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COMPONENTS OF TOTAL ELECTRIC ENERGY LOSSES POWER IN PQR SPATIAL COORDINATES

***Purpose.** To obtain relations determining the components of the total losses power with p-q-r power theory for three-phase four-wire energy supply systems, uniquely linking four components: the lowest possible losses power, losses power caused by the reactive power, losses power caused by the instantaneous active power pulsations, losses power caused by current flowing in the neutral wire. **Methodology.** We have applied concepts of p-q-r power theory, the theory of electrical circuits and mathematical simulation in Matlab package. **Results.** We have obtained the exact relation, which allows to calculate the total losses power in the three-phase four-wire energy supply system using three components corresponding to the projections of the generalized vectors of voltage and current along the pqr axis coordinates. **Originality.** For the first time, we have established a mathematical relationship between spatial representation of instantaneous values of the vector components and the total losses power in the three-phase four-wire energy supply systems. **Practical value.** We have elucidated an issue that using the proposed methodology would create a measuring device for determining the current value of the components of total losses power in three-phase systems. The device operates with measuring information about instantaneous values of currents and voltages.* References 15, tables 1, figures 3.
Key words: **energy supply system, p-q-r power theory, the minimum possible losses, total losses power, Matlab-model of the three-phase energy supply system..**

***Цель.** Целью статьи является получение соотношений для определения составляющих суммарной мощности потерь с использованием p-q-r теории мощности для трехфазных четырехпроводных систем электроснабжения, однозначно связывающих четыре компоненты: минимально возможную мощность потерь; мощность потерь, обусловленную реактивной мощностью; мощность потерь, обусловленную пульсациями мгновенной активной мощности; мощность потерь, обусловленную протеканием тока в нулевом проводе. **Методика.** Для проведения исследований использовались положения p-q-r теории мощности, теория электрических цепей, математическое моделирование в пакете Matlab. **Результаты.** Получено точное расчетное соотношение, позволяющее рассчитать суммарную мощность потерь в трехфазной четырехпроводной системе электроснабжения через три составляющие, соответствующие проекциям обобщенных векторов тока и напряжения на оси pqr системы координат. **Научная новизна.** Впервые установлена математическая связь между пространственным векторным представлением мгновенных величин и составляющими мощности суммарных потерь в трехфазных четырехпроводных системах электроснабжения. **Практическое значение.** Использование предложенной методики позволит создать измерительный прибор для определения текущего значения составляющих мощности суммарных потерь в трехфазных системах, оперирующий измерительной информацией о мгновенных значениях токов и напряжений.* Библ. 15, табл. 1, рис. 3.
Ключевые слова: **система электроснабжения, p-q-r теория мощности, минимально возможные потери, мощность суммарных потерь, Matlab-модель трехфазной системы электроснабжения.**

Introduction. The development of the modern theories of instantaneous active and reactive power in 1983, 1984 [1, 2] has allowed experts in the area of electrical engineering to change their views on such concepts as «reactive power», «apparent power», «unbalance power», «distortion power» [1-5]. On the basis of new theories the active filter control devices methods for energy supply systems (ESS), using the conversion of spatial coordinate systems were further developed, which opened up new directions and was the development of power electronics. The developed theories, operating with spatial vectors of currents and voltages, among which are the p-q theory, improved p-q theory of power, i_d-i_q method, cross-vector theory and p-q-r theory of instantaneous power [6-9], inspired the creation of conversion system control algorithms with near to unity power factor. [10] The principal possibility of the energy efficiency increasing of the ESS with nonlinear consumers at the connection of the power active filter (PAF) is shown [6, 10, 11, 15]. Currently there is no completed general theory linking the losses of electrical energy in the ESS with the provisions of the modern theories of instantaneous active and reactive power. Improving the energy efficiency of energy supply systems by PAF measures for specific operation modes solves a number of practical problems, such as determining the need for and the installation location of the power compensator, the creation of active power filter control algorithms, ensuring the work of distributed energy supply systems with the highest possible efficiency.

The goal of the paper is to develop the principles of the modern theories of instantaneous active and reactive power and to obtain calculation relations for determining the components of the additional electrical energy losses power in three-phase ESS by using pqr spatial coordinates.

An equivalent circuit of the three-phase ESS with PAF. The complex branched power supply system circuit of low and medium voltage consumers may be represented as a simple equivalent circuit shown in Fig. 1. The three-phase sinusoidal voltage source *Source* through line *Line* with resistors R_s is connected to the load unit *Load* which can include resistors, reactors, capacitor banks, nonlinear elements, current and voltage sources. The resistance of the neutral conductor is taken into account by the resistance R_n. If we take into account that the source and the load can operate in symmetric and unbalanced modes, then at the unidirectional flow of energy in the ESS from the source to the load may be 96 different variants of combinations of parameters «source-load» system, in which there are additional losses [13].

In the equivalent circuit (see Fig. 1) line inductance L_s is moved into the load, which is generally a reasonable assumption and facilitates further analysis of the ESS. In the unit load connection point parallel to the PAF is connected, the power circuit of which is an autonomous PWM inverter on transistor-diode modules, with the capacitor bank in the DC link. To monitor the status of the ESS and generating control actions in the circuit according to Fig. 1, sensors of currents and voltages are used, by which the phase voltages at the terminals of the connection source u_{sa}, u_{sb}, u_{sc}, the phase voltages at the load connection terminals u_{La}, u_{Lb}, u_{Lc}, as well as the phase load currents i_{La}, i_{Lb}, i_{Lc} and power compensator i_{ca}, i_{cb}, i_{cc} are measured.

