exclusion for a given parameter B_l

$$\pm X_s = \sqrt{\frac{\mu_0 \cdot I \cdot d_{rms}}{2\sqrt{2} \cdot \pi \cdot B_l} - (h-h_0)^2}. \quad (15)$$

This relationship establishes a mutual relationship between the size $\pm X_s$ of the strip of alienation and TL characteristics – its current (I) loading and designs (profile) of its pillars, namely the average height h of wires suspension points and mean square distance d_{rms} between them.

Underground cable TL. Magnetograms of underground cable lines, similar to the single-circuit air TL are determined by the first (n = 1) harmonic of their magnetic field strength.

Below relations for magnetograms for two most commonly used cable laying (Fig. 4) obtained by taking into account (8) and (14) are presented.

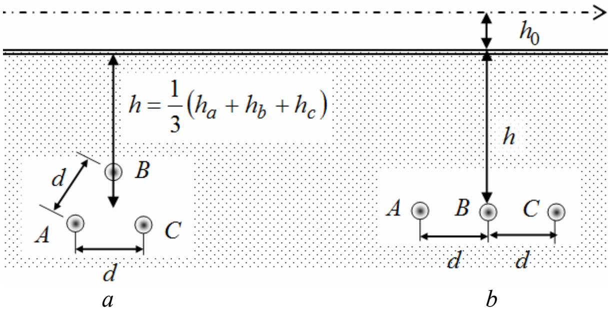

Fig. 4. «Triangle» (a) and «flat» (b) cable line laying

For the cable «flat laying»

$$B_l(x) \approx \mu_0 I \frac{\sqrt{3} \cdot d}{2\,\pi \cdot \left((h+h_o)^2 + x^2\right)}. \quad (16)$$

For the cable laying by «triangle»

$$B_l(x) \approx \mu_0 I \frac{\sqrt{3} \cdot d}{2\sqrt{2} \cdot \pi \cdot \left((h+h_o)^2 + x^2\right)}. \quad (17)$$

Conclusions.

1. It is shown that for the calculation of the magnetic field strength of transmission lines on the border of protected zones with limited accuracy (less than 10%), the first cylindrical space harmonic of its magnetic field can be used.
2. The simplified calculation relations of the magnetic field strength of the TL based on cylindrical spatial harmonics, allowing to simplify the calculation of the TL magnetic field distribution and assess the impact of the TL design peculiarities on the width of the land rights of way to ensure environmental safety are proposed.

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