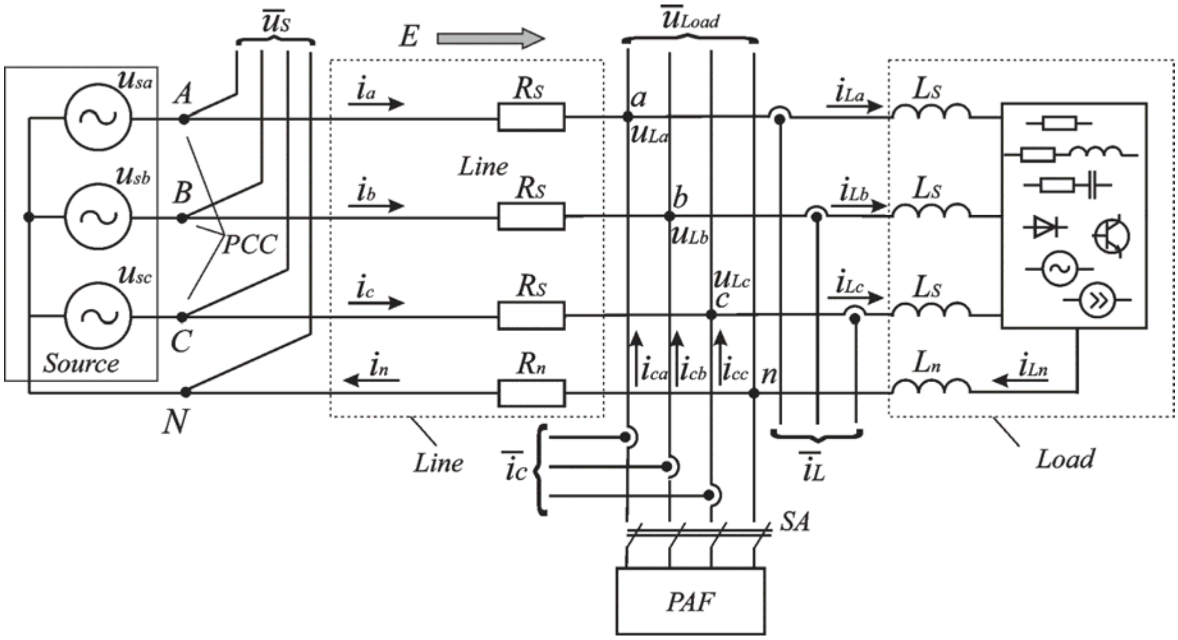

Fig. 1. An equivalent circuit of the three-phase ESS with PAF

When the switch SA is opened phase load currents are equal to the corresponding source phase currents.

The measured instantaneous values allow at any time to obtain information on the value of the instantaneous active power and the instantaneous reactive power. The first one is defined as the scalar product of two spatial vectors of voltage and current of the three-phase ESS presented, for example, in a coordinate system a, b, c, and the second one is the vector product of the same vectors:

$$p_S = \left|\vec{u}_S\right| \cdot \left|\vec{i}\right| \cdot \cos\varphi , \qquad (1)$$

$$\vec{q}_S = \vec{u}_S \times \vec{i}_S = \begin{bmatrix} q_{sa} \\ q_{sb} \\ q_{sc} \end{bmatrix} = \left[\begin{vmatrix} u_{sb} & u_{sc} \\ i_b & i_c \end{vmatrix} \cdot \begin{vmatrix} u_{sc} & u_{sa} \\ i_c & i_a \end{vmatrix} \cdot \begin{vmatrix} u_{sa} & u_{sb} \\ i_a & i_b \end{vmatrix} \right]^T , \qquad (2)$$

where

$$\vec{u}_S = \left[\vec{i}\, u_{sa} \;\; \vec{j} u_{sb} \;\; \vec{k} u_{sc} \right]^T \qquad (3)$$

is the spatial vector of the network voltage in the coordinate system a, b, c, $\vec{i}, \vec{j}, \vec{k}$ are orts of directions by the axes a, b, c of the coordinate system;

$$\vec{i} = \left[\vec{i}\, i_a \;\; \vec{j} i_b \;\; \vec{k} i_c \right]^T \qquad (4)$$

is the spatial vector of current in the coordinate system a, b, c.

Components of the additional energy losses power in three-phase ESS. In the absence of the three-phase ESS of calculated reactive power and constant in time instantaneous active power schedule the system is operating with the highest possible efficiency. The value of the highest possible efficiency is determined by the ratio of the power of three-phase resistive short circuit P_{sc} to the average calculated in the repetition time period, useful active load power P_{usf} [13]:

$$\eta_{\max} = \frac{1}{2} + \sqrt{\frac{1}{4} - \frac{1}{k_{sc}}} , \qquad (5)$$

where

$$k_{sc} = \frac{P_{sc}}{P_{usf}} . \qquad (6)$$

The indicated condition

$$p_{puls} = 0 \wedge P_{usf} = \text{const}, \quad q = 0, \qquad (7)$$

is performed in the ESS with symmetrical three-phase source and symmetrical resistive load. Violation of condition (7) leads to a rise in the ESS of additional losses power

$$\Delta P_{\Sigma} = \Delta P_{\min} + \Delta P_{add} , \qquad (8)$$

where $\Delta P_{\min}$ is the minimal possible losses power determined by the relation (5); ΔP_{add} is the additional losses power.

In [13] following the adoption of a number of assumptions an universal calculation ratio determined the power of total losses as the sum of four components, presented in parts of useful active power P_{usf} was obtained

$$\Delta P_{\Sigma *} = \Delta P_{\min *} \times \times \left(1 + Q^2_{RMS*} + P^2_{pulsRMS*}\right) + \Delta P_{n*} \Bigg|_{P_{usf} = const}, \quad (9)$$

where

$$\Delta P_{puls*} = \Delta P_{\min *} \cdot P^2_{pulsRMS*} \quad (10)$$

is the relative component of power additional losses due to the variable component of the instantaneous power of the three-phase ESS, $P_{pulsRMS*}$ is the relative mean-square value of the variable active power component calculated in the repetition time period;

$$\Delta P_{Q*} = \Delta P_{\min *} \cdot Q^2_{RMS*} \quad (11)$$

is the relative component of the power of additional losses due to the instantaneous reactive power of the three-phase ESS, Q_{RMS*} is the relative mean-square value of the module of the reactive power vector $|\vec{q}|$ calculated in the repetition time period;

$$\Delta P_{n*} = \frac{\Delta P_n}{P_{usf}} = \frac{R_S}{T \cdot P_{usf}} \int_t^{t+T} i_n^2 dt \quad (12)$$

is the relative losses power in the neutral conductor, calculated in the repetition time period T, due to the current i_n flow.

Check of the formula (9) on a specially created mathematical model showed the high accuracy in determining the total losses power for three-phase three-wire ESS in a symmetrical three-phase operation mode of the three-phase source. Using the formula (9) for four-wire ESS under certain combinations of parameters leads to considerable error arising from the lack of accounting (9) the mutual influence of the electromagnetic processes in the phase conductors and the neutral conductor.

In [14] it was proposed the refinement of the formula (9) by introducing an additional fifth component of the power of the additional losses power due to the mutual influence of the electromagnetic processes in the phase wires and the neutral wire of the three-phase ESS, ΔP_{mut*}

$$\Delta P_{\Sigma *} = \frac{\Delta P_\Sigma}{P_{usf}} = \Delta P_{\min *} + \Delta P_{puls*} + + \Delta P_{q*} + \Delta P_{n*} + \Delta P_{mut*} \Bigg|_{P_{usf} = \text{const}}. \quad (13)$$

The indicated way could minimize the error of calculation of the total losses power for four-wire ESS, however the calculation algorithm become more complicated, and any practical difficulties in the use of the adjusted ratio arose.

Representation of power components of additional losses of the three-phase ESS in pqr spatial coordinates. The greatest opportunities to extract components of the instantaneous losses power and components, requiring compensation in three-phase four-wire systems are presented by the p-q-r theory of instantaneous active and reactive power [9]. The mathematical apparatus of p-q-r theory, described in detail in the literature, is associated with a spatial transition from the Cartesian coordinate system *abc* to the pqr system. Transformation of coordinate systems is carried out in two stages: generalized spatial vectors of voltages and currents from the *abc* coordinate system using the direct Clark Transformation matrix are transferred to a fixed spatial αβ0 system:

$$\begin{bmatrix} u_\alpha \\ u_\beta \\ u_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} u_{Sa} \\ u_{Sb} \\ u_{Sc} \end{bmatrix}, \quad (14)$$

$$\begin{bmatrix} i_\alpha \\ i_\beta \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \cdot \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \cdot \begin{bmatrix} i_{La} \\ i_{Lb} \\ i_{Lc} \end{bmatrix}, \quad (15)$$

followed by a transition from αβ0 coordinate system into a rotating coordinate system pqr.

$$\begin{bmatrix} i_p \\ i_q \\ i_r \end{bmatrix} = \frac{1}{u_{\alpha\beta 0}} \begin{bmatrix} u_0 & u_\alpha & u_\beta \\ 0 & -\frac{u_{\alpha\beta 0} u_\beta}{u_{\alpha\beta}} & \frac{u_{\alpha\beta 0} u_\alpha}{u_{\alpha\beta}} \\ u_{\alpha\beta} & -\frac{u_0 u_\alpha}{u_{\alpha\beta}} & -\frac{u_0 u_\beta}{u_{\alpha\beta}} \end{bmatrix} \begin{bmatrix} i_0 \\ i_\alpha \\ i_\beta \end{bmatrix}, \quad (16)$$

where

$$u_{\alpha\beta 0} = \sqrt{u_\alpha^2 + u_\beta^2 + u_0^2}, \quad (17)$$

$$u_{\alpha\beta} = \sqrt{u_\alpha^2 + u_\beta^2}. \quad (18)$$

In the symmetrical mode of the three-phase source of the four-wire ESS the system pqr allows to extract four components of the instantaneous power:

$$\begin{bmatrix} P_{AV} + p_{puls} \\ q_q \\ q_r \end{bmatrix} = u_p \cdot \begin{bmatrix} i_{p-} + i_{p\sim} \\ i_r \\ i_q \end{bmatrix}, \quad (19)$$

where

$$u_p = u_{\alpha\beta 0} = u_s = \sqrt{u_\alpha^2 + u_\beta^2 + u_0^2} = = \sqrt{u_a^2 + u_b^2 + u_c^2} = const \quad (20)$$

is the module of the voltage space vector, which in pqr coordinates coincides with the direction of the axis p; P_{AV} and p_{puls} are, respectively, the constant calculated in the repetition time period, and the variable components of the instantaneous active power of the ESS; i_{p-} and $i_{p\sim}$ are, respectively, the constant and variable components of the projection of the generalized space vector of the current on the p-axis of the pqr coordinate system; q_q and q_r are, respectively, the instantaneous reactive power with respect to r-axis and q-axis.

Transfer of electrical energy from the source to the load with the least possible losses causes a DC component of the instantaneous active power, the other three components generally to be compensated. Exclusion from the system of the variable component of the instantaneous power will allow compensating amplitude asymmetry of network currents. Exclusion from the system of reactive power by the q-axis current will permit compensating the neutral wire. Exclusion from the system of reactive power by the r-axis will permit compensating the phase angle between the corresponding phase voltages and currents.

We express additional losses power components in the pqr coordinates. The total losses power in the three-phase four-wire ESS by the equivalent circuit (see Fig. 1) can be represented by two components:

$$\Delta p_{\Sigma} = \Delta p_s + \Delta p_n = i^2 \cdot R_s + i_n^2 \cdot R_n , \quad (21)$$

where Δp_s and Δp_n are respectively, the instantaneous losses power in the three-phase line and instantaneous losses power in the neutral wire;

$$i^2 = \left[i_a^2 \;\; i_b^2 \;\; i_c^2 \right]^T = \left[i_p^2 \;\; i_q^2 \;\; i_r^2 \right]^T \quad (22)$$

is the square of the network current module;

$$i_n = i_a + i_b + i_c \quad (23)$$

is the instantaneous value of the zero conductor current.

Let us consider the case when the resistance of the zero conductor equals to the resistance of the line wire

$$R_n = R_s . \quad (24)$$

In the symmetric mode of the source the neutral conductor current in the pqr system can be expressed from (15) through the projection of the resulting current vector on the r-axis in accordance with the fact that the r-axis of the rotating pqr coordinate system is fixed and coincides with the direction of the axis 0 of the coordinate system αβ0:

$$i_n = \sqrt{3} \cdot i_r . \quad (25)$$

Then substituting (22)-(25) into (21) we obtain

$$\Delta p_{\Sigma} = R_s \cdot \left(i_p^2 + i_q^2 + 4 \cdot i_r^2 \right). \quad (26)$$

Expressing projections of currents in the prq system through the respective power (19) and passing to relative units, the ratio can be written to determine the relative total instantaneous losses power in the coordinates pqr

$$\Delta p_{\Sigma *} = \frac{1}{k_{sc}} \cdot \left(p_*^2 + q_{r*}^2 + 4 \cdot q_{q*}^2 \right). \quad (27)$$

or for the average, calculated in the repetition time period, value

$$\Delta P_{\Sigma *} = \frac{1}{k_{sc}} \cdot \left(P_{RMS*}^2 + Q_{rRMS*}^2 + 4 \cdot Q_{qRMS*}^2 \right). \quad (28)$$

Thus, the relative total losses power in the pqr coordinate system can be represented by the sum of three components corresponding to the losses power by each of the coordinate axes

$$\Delta P_{\Sigma *} = \Delta P_{p*} + \Delta P_{q*} + \Delta P_{r*} . \quad (29)$$

Let us compare the relation (28) with the previously obtained relation (13). The square of the RMS active power value by the p-axis of the pqr coordinate system can be decomposed into two components

$$P_{RMS*}^2 = P_{AV*}^2 + P_{pulsRMS*}^2 = \left(1 + \Delta P_{\Sigma *} \right)^2 + P_{puls*}^2 . \quad (30)$$

The coefficient, which expresses the ratio of the power of the resistive short circuit of the three-phase ESS, can be determined by the relative power of the minimum possible losses

$$\frac{1}{k_{sc}} = \frac{\Delta P_{\min *}}{\left(1 + \Delta P_{\min *} \right)^2} . \quad (31)$$

The square of the modulus of the vector of the relative RMS reactive power

$$Q_{RMS*}^2 = Q_{qRMS*}^2 + Q_{rRMS*}^2 . \quad (32)$$

Relative average losses in the neutral wire

$$\Delta P_{n*} = \frac{3 \cdot Q_{qRMS*}^2}{k_{sc}} . \quad (33)$$

Substituting (30)-(33) to (28) and calculating the roots of a quadratic equation, we can write the ratio to calculate the relative total losses power through the components adopted previously

$$\Delta P_{\Sigma *} = \frac{1 + \Delta P_{\min *}^2 - \sqrt{\left(1 - \Delta P_{\min *}^2 \right)^2 - 4 \cdot \Delta P_{\min *} \times}}{2 \cdot \Delta P_{\min *}}$$
$$\left. \frac{\overline{\times \left(\Delta P_{puls*} + \Delta P_{q*} + \Delta P_{n*} \cdot \left(1 + \Delta P_{\min *} \right)^2 \right)}}{2 \cdot \Delta P_{\min *}} \right|_{P_{usf} = const} \quad (34)$$

The exact calculation expression (34) with a slight error may be replaced by a simplified relationship

$$\Delta P_{\Sigma *} = \Delta P_{\min *} + \Delta P_{puls*} + \Delta P_{q*} +$$
$$\left. + \Delta P_{n*} \cdot \left(1 + \Delta P_{\min *} \right)^2 \right|_{P_{usf} = \text{const}.} \quad (35)$$

Comparing (35) with (13) proposed earlier allows us to express additional fifth component, due to the mutual influence of the electromagnetic processes in the lines and the neutral conductor,

$$\Delta P_{mut*} = \Delta P_{n*} \cdot \left(\Delta P_{\min *}^2 + 2 \cdot \Delta P_{\min *} \right). \quad (36)$$

We write the relations expressing the components of additional losses power of the universal equation, through the relevant components in the pqr coordinates

$$\Delta P_{Q*} = \left(1 + \Delta P_{\min *} \right)^2 \cdot \left(\Delta P_{r*} + \frac{\Delta P_{q*}}{4} \right), \quad (37)$$

$$\Delta P_{puls*} = \left(1 + \Delta P_{\min *} \right)^2 \cdot \Delta P_{p*} -$$
$$- \Delta P_{\min *} \cdot \left(1 + \Delta P_{p*} + \Delta P_{q*} + \Delta P_r \right)^2 , \quad (38)$$

$$\Delta P_{n*} = \frac{3}{4} \cdot \Delta P_{q*} , \quad (39)$$

$$\Delta P_{mut*} = \frac{3}{4} \cdot \left(\Delta P_{\min *}^2 + 2 \cdot \Delta P_{\min *} \right) \cdot \Delta P_{q*} . \quad (40)$$

Components of additional losses power by the universal relation (37)-(40) in the coordinate representation (28) depend on the lowest possible losses power, which in its turn is a function of the active resistance of the line. Due to the fact that the measurement of resistance in the line is difficult to implement real-time task, we express the lowest possible losses power through the instantaneous values of currents and voltages measured according to Fig. 1. Substituting equation (31) to (28) and performing the transformations, we obtain the formula for the calculation of the relative power of the minimum possible losses

$$\Delta P_{\min*} = \frac{P_{pqr*} - 2\cdot \Delta P_{\Sigma*} - \sqrt{P^2_{pqr*} - 4\cdot \Delta P_{\Sigma*}\cdot P_{pqr*}}}{2\cdot \Delta P_{\Sigma*}}, \quad (41)$$

where

$$P_{pqr*} = P^2_{RMS*} + Q^2_{rRMS*} + 4\cdot Q^2_{qRMS*}. \quad (42)$$

The relative power of total losses can be determined by the instantaneous values of currents and voltages, measured in accordance with Fig. 1, or by projections on the p-axis of generalized spatial vectors of currents and voltages in the pqr coordinate system:

$$\Delta P_{\Sigma*} = \frac{1}{T}\int\limits_{t}^{t+T}\left(\left(i_{pL} + i_{pc}\right)\cdot u_{ps} - i_{pL}\cdot u_{pL}\right)dt, \quad (43)$$

where i_{pL}, i_{pc} are respectively projections on the p-axis of the pqr coordinate system of generalized space vectors of the load current and compensator current; u_{ps}, u_{pL} are respectively projections on the p-axis of the pqr coordinate system of generalized spatial vectors of the network voltage and the voltage at the terminals of the load connection.

With the principles of the p-q-r theory of instantaneous active and reactive power, as well as the relations (37)-(43) the total losses power in the ESS can be extracted into separate components, describing the universal calculation expression (13). In order to take advantage of the proposed method it is enough to have information about the instantaneous values of currents and voltages measured in the ESS using PAF.

The increase reserve of the ESS efficiency when PAF connecting. The economic efficiency of PAF connection from the point of the reduce of losses power in the ESS will be achieved when the total losses power in the ESS after connecting of the compensator will be smaller than before its connection

$$\Delta P_{on*} < \Delta P_{\Sigma*}. \quad (44)$$

If after connecting PAF the active load power remains unchanged, then the inequality (44) can be represented as

$$\Delta P_{c*} + \Delta P_{saf*} < \Delta P_{add*}, \quad (45)$$

where ΔP_{c*} is the power of losses, required to maintain the voltage on the capacitor of the DC link of the PAF above peak value of the network voltage; ΔP_{saf*} is the losses power of the power compensator.

Define the maximum possible effect of increasing efficiency by adopting the ideal compensator and useful power unchanged before and after the connection of the PAF. Fig. 2 shows a Matlab-model of the equivalent circuit of a three-phase ESS with PAF, with characteristics corresponding to the circuit of Fig. 1. The model consists of a power circuit, voltage and current sensors, measurement subsystem, subsystem of the ESS mode setting, the subsystem of total losses power components calculation, and virtual instrumentation. The Matlab-model permits investigating the operation of the three-phase ESS in 96 indicated variants in which additional losses can arise. For the modelling a three-phase four-wire ESS with symmetrical three-phase voltage source at $R_n = R_s$ was selected. The parameters of the model's elements: k_{sc} = 5 ÷ 30; U_m = 311.13 V; f_s = = 50 Hz; P_{usf} = const = 400.1 kW.

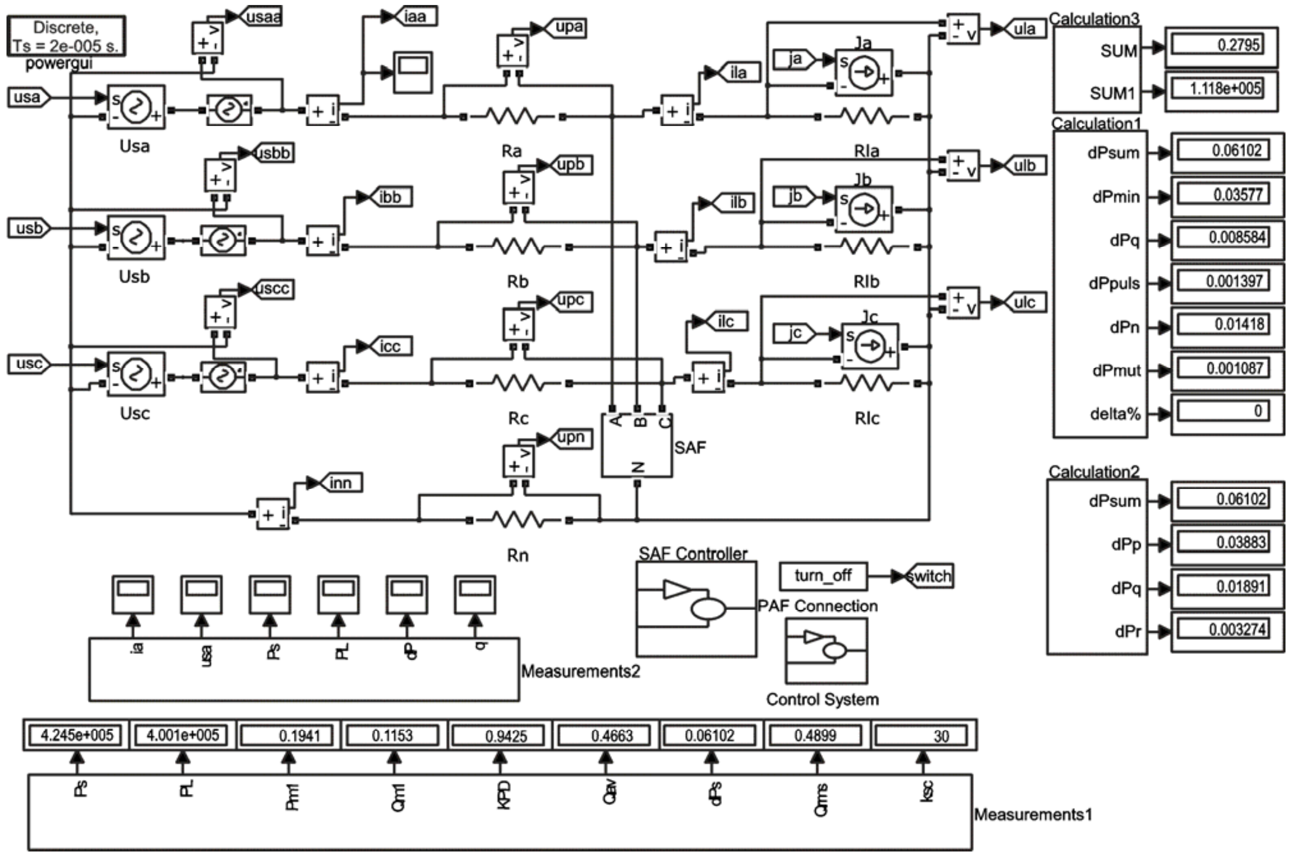

Fig. 2. A Matlab-model of the equivalent circuit f the three-phase ESS with PAF

As an example we consider three separate factors of the arising additional losses in the ESS:

1. Symmetrical active-reactive load. We accept $\varphi = 20°$.

2. Asymmetrical resistive load. We accept the active resistances of three phases of the load

$$\begin{aligned} R_{La} &= k_{la} \cdot R_L, \\ R_{Lb} &= k_{lb} \cdot R_L, \\ R_{Lc} &= k_{lc} \cdot R_L, \\ k_{La}^2 &+ k_{Lb}^2 + k_{Lc}^2 = 3, \end{aligned} \tag{46}$$

$k_{la} = 1$, $k_{lb} = 1.3$, $k_{lc} = 0.5568$.

3. Symmetrical nonlinear load. We accept the current of the phase A

$$i_a = \sum_{n=2k\mp1} i_k = \sum_{n=2k\mp1} \frac{U_m}{n \cdot (R_s + R_L)} \cdot \sin(n \cdot \vartheta), \quad k = 1, 2, 3...18. \tag{47}$$

Six modes of the ESS were taken to summarize the simulation results corresponding to the combination of three of indicated factors:

Mode 1 – symmetrical active-reactive load.

Mode 2 – asymmetrical resistive load.

Mode 3 – symmetrical nonlinear load.

Mode 4 – asymmetrical active-reactive load.

Mode 5 – symmetrical mixed (active-inductive and nonlinear) load.

Mode 6 – asymmetrical nonlinear load.

Using this model, the total losses power components by the universal formula (13) and in the pqr coordinates (29) were calculated. And the results of the calculation for the six modes adopted, in percentage terms, are summarized in Table 1.

The Table 1 shows that in these modes the greatest contribution to the total losses power two components make: component of the additional losses power due to the instantaneous reactive power, and the components of additional losses power due to current flow in the neutral wire.

Fig. 3 shows a reserve for increasing the efficiency for the considered six modes of operation of the ESS: a large area of the zone of the increase of efficiency, bathed in a dark color in the Figure, corresponds to more favorable technical and economic conditions when using the PAF. The economic feasibility of the PAF use increases for the ESS, where several factors leading to additional electric power losses can simultaneously take place. As an example of such ESS municipal networks at the level of individual consumers or consumer groups can serve.

Conclusions.

1. A technique of representation of components of total losses power in three-phase ESS based on the use of the pqr theory of instantaneous active and reactive power is substantiated. According to the proposed technique the total losses power can be represented as the sum of three components ΔP_{p*}, ΔP_{q*}, ΔP_{r*} defined by the projections of generalized spatial vectors of current and voltage on the axis of the pqr coordinate system.

2. Using spatial coordinate transformations of p-q-r coordinates of the theory of instantaneous active and reactive power, an exact relation (34) is obtained, taking into account the four components of the total losses power: power of the lowest possible losses; additional losses power due to the instantaneous reactive power; additional losses power due to fluctuations of the instantaneous active power; additional losses power due to the current flowing in the neutral wire.

3. Comparison of the exact calculation relation (34) with the previously proposed universal formula (15) made it possible to determine the fifth component of the additional losses power due to the mutual influence of electromagnetic processes in the lines of the three-phase ESS and the neutral wire.

4. The method of calculating the additional components of the total losses power is determined. The method based on the use of measuring information about the values of instantaneous current and voltage in the ESS with PAF. Using this method will allow developing a measuring device that registers the components of the losses power at the current time, the scope of which may be associated with the development of mode control algorithms of the ESS with minimal losses of electric energy.

5. The method of determining the reserve of the increasing the efficiency of the ESS at using PAF to substantiate the economic efficiency of its installation is proposed.

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Table 1

Results of determination of components of the total losses power

k_{sc}	$\Delta P_{\Sigma *}$	Components of the total losses power by the universal formula (13), %					In the coordinates pqr, %		
		$\Delta P_{\min *}/\ \Delta P_{\Sigma *}$	$\Delta P_{q*}/\ \Delta P_{\Sigma *}$	$\Delta P_{\mathrm{puls}*}/\ \Delta P_{\Sigma *}$	$\Delta P_{n*}/\ \Delta P_{\Sigma *}$	$\Delta P_{mut*}/\ \Delta P_{\Sigma *}$	$\Delta P_{p*}/\ \Delta P_{\Sigma *}$	$\Delta P_{q*}/\ \Delta P_{\Sigma *}$	$\Delta P_{r*}/\ \Delta P_{\Sigma *}$
Mode 1									
5	0.4792	79.71	16.60	0	0	3.72	91.32	0	8.69
10	0.1514	83.89	15.79	0	0	0.00	87.58	0	12.43
15	0.09199	84.12	15.75	0	0	0.00	86.43	0	13.57
20	0.0662	84.18	15.74	0	0	0.00	85.89	0	14.12
25	0.05172	84.22	15.72	0	0	0.00	85.56	0	14.44
30	0.04245	84.24	15.71	0	0	0.00	85.35	0	14.65
Mode 2									
5	0.4465	85.55	4.32	2.64	2.64	4.87	95.10	3.52	1.38
10	0.1559	81.47	5.89	2.67	7.58	2.41	87.81	10.10	2.10
15	0.09752	79.35	6.44	2.65	9.81	1.73	84.64	13.07	2.28
20	0.07126	78.20	6.74	2.63	11.06	1.35	82.89	14.75	2.36
25	0.05621	77.50	6.92	2.62	11.85	1.11	81.64	15.79	2.40
30	0.04644	77.00	7.04	2.61	12.39	0.94	81.05	16.52	2.43
Mode 3									
5	0.5046	75.70	10.29	0.15	4.81	9.06	89.79	6.41	3.79
10	0.1801	70.53	12.12	0.14	13.04	4.19	77.40	17.39	5.19
15	0.1141	67.82	12.62	0.13	16.52	2.94	72.65	22.02	5.37
20	0.084	66.34	12.86	0.12	18.39	2.27	70.07	24.51	5.40
25	0.06655	65.46	12.99	0.12	19.58	1.84	68.50	26.07	5.41
30	0.05514	64.85	13.10	0.12	20.37	1.55	67.43	27.20	5.41
Mode 4									
5	0.6077	62.85	23.33	1.87	1.74	10.21	86.05	2.32	11.64
10	0.1902	66.78	22.13	2.05	6.47	2.55	76.08	8.63	15.27
15	0.1185	65.30	22.26	2.00	8.67	1.71	72.11	11.55	16.30
20	0.08644	64.47	22.32	1.97	9.92	1.31	70.06	13.22	16.72
25	0.06816	63.91	22.36	1.95	10.72	1.07	68.76	14.29	16.96
30	0.0563	63.52	22.38	1.93	11.27	0.89	67.87	15.03	17.10
Mode 5									
5	0.5046	75.70	9.45	1.05	4.81	9.00	90.25	6.41	3.35
10	0.1801	70.53	11.35	0.96	13.04	4.14	78.01	17.39	4.59
15	0.1141	67.82	11.89	0.91	16.52	2.89	73.27	22.02	4.74
20	0.084	66.34	12.15	0.88	18.39	2.22	70.70	24.51	4.77
25	0.06655	65.46	12.31	0.86	19.58	1.79	69.12	26.09	4.78
30	0.05514	64.85	12.42	0.84	20.37	1.50	68.04	27.20	4.78
Mode 6									
5	0.5663	67.45	11.07	3.53	5.72	12.24	88.49	7.62	3.89
10	0.2	63.51	13.10	2.87	15.42	5.09	74.25	20.56	5.17
15	0.1266	61.12	13.61	2.58	19.23	3.47	69.07	25.63	5.32
20	0.0931	59.86	13.85	2.44	21.21	2.64	66.36	28.27	5.35
25	0.0737	59.11	13.98	2.35	22.43	2.13	64.74	29.88	5.36
30	0.06102	58.60	14.07	2.29	23.24	1.78	63.63	30.99	5.37

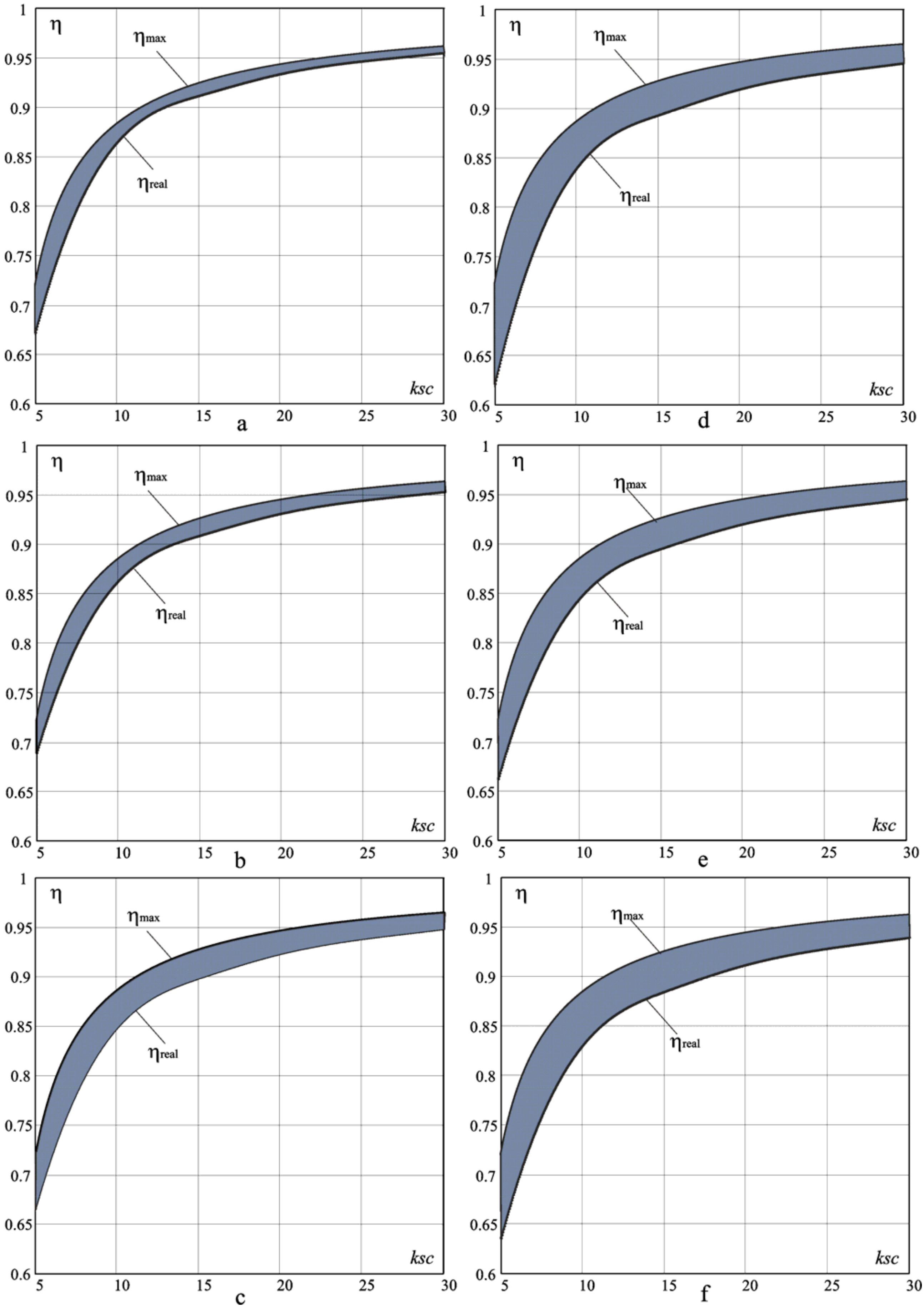

Fig. 3. Reserve of the increase of the efficiency of the three-phase four-ware ESS:
a – in the mode 1; *b* – in the mode 2; *c* – in the mode 3; *d* – in the mode 4; *e* – in the mode 5; *f* – in the mode 6

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